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Tokamak systems analyses using a fast 1D transport solver: models and assessments

Jose Carlos Rivas[a], Emiliano Fable[b], Richard Kemp[a], Hanni Lux[a], Ronald Wenninger[b,c]

[a]*CCFE/UKAEA, Culham Science Centre, Abingdon, Oxon OX14 3DB, UK*
[b]*Max-Planck-Institute for Plasma Physics, D-85748 Garching, Germany*
[c]*EUROfusion Programme Management Unit, Garching, Germany*

During the pre-conceptual design stage of nuclear fusion power plants, systems codes provide rapid evaluation of feasible design points and consequences of major design choices. They do this by optimizing simple models of all relevant power plant subsystems within physical laws and engineering limits. Typically these models use 0D scaling relations to describe the interactions between the physics and engineering design parameters. This can lead to oversimplifications due to the high non-linearity of the underlying physics models involved.

In this work, we describe a novel approach that integrates a fast 1D transport and equilibrium solver in the PROCESS systems code. Validation of both the standalone version as well as the version integrated into PROCESS against simulations run with the ASTRA transport code show good agreement. The comparison between the previous 0D version of the PROCESS code and the new 1D version show logical differences due to the different models. In the near future, more detailed investigations of the effects of this model on the European demonstration power plant (DEMO) design are planned.

Keywords: Systems Studies, Transport Model, Fusion Power Plant

1. Introduction

Systems codes are used in providing an evaluation of feasible design points and consequences of major design choices. They aim to describe all relevant parts of a fusion power plant and their relations using simplified models. This allows a faster run time, while being able to model the interactions of the power plant subsystems with enough accuracy to assure an overall consistent result.

To find a suitable design point, a constrained optimisation of the power plant parameters is carried out that assures the stakeholder requirements are fulfilled, as well as the engineering limits and physical laws.

This makes them suitable tools for initial design evaluations during the pre-conceptual design phase. These initial results allow to select promising design points and are then followed up by more detailed 3D physics and engineering analysis.

There are two main sources for uncertainties in these DEMO design point solutions: 1) in the current pre-conceptual phase, there is a large extrapolation beyond experimental data, 2) many systems codes models are at high risk of oversimplification.

Regarding the first source, uncertainty analysis can be performed assessing their influence on the solution [1]. Regarding the second source, comparisons with experiments or more complex codes can be conducted. In particular in the case of plasma physics models, due to their high nonlinearity and their strong influence on plant design, oversimplification is always a concern.

In this work we use the PROCESS systems code [1] [2]. For plasma physics a 0D balance is solved where a global confinement time result can be obtained from a variety of scalings. The plasma current, fusion power, plasma heating or radiative losses are obtained from global models or doing volume calculations by assuming parabolic radial profiles (with or without pedestal) for plasma density and temperature. This is similar to the approach taken in other systems codes [3] [4].

However, due to the strong influence of the plasma radial profiles in the plasma results and hence in the final solution, it was decided to develop a version of PROCESS code where the plasma physics results are calculated by means of a transport and MHD equilibrium solver. The motivation is to provide intrinsically consistent plasma profiles and confinement parameters derived from fundamental physics, for the purposes of assessing previous results and providing more confidence in future studies.

In this work, we describe the novel approach of implementing a fast 1D transport and equilibrium solver into the PROCESS systems code which is aimed to provide more consistent plasma profiles and confinement parameters derived from fundamental physics. In Section 2, the implemented models are described, in Section 3, the integration within PROCESS code is described, in Section 4 the validation strategy and some results are shown and discussed; finally, in Section 5 the work is summarized and future tasks are outlined.

2. Model description

In this section, we describe our fast 1D transport and equilibrium model and how it is linked to PROCESS.

