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Temperature gradient driven Alfvén instability producing inward energy flux in stellarators

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Abstract

Destabilizing influence of plasma inhomogeneity on Alfvén eigenmodes in stellarators is considered. It is found that the diamagnetic frequency can strongly increase in a plasma with inhomogeneous temperature due to the resonance interaction of particles and Alfvén eigenmodes. This occurs when the particle resonance velocity exceeds the thermal velocity, in which case the role of superthermal particles enlarges. Then Alfvén eigenmodes can be destabilized even in the absence of the energetic ion population. It is shown that in the case of the temperature distribution with large gradient at the periphery, the destabilized mode can channel the energy from the peripheral plasma region to the inner region. A stability analysis with using a model temperature profile was carried out for the Wendelstein 7-X stellarator. It indicates that the considered mechanism could lead to an Alfvén instability accompanied with the inward energy flux in the first W7-X experiments where long-lasting high-frequency oscillations were observed.

Keywords: Stellarator, Alfvén eigenmodes, destabilization, energy flux, inward spatial channelling
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1. Introduction

The energy and momentum transfer across the magnetic field realized by the destabilized Alfvénic Eigenmodes (the phenomenon named the Spatial Channelling, SC) can be an important factor affecting the plasma performance [1]. Presumably, it was responsible for the degradation of plasma heating by neutral beam injection (NBI) in the NSTX spherical torus, by delivering a part of the NBI power from the plasma core to the periphery (an alternative explanation of the experiment was that the destabilized Alfvén eigenmodes strongly deteriorated the electron energy confinement [2]). On the other hand, if the SC were directed inwards, it would have a positive effect on the plasma energy balance. One can suppose that this took place in the first experiments on the Wendelstein 7-X stellarator: First, the ion temperature profile, $T_i(r)$, was rather flat in the plasma core but steep at the periphery [3, 4, 5]; second, a long-lasting high frequency oscillations were observed, which were identified as an Ellipticity induced Alfvén Eigenmode (EAE) located in the region $r/a \gtrsim 0.5$, with a the plasma radius [6]. These facts suggest that the inhomogeneity of the ion temperature at the periphery drove an Alfvén instability which adjusted the ion temperature profile in a way that a destabilizing effect of the peripheral region was compensated by the damping in the region where $T_i(r) \approx$ const.

The purposes of this work are to see whether Alfvén eigenmodes can indeed be destabilized in a plasma with Maxwellian velocity distribution (usually Alfvén instabilities are driven by fast ions produced by NBI or other sources of plasma heating) and whether instabilities caused by the temperature gradient can lead to the inward SC. In addition to a general analysis, a specific example relevant to the W7-X experiment described in reference [6] will be considered.

We will employ the fact that the so-called non-axisymmetric resonances arising because of the lack of axial symmetry in stellarators can lead to the interaction of Alfvén modes with the ions having energies $\mathcal{E} \gtrsim T_i$ [7].

2. Enhanced destabilization of Alfvén modes by the temperature gradient

In the absence of the energetic ions, the growth / damping rate of Alfvén instabilities can be described by

$$\gamma = \frac{1}{2\mathcal{W}} \mathrm{Re} \int d^3x \tilde{j}_\perp^{kin} \cdot \nabla_\perp \tilde{\Phi}, \tag{1}$$

where $\gamma = \sum_{\sigma=e,i} \gamma_\sigma$, $\tilde{j}_\perp^{kin} = \sum_{\sigma=e,i} \tilde{j}_{\sigma,\perp}^{kin}$ is the transverse current, $\mathcal{W} = \int d^3x c^2 (\nabla_\perp \tilde{\Phi})^2/(4\pi v_A^2)$ is the mode energy, Φ is a scalar potential of the electromagnetic field, tilde labels perturbed quantities, v_A is Alfvén velocity, the subscripts e and i label electrons and ions, respectively. When the transverse current is associated with the particle drift in the equilibrium magnetic field and effects of the trapped

