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J. Nuehrenberg et al.

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Elimination of current sheets at resonances in three-dimensional toroidal ideal-mhd equilibria

M.I.Mikhailov[1], **J.Nührenberg**[2], **and R.Zille**[3]

[1] National Russian Research Centre "Kurchatov Institute", Moscow, Russia
[2] Max-Planck-Institut für Plasmaphysik, Wendelsteinstr. 1, 17491 Greifswald, Germany
[3] Max-Planck-Institut für Plasmaphysik, Boltzmannstr. 2, 85748 Garching, Germany

E-mail: `Juergen.Nuehrenberg@ipp.mpg.de`

Abstract. On the basis of the ideal-mhd equilibrium code VMEC [1] supplemented by transformation of its results into magnetic coordinates it is shown how current sheets, radially widened by the finite grid size, at rational values of rotational transform in 3d toroidal equilibria can be removed by variation of the equilibrium geometry.

1. Introduction

Recently it has been stated that the 3d equilibrium code VMEC cannot compute current sheets at rational values of rotational transform in 3d toroidal mhd equilibria [2]. On the other hand, the existence of magnetic islands and current sheets in three-dimensional slab-geometry equilibria has been proven [3] and, more recently [4], VMEC has been verified for force-free large-aspect-ratio circular cross-section equlibria against linear ideal-mhd equilibrium theory in the limit of nested surfaces. Here, it will be shown that an evaluation of the equilibrium results of general 3d toroidal equilibria obtained with VMEC can find the signature of current sheets, computationally radially widened by the finite radial grid size, in calculating the equilibrium.

Boozer's coordinates [5] s, θ, ϕ (flux label, poloidal and toroidal coordinate) are used for the evaluation [6] of the equilibrium results. In these coordinates, the covariant component of $\vec{B}$, B_s, has been named $\tilde{\beta}$ because the inhomogeneous part of its equation when formulated as a differential equation along fieldlines

$$\sqrt{g}\vec{B} \cdot \nabla\tilde{\beta} = p'(\sqrt{g} - V')$$

is proportional to the derivative of the pressure with respect to the flux coordinate. Here, g is the Jacobian, $V(s)$ the volume enclosed by the flux surface with label s.

The radial dependency of the homogeneous part of $\tilde{\beta}$ on rational magnetic surfaces, $\iota_{period} = n/m$, is proportional to $\delta(s - s_{res})$ and is related to the force-free current density sheet necessary for rational magnetic surfaces to exist [7]. In order to describe $\tilde{\beta}$ completely, an equivalent expression, calculating it from the geometry directly, is used

$$\tilde{\beta} = (-{F_T}' g_{\phi s} + {F_P}' g_{\theta s})/\sqrt{g}$$

where $g_{\phi s}$ and $g_{\theta s}$ are elements of the Jacobian and F_T and F_P toroidal and poloidal flux, respectively. This expression is computed for various radial grid sizes so that the contribution of the resonance can be extrapolated to infinite radial grid size. Then a suitable boundary coefficient of the equilibrium investigated can be selected to eliminate the resonant contribution.

2. Case studies

Three physically different types of configurations are investigated: a vacuum field, a force-free equilibrium, and a finite-β equilibrium.

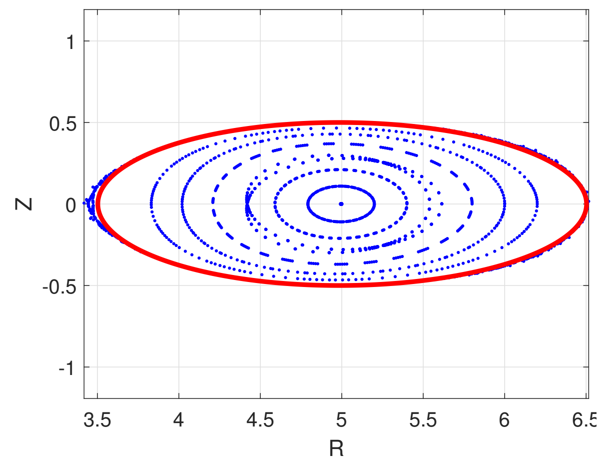

Figure 1. Poincaré plot of a vacuum field; solid line: VMEC.