2.1 Transport solver

A simplified set of transport equations based on those

author's email: jose.rivas@ukaea.uk

solved by ASTRA code [5] for the steady state was set up for implementation first as a standalone code and later in PROCESS code. The transport flux $Q_{tr,j}$ of a quantity y_j, where y_j can be either the electron temperature T_e, the ion temperature T_i, or the electron density n_e is calculated in two ways. First, from a prescribed transport model:

$$Q_{tr,j} = -\chi_j \partial_\rho y_j + V_j y_j \quad (1)$$

where χ_j are the diffusivities and V_j the convective velocity. Second, from the particle and power balance, and because both must be equal in the converged solution:

$$Q^{tr,j} = f(P_e, P_i, S_e) \quad (2)$$

with P_e the electron heat flux, P_i the ion heat flux and S_e the electron flux traversing the generic flux surface. The respective sources and sinks taken into account are the heating from alpha particles, auxiliary heating, radiation losses as well as fueling from pellets and beams. All of them are calculated and applied as radial profiles. The solution will be a set of radial profiles T_e, T_i, n_e such that:

$$Q_{tr,j} = Q^{tr,j} \quad (3)$$

The initial guess as starting point for the numerical scheme solving the transport equations can be either taken from 0D predictions or some arbitrary input. The numerical solver uses a generalized fixed point iteration scheme, with a weighting relaxation constant. That is:

$$\left[\frac{\partial_\rho y_j}{y_j}\right]^{n+1} = c\left[\left(V_j y_j - Q_j\right)/\left(\chi_j y_j\right)\right]^n + (1-c)\left[\frac{\partial_\rho y_j}{y_j}\right]^n \quad (4)$$

where c is set $\ll 1$ to make the scheme absolute convergent even for strong nonlinear transport models. In equations 4 and 5, y_j=1,2,3 corresponds to T_e, T_i, n_e .

Then, the radial profiles of density and temperatures for the next iteration are calculated from the gradients and the pedestal value:

$$y_j(\rho) = y_j^{pedestal} exp\left(\int dx \left[\frac{\partial_\rho y_j}{y_j}\right](x)\right) (5)$$

The boundary condition for the core transport solution is the pedestal top temperatures and density.

2.1.2 Power and particle balance

Fuelling and heating

In the previous version of PROCESS code heating and current drive (HCD) models are either global scalings or calculated over parametric radial profiles for plasma density and temperature. With this new version, fuelling and external heating are radial profiles, but in this stage of development are assumed to follow a gaussian distribution, in order to provide a basic functionality until specific 1D subroutines are implemented. By varying the position and the standard deviation, the effect of these contributions can be modulated.

The power partition between ions and electrons, both for external heating and for alpha heating, are calculated using 0D scalings, again to provide basic functionality until 1D subroutines are developed.

Fusion power is now calculated for any radial position, using the same set of fusion reactors considered for 0D calculations. Also equilibration power and ohmic power are using the same models as in PROCESS 0D, but now applied to radial profiles.

Power losses

Since 2015 PROCESS has a temperature dependent 1D model for the line and bremsstrahlung radiation from a range of seeded impurities and the fuel based on loss functions from the ADAS database [6]. The model is only used for radiation from inside the separatrix which is considered to be in local ionization equilibrium.

For the synchrotron radiation, the Albajar global scaling that is used for the 0D plasma balance calculation is replaced here by a locally applied Trubnikov model (LATF), [6] which allows the calculation of a radial profile but does not take into account the wall reflection effects.

Conductive losses are calculated from the balance equations, considering steady state. These values must fit with the calculation from the transport model, in order to find a solution.

2.1.3 Transport model

The transport model used for the calculations in this work employs a mixed scaling including Bohm and gyro-Bohm models, with a simple radial dependence [7].

2.2 Equilibrium solver

Additionally to the equilibrium of particle and energy fluxes, the equilibrium between magnetic and thermal plasma pressure is calculated to find a solution consistent with the MHD model. The equilibrium is found by solving the Grad-Shafranov equation:

$$\Delta^* \psi = -R j_\phi \quad (7)$$

where the toroidal current density is given as:

$$j_\phi = -R\left(\mu_0 \frac{dP}{d\psi} - \frac{F}{R^2}\frac{dF}{d\psi}\right) \quad (8)$$

$$F = R\, B_T \quad (9)$$

The Grad-Shafranov equation is solved using a 3-moment method using the well-established EMEQ code, described briefly by the ASTRA Manual [5]. The boundary shape is specified via 3 moments (triangularity, elongation, Shafranov shift of the plasma boundary) that are input to the solver. In addition, the pressure and the current profile are also input to the solver.

In finding a valid solution, MHD stability criteria must be met. For the previous 0D version, only global density and beta limits are taken into account. In this version, additionally to the previous, pressure and pressure gradient radial profiles are modified according to a simple MHD stability relation for a cylindrical tokamak

model. Normalized beta is kept under 0.8 of the Troyon limit, whereas line averaged plasma density is allowed to go up to 1.5 the Greenwald limit.