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particles are negligible, calculations lead to [7, 8]

$$\frac{\gamma^{(\sigma)}}{\omega} = \frac{\sqrt{\pi}M_\sigma}{8\delta_0 M_i}\left[\sum_{mn\mu\nu}\int_0^a drrn_\sigma(r) \times \epsilon^{-2}\left|\mu\epsilon_{\mu\nu}\Phi'_{mn} - m\epsilon'_{\mu\nu}\Phi_{mn}\right|^2 Q(u_\sigma)\bar{k}_{res}^{-2}\right] \times \left[\sum_{mn}\int_0^a drr^{-1}n_i(r)\left(r^2|\Phi'_{mn}|^2 + m^2|\Phi_{mn}|^2\right)\right]^{-1}, \quad (2)$$

where Φ_{mn} and $\epsilon_{\mu\nu}$ are Fourier harmonics defined by $\tilde{\Phi} = \sum_{m,n}\Phi_{m,n}(r)\exp(im\vartheta - in\varphi - i\omega t)$ and $B = \bar{B}[1 + 0.5\sum_{\mu,\nu}\epsilon_{\mu\nu}\exp(i\mu\vartheta - i\nu N\varphi)]$, $\bar{B}$ is the average magnetic field at the magnetic axis, the radial coordinate r is defined by $\psi = \bar{B}r^2/2$, ψ is the toroidal magnetic flux, ϑ and φ are the poloidal and toroidal Boozer angles, respectively, N is the number of periods of the equilibrium magnetic field, $\epsilon = r/R$, n_σ is the particle density, $\bar{k}_{res} \equiv k_{res}R = (m+\mu)\iota - (n+\nu N)$, ι is the rotational transform of the field lines, R is the major radius of the torus, prime denotes the radial derivative, $\delta_0 \gtrsim 1$ is determined by the plasma shaping [9], $Q(u)$ is defined by

$$\int d^3v(v_\parallel^2 + 0.5v_\perp^2)^2\delta(\omega - k_{res}v_\parallel)\hat{\Pi}F_\sigma = \frac{\sqrt{\pi}n_\sigma\omega}{k_{res}^2}Q(u_\sigma), \quad (3)$$

where F_σ is the particle distribution function, $u \equiv |v_\parallel^{res}|/v_T$, $v_\parallel^{res} = \omega/k_{res}$, $v_T = \sqrt{2T/M}$ is the particle thermal velocity, M is the particle mass, $\hat{\Pi}$ in the case of a plasma with isotropic velocity distribution is

$$\hat{\Pi} = \frac{1}{v}\frac{\partial}{\partial v} + \left(\frac{\omega R}{v_\parallel} + n\right)\frac{1}{\iota\omega\omega_B}\frac{1}{r}\frac{\partial}{\partial r}. \quad (4)$$

Due to (4), we can write $Q = Q_v + Q_r$, where Q_v and Q_r are associated with the first term and second term in the RHS of equation (4), respectively.

It is clear that $Q_v < 0$ and, hence, $\gamma < 0$ for a plasma with Maxwellian velocity distribution, F_M. Therefore, an instability arises when $Q_r > |Q_v|$. The magnitude of Q_r essentially depends on the resonance velocity. Because

$$\left.\frac{\partial F_M}{\partial r}\right|_{v_\parallel = v_\parallel^{res}} = \frac{n_\sigma}{\pi^{3/2}v_T^3}e^{-u_\sigma^2 - \mathcal{E}_\perp/T_\sigma} \times \left[\frac{n'_\sigma}{n_\sigma} + \left(u_\sigma^2 + \frac{\mathcal{E}_\perp}{T_\sigma} - \frac{3}{2}\right)\frac{T'_\sigma}{T_\sigma}\right], \quad (5)$$

the ratio of the ∇T_σ term to the ∇n_σ term grows when u_σ increases, at least for $u^2 \geq 3/2$. Therefore, one can expect that the condition $Q_r > |Q_v|$ is satisfied most easily at large u. For this reason, we assume that $u > 1$. Then the integral over transverse velocities in (3) can be taken in the limits $(0, \infty)$ because the region of trapped particles lies at $\mathcal{E}_\perp \gg T_\sigma$ when $u > 1$. As a result, we have:

$$Q_v(u) = -\frac{1}{u}(2u^4 + 2u^2 + 1)e^{-u^2}, \quad (6)$$

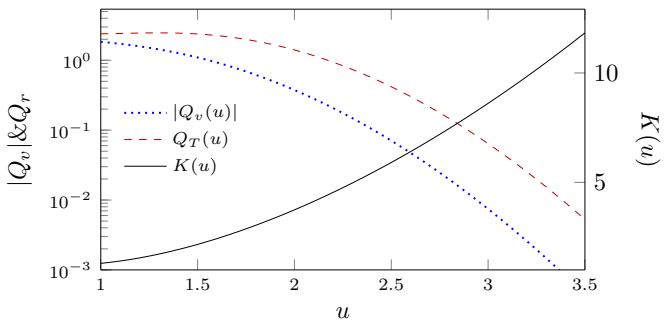

Figure 1: Functions $Q_v(u)$, $Q_T(u)$, and the enhancement factor $\mathcal{K}$ versus $u \equiv |v_\parallel^{res}|/v_T$.