a) A vacuum field

If a vacuum field with an outer smooth magnetic surface and an inner island chain at a resonance is recomputed as a zero-β equilibrium with nested surfaces, a surface current density appears at the resonance. Here, this is demonstrated starting from a simple l=2 stellarator with 9 periods and rotational transform unity occurring in the confinement region. This configuration is changed into a 3-period configuration by a resonant $m = 3, n = 3$ perturbation so that an $m = 3$ island chain appears at $\iota = 1$. Figure 1 shows a cross-section of the VMEC boundary and the vacuum field solution for this boundary as obtained from an outside surface current density [8].

Figure 2 shows the resonant component of $\tilde{\beta}$ as obtained from VMEC. The δ-function character of the resonant $\tilde{\beta}$ component is apparent and verified by integrating $\tilde{\beta}_{31}$ between its zeroes closest to the resonance. These integrals depend only weakly on the mesh-width and their convergence is linear with decreasing mesh-widths, see Fig.3.

b) Force-free equilibrium

A two-period quasi-axisymmetric configuration [9], a tokamak-stellarator hybrid, is selected because low-order resonances (e.g. ι per period, $\iota_p = 1/3$) occur which should be of particular importance. Its $\beta = 0$ equlibrium is investigated at its $\iota = 2/3$ resonance. Again, the signature of a δ-function behaviour of $\tilde{\beta}_{31}$

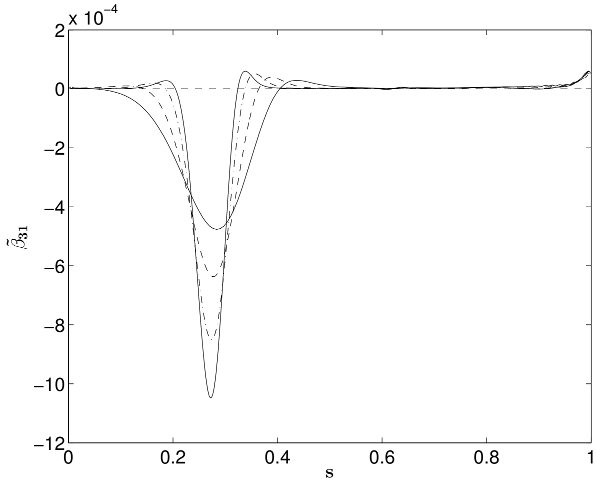

Figure 2. $\tilde{\beta}(m=3, n=1)$ as a function of s for various gridsizes (149, 307, 613, 1223) in VMEC.

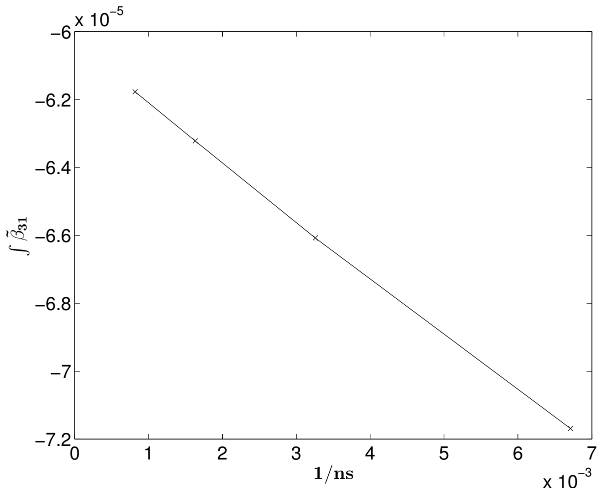

Figure 3. The values of the integrals between the zero values adjacent to the resonance as a function of mesh-widths in the vacuum field case.