2.3 Pedestal scaling

PROCESS has the option to set the electron pedestal top temperature to a value of a DEMO relevant scaling

$$T_{e,ped} = 2.16\,\delta^{0.82} I_P^{0.26} R^{-0.39} \kappa^{0.5} \beta_N{}^{0.43} a^{0.88} \qquad (5)$$

based on a large number of EPED runs [8]. This scaling assumes the toroidal magnetic field is $B_T = 5.7T$.

The pedestal top density can be configured either as a fraction of the Greenwald density limit $\frac{n_{e,ped}}{n_{GW}}$ or also can be set by the user.

3 Integration in PROCESS code

3.1 Flow diagrams

In the Fig.1, the flow diagrams of PROCESS code and transport solver execution are shown

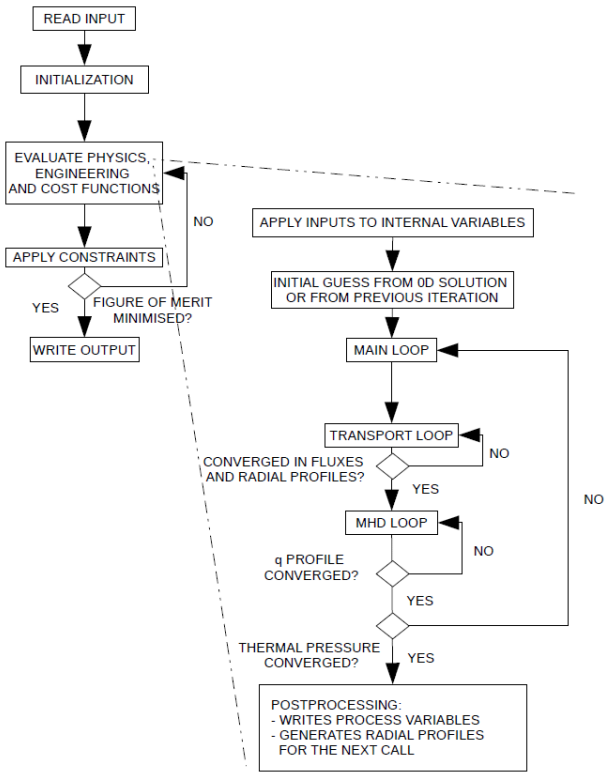

Figure 1: Flow diagrams of PROCESS (top left) and transport solver (bottom right). The dashed lines signal where the transport solver is called inside PROCESS execution flow.

The PROCESS code uses a non-linear constrained optimization solver called VMCON [9] which iterates a set of scalar variables inside pre-defined intervals to optimise a figure of merit within user defined equality and inequality constraints.

The transport solver first calculates an initial guess, usually from the 0D solution of PROCESS, which consists in parabolic radial profiles and their gradients. After that, it executes the main loop, which continues until the solution converges. The main loop contains a transport loop and a loop for the MHD equilibrium.

The transport loop calculates the fluxes from the power and particle balance, and the transport coefficients from the transport model. From them, new gradients are estimated and used to change the gradients vector as described in Eq.3.

With the new gradients, the core part of the radial profiles is updated. After that, the pedestal is updated. The loop operates until the profiles converge.

The MHD loop uses the EMEQ code to solve the Grad-Shafranov equation. As some of the inputs, namely q profile and Ψ profile are also outputs, it must be iterated until convergence is achieved.

Because the transport solver is executed inside a VMCON iteration, it must produce a solution which is compatible with the values of VMCON iteration variables. There are two iteration variables whose consistency is currently enforced: the volume averaged density and the safety factor at the edge.

3.2 Development work and computational needs

The work was developed through the following process:

1. Initial standalone code, developed on the basis of ASTRA models, and validated also with ASTRA results.

2. Initial integration work. The code was integrated inside PROCESS as a subroutine. At this stage, it was treated as a black box with inputs and outputs.

3. Re-engineering of the code, in order to make the models and variables compatible with PROCESS. It was decided to keep the MHD solver unchanged, because it had a record of use in ASTRA code and also some publications documenting the models.