$$Q_r(u) = -\frac{\rho_\sigma v_{T\sigma}}{2\iota\omega r}[(m+\mu)\iota - \nu N]\left[Q_v\frac{n'_\sigma}{n_\sigma} + Q_T\frac{T'_\sigma}{T_\sigma}\right], \quad (7)$$

where

$$Q_T(u) = \frac{1}{u}(2u^6 + u^4 + 2u^2 + 1.5)e^{-u^2}. \quad (8)$$

We assume that $Q_r > 0$ (otherwise plasma is stable), which imposes a restriction on the mode numbers, the resonance numbers (μ, ν), and radial derivatives of the plasma temperature and density. Then the ratio of the driving term to the stabilizing one is negative and given by

$$\frac{Q_r}{Q_v} = -\frac{\rho_\sigma v_{T\sigma}}{2\iota\omega r}[(m+\mu)\iota - \nu N]\left(\frac{n'_\sigma}{n_\sigma} + \mathcal{K}\frac{T'_\sigma}{T_\sigma}\right), \quad (9)$$

where $\mathcal{K} \equiv Q_T/|Q_v|$ is the "enhancement factor",

$$\mathcal{K} = \frac{2u^6 + u^4 + 2u^2 + 1.5}{2u^4 + 2u^2 + 1}. \quad (10)$$

For instance, $\mathcal{K}(2) = 3.44$, $\mathcal{K}(3) = 8.61$. On the other hand, the function $Q_T(u)$ is decreasing rather weakly in the range $1 < u < 3$, see figure 1. Therefore, this range of the resonance velocities may play the main role in instabilities.

Equations (6)-(10) remain valid during the instability provided that Coulomb collisions are strong enough to sustain Maxwellian velocity distribution.

3. Inward spatial channelling and its consequences

Let us assume that $\mathcal{K} \gg 1$ and the temperature gradient term in the ratio Q_r/Q_v dominates. Then $\mathcal{K} \approx u^2 \propto 1/T_\sigma$ (for $k_{res} \approx$ const within the mode width when the magnetic shear is not large), which leads to $Q_r/Q_v \propto T'_\sigma/(rT_\sigma)$. It follows from here that the instability condition $Q_r/|Q_v| > 1$, with $Q_r > 0$, is most easily satisfied at the plasma periphery where the temperature is low. For instance, when $T_\sigma = T_0(1-r^2)^\tau$, $Q_r/Q_v \propto (1 - r^2/a^2)^{-1}$. It follows from here that it may happen that $Q_r/|Q_v| < 1$ in the region $r_1 < r < r_0$, but $Q_r/|Q_v| > 1$ in the region $r_0 < r < r_2$, where r_0 is defined

by equation $Q_r/|Q_v| = 1$, $r_1 < r < r_2$ is the region where the mode is located. The instability growth rate then is $\gamma = \gamma_+ - |\gamma_-|$, where $\gamma_+ > 0$ and $\gamma_- < 0$ are the drive and damping determined by equation (2) with the integral in nominator taken in the limits $(0, r_0)$ and (r_0, a), respectively. If a steady state takes place, $\gamma_+ = |\gamma_-|$, so that $\gamma = 0$.

In the considered case, the mode will transfer the energy from the peripheral region to the central region, i.e., the inward spatial channelling of the plasma energy will be realized within the mode width, which improves the plasma energy confinement.

The presence of additional damping mechanisms may lead to absorption of the mode energy beyond the region $r_1 < r < r_0$, decreasing the energy flux.

The inward SC is significant if the power density received by the mode in the region $r_0 < r < r_2$ (P_+) and the plasma heating power by the mode in the region $r_1 < r < r_0$ (P_-) are sufficiently large to produce a considerable the inward energy flux. The power density $P_\pm$ can be evaluated as follows:

$$P_\pm = 2\omega \frac{\gamma_\pm}{\omega} \frac{B^2}{4\pi} \left(\frac{\tilde{B}}{B} \right)^2 . \qquad (11)$$