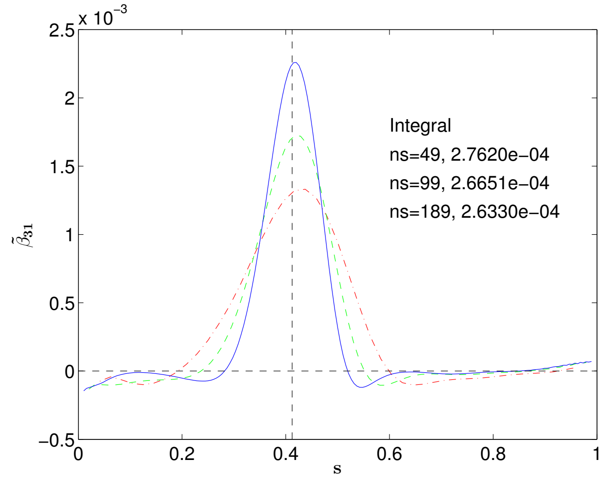

Figure 4. The $m = 3, n = 1$ Fourier component of $\tilde{\beta}$ as a function of the flux label for various grid sizes in the force-free case. The insets show the values of the integrals between the zero values adjacent to the resonance 2/3 at 0.4123.

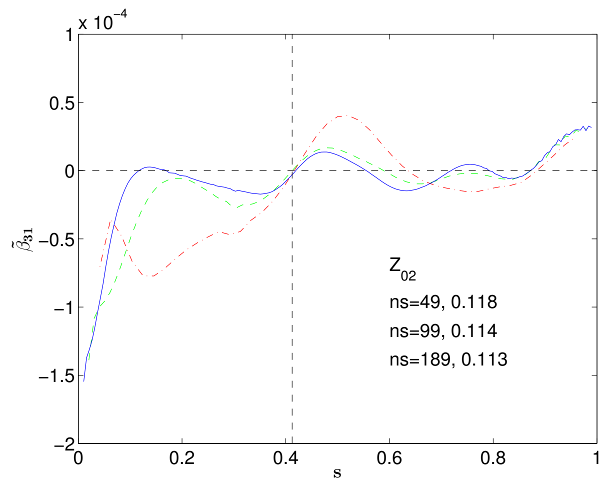

Figure 5. The same as in Fig. 4, now for various grid sizes and associated values of $Z_{m=2,n=0}$ which render $\tilde{\beta}$ a locally odd function at the resonance. Please note the difference in vertical scale between Fig. 4 and here.

smoothed by finite radial grid sizes is clearly seen in Fig. 4.

A search for a parameter of the equilibrium boundary [9] which is effective in reducing the amplitude of $\tilde{\beta}_{31}$ at the resonance yields the boundary coefficient $Z(m = 2, n = 0)$ whose values for eliminating the singularity in $\tilde{\beta}(m = 3, n = 1)$ only weakly depend on radial grid size, see Fig. 5, and turn out to converge quadratically with the grid-point distance to 0.1126.

One expects that the shape of the magnetic surfaces should reflect a resonant current sheet and its elimination. This is seen in Fig. 6 where the two cases are compared with respect to $|\nabla s|^2$: elimination of the current sheet eliminates the resonance signature in the m=3, n=1 Fourier component of $|\nabla s|^2$.

The change of the geometry of the configuration is shown in Fig. 7 where the flux surface cross-section is shown in that symmetry plane in which the smallest deviation from a circular cross occurs: a small reduction in triangularity is seen.

c) Finite-pressure equilibrium

For 3D equilibria obtained under the assumption of flux surfaces at finite β, a stronger divergence than above dominates the behavior of the equilibrium at a resonance. The parallel current density exhibits a $1/(s - s_{res})$ singularity if the pressure gradient does not vanish [6]. MHD-stable equilibria require

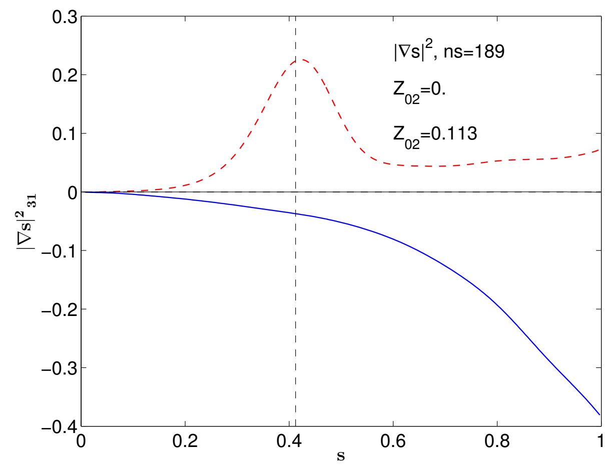

Figure 6. The $m = 3, n = 1$ Fourier component of $|\nabla s|^2$ as a function of the flux label without (dashed) and with (solid) elimination of the resonance in the force-free case.