The rest of the transport solver was re-engineered, through a staggered, yet iterative process of refactoring and integration, consisting in:

- Substitution of models and variables in the original code by PROCESS models and variables when possible, to make the solution compatible with the rest of models in PROCESS.

- Configuration analysis, of the interaction between PROCESS and the solver, and between the user and the solver through the PROCESS input file.

4. PROCESS optimizer debugging.

This activity involved checking the evolution of the transport solver solution through the optimization process and its influence in the rest of the code.

After this development cycle, the resulting subroutine integrated reliably inside PROCESS. At present, the solutions of PROCESS with the transport solver are

meaningful, even if it is expected to add improved functionality in some models.

Finally, it is worth to mention the increase in the computational power that the new PROCESS code needs, in contrast with the 0D version of PROCESS, which allowed obtaining results in some seconds with modest computational power.

The overheads are due to using 1D models, and more complex calculations and check ups. Therefore, a batch framework for the execution in parallel of the new version of PROCESS has been set up. This way, several instances of PROCESS can be executed simultaneously, thus speeding up the parametric analyses.

4. Validation results and discussion

The strategy for code validation was to perform separate validations for the transport solver and for the new PROCESS code version. The validation of the transport solver solution was done against an ASTRA code solution. An initial validation of PROCESS code was done against a previous solution with the 0D version using a recent EUROfusion DEMO baseline configuration.

For the transport solver validation, the volume averaged and pedestal top density were fixed to the ASTRA simulation values while the other values were taken from the DEMO baseline configuration.

As shown in Figures 2-4, there is a good agreement in the radial profiles of the bootstrap current, the electron density and temperature between both ASTRA and the PROCESS transport solver. The differences are dominantly related to the use of simpler models in the PROCESS transport solver. In particular for the bootstrap current and the electron density the different transport models used in both cases have a significant influence. Furthermore, the flat part in the temperature profile of the PROCESS solution is due to the use of a simpler model to enforce the MHD stability of the profile (Suydam cylindrical criterion).

There is a good agreement in the radial profiles of the bootstrap current, the electron density and temperature between both ASTRA and the PROCESS transport solver (green and yellow lines in figures 2, 3 and 4). The differences are dominantly related to the use of simpler models in the PROCESS transport solver. In particular for the bootstrap current and the electron density the different transport models used in both cases have a significant influence. Furthermore, the flat part in the temperature profile of the PROCESS solution is due to the use of a simpler model to enforce the MHD stability of the profile (Suydam cylindrical criterion).

The red line in the figures corresponds to the 1D transport run with PROCESS that was conducted to compare with the 0D DEMO baseline configuration. Predominantly due to the variation in transport solver inputs this solution is different from the other two cases. The dependency on the density gradient of the bootstrap current profile can be noted here clearly. This is a relevant difference with the previous 0D version of PROCESS, where the radial profiles of density and temperature are calculated as parabolic profiles depending on one or two parameters plus the pedestal, and are kept fixed during the simulation.

Some values corresponding to both 0D and 1D solutions of the DEMO baseline configuration are shown in the table. The fact that the solution is ignited is due to having slightly higher values of beta and confinement, which in turn can be attributed to the small differences in the radial profiles. These differences account for the difference in the plasma performance in terms of fusion power generation and radiative losses.

Still, these are only initial results intended to test the functionality and the integration of physical and engineering models. It is expected that a cleaner comparison with less differences in e.g. the transport model or the input parameters will yield closer consistency between the results.