In the steady state, the inward power flux across the r_0 radius is $P_+ V_+$, where V_+ is the volume of the region giving the energy to the mode. Assuming, for instance, that $r_0/a = 0.6$ and $r_2/a = 0.7$ and taking the plasma volume $V_p = 30$ m^3 we obtain $V_+ = 0.13 V_p = 3.9$ m^3. The mode amplitude is not determined by our theory. We evaluate it in the assumption that the inward energy flux equals to the outward neoclassical ion thermal flux at $r/a \sim 0.6$ (~ 0.05 MW), calculated for a W7-X plasma with the ECRH power $\mathcal{P} = 2$ MW in reference [4]. Taking $\gamma_+/\omega = 10^{-3}$, $B = 2.3$ T, $f = 200$ kHz (this frequency lies in the range of the EAE gap in the Alfvén continuum), we obtain $\tilde{B}/B \sim 10^{-3}$. This may be unrealistically large and, therefore, the energy flux is expected to be smaller than we assumed. The best way to make a reliable conclusion is to determine the mode amplitude and its radial structure experimentally.

4. Estimates for a W7-X experiment

The instability we are interested in started at $t_1 \sim 100$ ms when it had the frequency $f \sim 220$ kHz, see figure 11 in [6]. The frequency was decreasing (which indicated on Alfvénic nature of the instability, because the plasma density was growing) whereas the mode amplitude was growing till $t_2 \sim 200$ ms. After that a quasi-steady-state wave activity with the frequency around 180 kHz and the poloidal mode number $|m| = 16$ lasted for 600 ms (the plasma density was slowly growing). This suggests that the mode amplitude grew before the moment t_2 due to $\gamma_+ > |\gamma_-|$ and, after it became large enough, a steady state energy transfer from the region of strong temperature gradient to the region located at smaller radii might take place, with $\gamma_+ \approx |\gamma_-|$.

Below we consider the steady state phase of the instability.

Let us assume that the mode location includes partly the region with big T_i' (around $r/a \sim 0.6$) and that the mode interacts with particles through the $\mu = \nu = \pm 1$ resonance (the ϵ_{11} harmonic is rather large in W7-X). Then we can take $\iota = 0.8 - 0.82$, $T_i = 1 - 1.5$ keV, $|m| = 16$, $n = 12$, $f = 180$ kHz. The resonance velocity associated with the helical harmonic is $v_\parallel = \omega/k_{res}$, where $k_{res} = [(m+\mu)\iota - (n+\nu N)]/R$, $\mu = \nu = \pm 1$. We obtain that $|\bar{k}_{res}| = 5 - 5.3$ and $3.4 - 3.06$, which leads to $|v_\parallel^{res}| = 1.24 \times 10^8$ cm s^{-1} and 1.82×10^8 cm s^{-1}. Hence, $2.3 \le u_i \le 4.6$, and the enhancement factor for ions well exceeds unity, being in the range $5 < \mathcal{K} < 20$ [equation (10) was used]. On the other hand, u_e is much less than v_{Te} and, therefore, $Q_r(u_e)$ can hardly exceed $|Q_v(u_e)|$. For this reason, and because the fraction of resonant passing particles is small for $u \ll 1$, the interaction of the mode and electrons can be neglected (at least, in our formalism).

It can be shown by using equation (9) that due to large $\mathcal{K}$ a necessary condition of the instability, $Q_r > |Q_v|$ in the region of large T' can be satisfied. However, it does not mean that the drive exceeds the damping in the region of the mode location, leading to instability.

In order to see whether $\gamma \ge 0$ we have to calculate integrals in equation (2). Therefore, it is necessary to specify the mode structure and radial profiles of plasma parameters.

Following reference [6], we assume that the mode is an EAE (located at $0.5 \lesssim r/a \lesssim 0.7$, the mode numbers $|m_1| = 16$, $|m_2| = 14$, and $|n| = 12$), $n_i(r) = n_e(r)$ and take $n_e(r)$ according to figure 10 of [6]. In addition, because the exact profile of the ion temperature is not available, we restrict ourselves to the following model: $T_i(r) = const = T_i(r_1)$ for $0 \le r \le r_1$; $T(r) = T(r_1)[1 - (r - r_1)/(r_2 - r_1)]$ for $r_1 \le r \le r_2$; $T(r) = T(r_2)[1 - (r - r_2)/(a - r_2)]$ for $r_2 \le r \le a$. This profile is in a qualitative agreement with the one in a W7-X discharge with ECRH power $\mathcal{P} = 2$ MW, shown in figure 6 of reference [3]; it has a strong gradient of $T_i(r)$ at $r/a \sim 0.6$, but data near the plasma edge were not available. A similar profile of $T_i(r)$, but with much larger drop of the ion temperature in the region $r/a = 0.6 - 0.8$ (by ~ 2.5 keV) was predicted numerically for a discharge with $\mathcal{P} = 4$ MW, see figure 4 in reference [10]. The experiment described in [6] was carried out at the intermediate power, $\mathcal{P} = 2$ MW. Therefore, it is reasonable to assume that $T_i(r)$ profile was of the same character as those in the discharges mentioned above.

