EUROfusion

WPJET1-CPR(18) 18689

R Martone et al.

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Preprint of Paper to be submitted for publication in Proceeding of 2018 International Applied Computational Electromagnetics Society (ACES) Symposium

This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014-2018 under grant agreement No 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

Approaches for Magnetic Sources Reconstruction in Controlled Thermo-Nuclear Fusion Technology

A. G. Chiariello, A. Formisano, R. Martone
Dept. of Ind. and Inform. Engineering
Univ. della Campania "L. Vanvitelli", Aversa (CE), Italy
Raffaele.Martone@unicampania.it

JET Contributors
See the author list of "X. Litaudon et al *2017* Nucl. Fusion *57 102001. EUROfusion Consortium, JET, Culham Science Centre, Abingdon, Oxfordshire, OX14 3DB, UK*

***Abstract*—The estimation of the unknown magnetic sources in Fusion devices by external measurements is a critical task due to its impact on several actions, including the assessment of the magnetic diagnostic system. Suitably treating the modelling of ferromagnetic materials, a linear model can be used also in presence of iron, based on the use of equivalent magnetizing currents. The several available numerical approaches provide quite different solutions while all preserving a good representation capability**

***Keywords*—inverse problems, fusion technology, magnetic sources identification.**

I. Introduction

The magnetic diagnostic is called to play a critical role in all the systems where the control of operation is based on a suitable knowledge of the magnetic field distribution. In these cases, the correct functioning of the system depends on the accuracy and resolution of the measurements. The fusion technology [1], a challenging scientific research for the energy of the future, presents a typical example. As a matter of fact, in the Tokamak [1], the most promising device for the commercial reactors, the feedback plasma shape and position control is based on a magnetic monitoring; in addition, also the counteraction of possible instabilities is performed taking benefit of a suitable magnetic detection of precursors.

In order to guarantee the required standard, the diagnostic system should be frequently assessed, possibly on line, with the aim to detect the failure probes and, possibly, to provide each of them with a suitable confidence qualification. Several assessments procedures have been proposed. A classical and reliable approach is based on the comparison, probe by probe, of the actual measurements (field components, magnetic fluxes linked with a closed line, or related quantities) with the evaluation of the same quantities from the numerical elaboration of the magnetic sources, including free currents and the current possibly modelling the effect of ferromagnetic materials. If the discrepancy between the measured and calculate quantity related to a probe overcame its error bar (depending on the noise and, in addition, on the modelling uncertainties and computation approximation), the probe should be included in the list of suspected to be wrong and, additional investigation to assess its actual performances are recommended. Unfortunately, in several cases, the sources (or part of them) are not known; but when the number of probes are high enough and their distribution is suitable, an estimation of the sources can be effectively performed, taking advantage of the system's own measuring. In the general frame of fusion technology, this is the case of the Tokamak; in fact, the plasma current profile is not known because the position and shape can be estimated only in a rough way by specific diagnostics; a possible solution is based on the equivalent plasma current approach [2]. In addition, in case of iron presence treated by means of magnetizing currents, also the equivalent iron currents can be suitably included in the list of unknown sources.

In any case, the assessment of the diagnostic system is articulated in two stages: (i) a preliminary step aimed to estimate the unknown (free and magnetizing) currents; (ii) the comparison between calculated and measured quantities, aimed to assess the performance of each probe. The first step falls in the class of inverse problems because it should identify the sources of a magnetic field starting from its effect on a sampled domain; its solution can be found in the general approaches based on the minimization of a suitable error function or, in the case of linear behaviour, can benefit from the well assessed approaches of the matrix algebra.

This paper is focused on the first step of the procedure, with the specific application to the diagnostic systems of the JET [1] a Tokamak presently operating in U.K., that includes a ferromagnetic core in the magnetic poloidal system in order to drive the toroidal loop voltage. Several approaches to estimate the unknown are considered and the results are compared in terms of both identified currents and their impact on the reconstructed magnetic field.