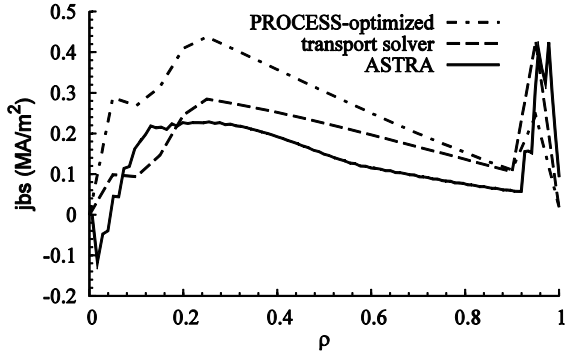

Figure 2: bootstrap current radial profile

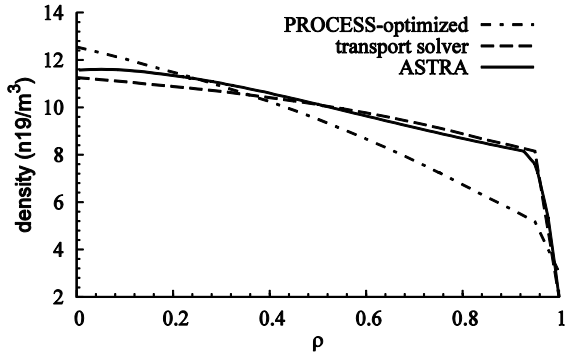

Figure 3: plasma density radial profile

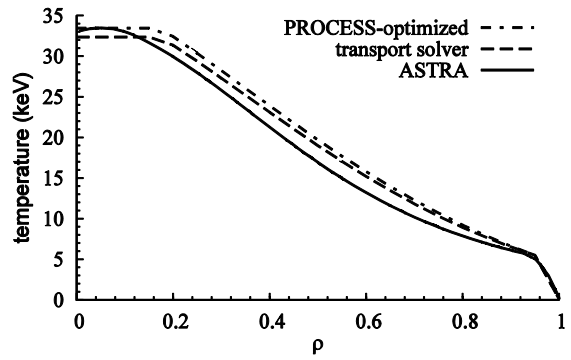

Figure 4: electron temperature radial profile

Table 1: Comparison of PROCESS 1D solution with a DEMO baseline solution from PROCESS 0D

	DEMO baseline	PROCESS 1D results
Plasma Geometry :		
Major Radius R [m]	9.5	9.22
Minor Radius a [m]	3.1	2.97
Aspect ratio A	3.1	3.1
Elongation κ_{95}	1.59	1.59
Triangularity δ_{95}	0.33	0.33
Plasma vol. [m^3]	2500	2202
Current and Field :		
Plasma current [MA]	19.6	16.89
Plasma internal inductance, li	1.11	1.1
Vacuum toroidal field at R(T)	5.7	4.91
Average poloidal field (T)	0.96	0.81
Total field (sqrt($bp^2 + bt^2$)) (T)	5.44	4.98
Safety factor on axis	1	1
Safety factor q_{95}	3.2	3
Beta Information :		
$\beta_{N,th}$ [% m T MA^{-1}]	2.3	2.97
$\beta_{N,tot}$ [% m T MA^{-1}]	2.6	3.3
Limit on thermal beta	5.6	5.07
Temperature and Density:		
Electron temperature (keV)	13.62	13.96
Electron temperature on axis (keV)	28.88	33.45
Electron density (/m3)	$7.78x10^{19}$	$7.71x10^{19}$
Electron density on axis (/m3)	$9.69x10^{19}$	$1.25x10^{20}$
Pedestal top density [m^{-3}]	$6.78x10^{19}$	$5.17x10^{19}$
Pedestal top temperature [keV]	5.5	5.5
Power Balance and confinement:		
Fusion power [MW]	2037	2000
Core radiation power (MW)	118.8	101.1
Edge radiation power (MW)	155.7	114.7
Ohmic heating power (MW)	1.1	0.214
Aux. heating [MW]	50	0.12
Ion electron equilibration power	0	40.71
Power into divertor zone via charged particles (MW)	158.2	165.8
Confinement H factor	1.1	1.41
Global energy confinement time (s)	4.583	4.43
Performance:		
Fusion gain factor Q	39.19	5992
Bootstrap fraction	0.344	0.512
Auxiliary current drive fraction	0.1	0.000035
Inductive fraction	0.556	0.488
Burn time [hrs]	2	2.19
Net electrical output (MW)	500	609.7

5. Summary and future work

This contribution shows the work done in the development of a simple transport and MHD equilibrium solver, its implementation in PROCESS systems code and the validations done.

However, this work is a significant milestone in the development of PROCESS as a systems code with transport modelling capabilities, which has the potential to provide results that are more consistent with the current understanding of fusion plasma behaviour.

An evolution of this model that is being considered to avoid some of the convergence problems of the current implementation is to substitute the transport solver by a set of iteration variables and constraints compatible with VMCON optimizer. This idea is still in an early stage of development, but if it is found to be feasible, it could result in a new approach for transport calculations in a systems code.

In the future, we plan to apply the model to the 2017 baseline design of the European DEMO (G. Federici, this conference) and investigate the effects of a more consistent and more complex physics model on the machine design.

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