Calculations were carried out for two cases: for $T_i(0) > 2$ keV and $T_i(0) < 2$ keV.

The results of calculations with $T_i(0) > 2$ keV of the "partial" growth / damping rate γ_r (defined by $\gamma =$

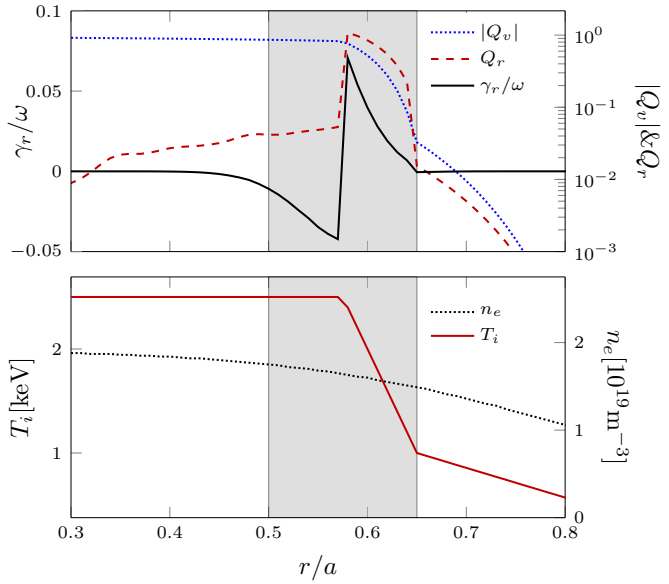

Figure 2: Radial dependence of the plasma and wave characteristics for which the plasma is close to the margin of stability ($\gamma_+/\omega = 1.40 \times 10^{-3}$, $\gamma_-/\omega = -1.27 \times 10^{-3}$), although the instability drive and the damping are rather strong within the mode width. Lower panel: the temperature and density of the ions adopted for calculations; the region of location of the EAE mode with $m_1 = -16$, $m_2 = -14$, and $n = -12$ [6] (shaded area). Upper panel: the r-dependent growth / damping rate γ_r is defined by $\gamma = a^{-2}\int_0^a drr\gamma_r$ (solid lines), $|Q_v|$ and Q_r (dashed lines).

$a^{-2}\int_0^a drr\gamma_r$), Q_r and $|Q_v|$ for selected u (for which these magnitudes are largest) are presented in figure 2. We observe that, as expected, the region where drive / damping dominates is located at larger / smaller radii. This means that the inward SC takes place. The system is close to the margin of stability, $\gamma_+ \approx |\gamma_-|$, with $(\gamma_+ - |\gamma_-|)/\gamma_+ = 9\%$. Hence, the mode could be in a steady state, which was the case in experiment. Slightly varying the ion temperature gradient region we obtain either a quite large growth rate or damping rate of the mode. It looks that the temperature gradient was self-regulated under the influence of the instability in the experiment. The mechanism of this is simple: because γ_r is largest near the border between the regions with $\gamma_r > 0$ and $\gamma_r < 0$, it is sufficient to reduce T' only in this region to make $\gamma \approx 0$. This could take place after the initial stage of the instability when the mode amplitude became sufficiently large.

Calculations with $T_i(0) < 2$ keV were carried out in the assumption that $T_i(0) = 1.8$ keV. It was found that the picture shown in figure 2 persists, but γ_+ and γ_- decrease. Note that the shape of $T_i(r)$ in the region $r/a > 0.65$ almost does not affect the results of calculations because the mode amplitude is negligible in this region.

A question may arise, why other Alfvén modes were not destabilized in W7-X? The answer for the Helicity induced eigenmodes (HAE) and Mirror induced eigenmodes (MAE) is clear: because their frequencies well exceed the EAE frequency, the inequality $Q_r > |Q_v|$ cannot be satisfied. On the other hand, frequencies of the Toroidicity induced Eigenmodes (TAE) are lower then the EAE frequency by a factor of two and, for the first sight, TAEs should be destabilized. However, $u_i \sim 1$ for TAEs. Hence, $\mathcal{K}^{TAE} \sim 1.3$, which is considerably less then the enhancement factor for the EAE mode. As a result, the ratio $Q_r/|Q_v|$ is slightly less then unity. This means that the damping of TAE modes exceeds their drive. Of course, this does not mean that a TAE instability driven by the temperature gradient is not possible. In particular, destabilizing effect of the ion temperature inhomogeneity on a TAE in a high-β plasma was observed numerically in reference [11].