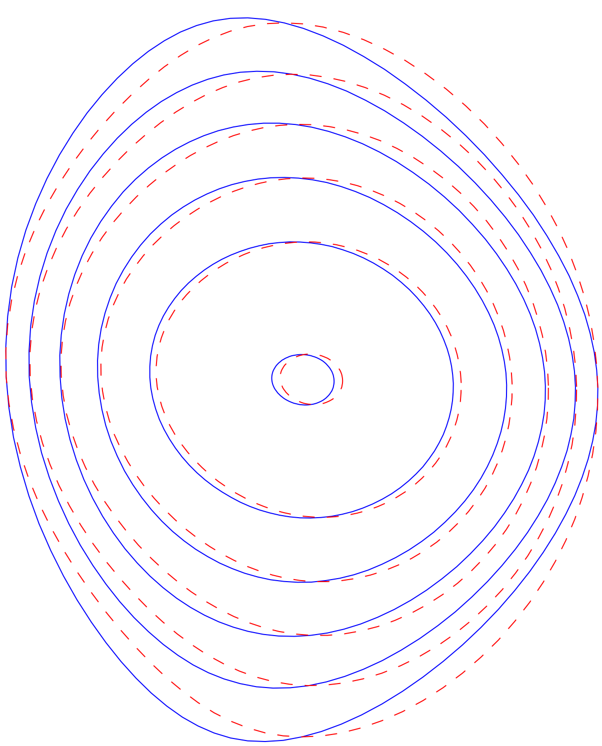

Figure 7. Flux surfaces without (red) and with (blue) the choice of Z(m=2, n=0) which eliminates the surface current density at the resonance

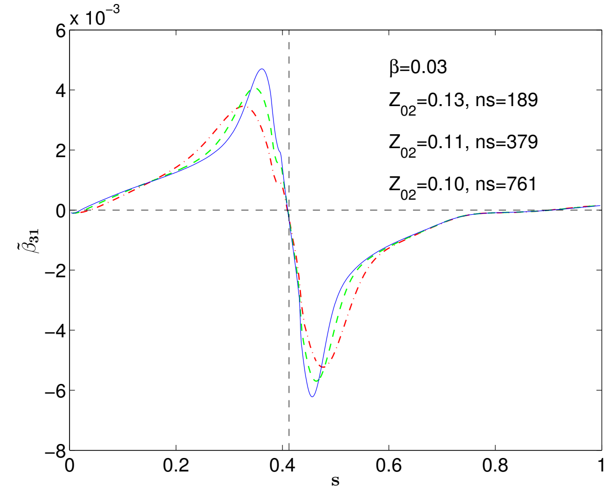

Figure 8. Various grid sizes and associated values of $Z_{m=2,n=0}$ render $\tilde{\beta}$ an odd function at the resonance. Please note the difference in vertical scale between Fig. 5 and here.

a pressure profile flattening at the resonance so that this procedure is routinely applied in stability studies of 3D equilibria. This procedure is applied here, too. The inhomogeneous solution for $\tilde{\beta}$ then exhibits a resonant Fourier component which vanishes in the immediate neighbourhood of the resonance and is approximately odd outside the small pressure-flattened region. Therefore, vanishing of the even part of the solution (the δ-function widened by the finite grid size) is indicated by the local oddness of the resonant Fourier component.

Figure 8 shows results at $< \beta > \approx 0.03$ for the same configuration as considered in case study b. The change of the boundary coefficient Z_{20} needed to obtain the above property is similar to the one for $\beta = 0$ and converges linearly to $Z_{20} = 0.09$.

3. Discussion

The results found above should be checked with 3d equilibrium codes which do not assume nested magnetic surfaces as, by way of example, current versions of PIES [10] and HINT [11] .

Acknowledgments

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