5. Summary

In summary, we found that the destabilizing influence of the spatial inhomogeneity of the bulk plasma with Maxwellian velocity distribution on Alfvén eigenmodes in toroidal systems can overcome Landau damping. If this is the case, Alfvén instabilities arise provided that other damping mechanisms are weak. A necessary condition of instabilities $[Q_r(r) > |Q_v|(r)]$ is obtained. The ratio $Q_r/|Q_v|$ grows when $u \equiv |v_r|/v_T$ increases (in the region $u > 1$). The reason is that the superthermal particles considerably contribute to the diamagnetic frequency (ω_*) due to a strong dependence of the coefficient at T' in ω_* on u. We refer to this coefficient given by equation (10) as the "enhancement factor" in the temperature gradient driven instability. However, when $u \gg 1$, the growth rate is exponentially small, so the case of practical interest is u in the range $u = 2-3$ for which $3.4 < \mathcal{K} < 8.6$. These resonance velocities can take place due to non-axisymmetric resonances in stellarators.

An important feature of the T'-driven instabilities is that they can lead to the inward SC, i.e., to the inward energy flux, when there are regions with $Q_r(r) > |Q_v|(r)$ and $Q_r(r) < |Q_v|(r)$ within the mode width, but $\gamma > 0$.

Note that the inward SC can also provide the transfer of the energy of fusion produced alpha particles by means of fast magnetoacoustic waves with frequencies above the alpha gyrofrequency [12].

It seems that an EAE or another Alfvén mode with $\omega \sim 200$ kHz can be destabilized due to the considered mechanism, leading to the inward SC of the ion energy, in Wendelstein 7-X due to the presence of the helical Fourier harmonic with $\mu = \nu = 1$ in the equilibrium magnetic field. This could explain long-lasting high frequency oscillations observed experimentally, which are described in reference [6]. The considered instability may affect the plasma performance in W7-X and deserves further experimental and theoretical studies.

In conclusion, we note that the destabilized mode may lead to anomalous thermal conductivity, which tends to decrease the temperature gradient. When studying this effect one has to take into account that the presence of energy sources and sinks within the mode width influences

the profile shape of eigenmodes [12]. This effect is significant for instabilities with sufficiently large drive (γ_+) and damping rate (γ_-).

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References

[1] Ya.I. Kolesnichenko, Yu.V. Yakovenko, V.V. Lutsenko, Phys. Rev. Lett. **104** (2010) 075001.

[2] N.N. Gorelenkov et al., Nucl. Fusion **50** (2010) 084012.

[3] T. Klinger et al., Plasma Phys. Control. Fusion 59 (2017) 014018.

[4] M. Hirsch et al., Nucl. Fusion **57** (2017) 086010.

[5] R.C. Wolf et al., Nucl. Fusion **57** (2017) 102020.

[6] T. Windisch, A. Krämer-Flecken, J.L. Velasco, A. Könies, C. Nührenberg, O. Grulke, T. Klinger, and W7-X team, Plasma Phy. Contr. Fusion, (2018) accepted for publication.

[7] Ya.I. Kolesnichenko, V. V. Lutsenko, H. Wobig, Yu.V. Yakovenko, Phys. Plasmas **9** (2002) 517.

[8] Ya.I. Kolesnichenko, A. Könies, A.V. Tykhyy, Plasma Phys. Contr. Fusion, to be submitted.

[9] Ya.I. Kolesnichenko, V.V. Lutsenko, H. Wobig, Yu.V. Yakovenko, O.P. Fesenyuk, Phys. Plasmas **8** (2001) 491.

[10] V. Erckmann et al., Fusion Science and Technology **52** (2007) 311.

[11] A. Könies, A. Mishchenko and R. Hatzky, 2008 Theory of Fusion Plasmas vol 1069, ed. X. Garbet et al (New York: AIP) p. 133 (2008).

[12] Ya.I. Kolesnichenko, V.V. Lutsenko, M.H. Tyshchenko, H. Weisen, Yu.V. Yakovenko, and JET Contributors, "Analysis of possible improvement of the plasma performance in JET due to the inward spatial channelling of fast-ion energy", submitted to Nucl. Fusion.