II. Mathematical Model

The magnetic field is modelled as the superposition of the effects of the active poloidal currents and the iron equivalent currents [3]; in addition, in the limit of high permeability in the external arms of the iron core, a vanishing tangential component of the magnetic field is supposed to affect the external iron surface (see [4] for more details):

$$\underline{\underline{G}_i}\ \underline{s_i} + \underline{\underline{G}_a}\ \underline{s_a} = \underline{n}\,, \tag{1}$$

where $\underline{s_a}$ and $\underline{s_i}$ represent the active and equivalent magnetizing currents, respectively; $\underline{n} = [\underline{n_m}\ \ \underline{n_c}]^T$ collects the actual $\underline{n_m}$ measurements and the $\underline{n_c}$ constraints on the vanishing tangential field imposed by the iron, respectively; finally, $\underline{\underline{G}_i}$ and $\underline{\underline{G}_a}$ are the matrixes of coefficients. It should be noticed that the $\underline{s_i}$ equivalent sources are not aimed to provide a realistic map of the actual iron magnetization, but just to assess the impact of iron on probes response. Here a vacuum pulse is considered (dry run) and, in addition, the eddy currents are supposed negligible; therefore, just the iron equivalent currents are unknown in a linear algebraic model. The active currents $\underline{s_a}$ are supposed known; therefore the N_i iron currents in $\underline{s_i}$ can estimated by suitably treating the N_n linear equations in (1). Among the different alternatives, here the following approaches [5] are considered and the results compared.

Single Values decomposition (SVD). The "inverse" matrix $\underline{\underline{G_i^\dagger}}$ is assumed in the form $\underline{\underline{G_i^\dagger}} = \sum_{k=1}^{r} \frac{1}{\sigma_k} \mathbf{v}_k \mathbf{u}_k^T$ where σ_k, $\mathbf{v}_k$ and $\mathbf{u}_k$ are singular values, right and left singular vectors, respectively and r is the rank of $\underline{\underline{\mathrm{G}_i}}$. The solution corresponds to the minimization of the quadratic norm of the residuals of the N_n equations (pseudoinversion). The SVD is very effective because provides the contribution of each singular vector; unfortunately it presents a poor regularization of the ill-conditioned problems due to the relevant effect of the small singular value; the most effective measures to mitigate such difficulties are based on the introduction of a suitable weighting function in the inverse matrix $\underline{\underline{G_i^\dagger}} = \sum_{k=1}^{r} \frac{W_\mu(\sigma_k)}{\sigma_k} \mathbf{v}_k \mathbf{u}_k^T$ [4].

Truncated SVD (TSVD). The weighting function $W_\mu(\sigma_k)$ is assumed vanishing when $\sigma_k^2 < \mu$, the unit otherwise, where μ is a suitable threshold. It should be noticed that the effect of the weighting function is to remove a suitable number of singular vectors from the reconstruction process.

Tichonov regularization (TichR). The weighting function is assumed $W_\mu(\sigma_k) = \sigma_k^2/(\sigma_k^2 + \mu)$ where μ is a regularization parameter. It can be shown that this approach minimizes the sum of the quadratic norm of the residuals of the N_n equations, and μ times the quadratic norm of unknown. An optimal value μ_{opt} is suggested by the L curve method, to suitably balance the norms of the solution and the residual error [5].

III. Numerical Results

The procedure has been assessed versus the experimental data of the dry run #86570 at t=52s operated by JET. Here the axi-symmetry assumption is done but the fully 3D extension can be easily provided. The list of probes considered 18x4 pick-up coils, 14x4 saddle loops and 6 flux loops [1]; all the signals have been elaborated according the standard JET procedure and normalized on the number of probes of each class. A set of $\mathrm{N}_i = 62$ unknown equivalent currents have been introduced to model the iron in (1). Apart from the 134 actual measurements, the set of available equations includes also vanishing tangential field constraints in 148 points. More details in [3]. The calculation of the unknown iron currents has been performed with the mentioned approaches, calibrating the parameters when the case. To effectively perform the diagnostic system assessment, the procedure should guarantee a suitable "generalization" capability; this ability has been checked by using just 60 (over 72) pickup coils in (1) for the reconstruction and the remaining 12 for the cross-check. For each approach, the following data have been collected (Table I): the norm of the evaluated sources $\mathrm{S_N}$ and the normalized norm of the residuals on the pickup coils measurements used for the inverse $\mathrm{R_I} = \left\|\underline{n_p} - \underline{n_p^{inv}}\right\| / \left\|\underline{n_p}\right\|$ where $\underline{n_p^{inv}}$ are the measurement estimated by $\mathrm{S_N}$. In addition, to evaluate the "generalization" capability. The same norm has been evaluated for just the cross-check probes confirms a good generalization capability.

TABLE I COMPARISON AMONG THE PERFORMANCE OF INVERSION PROCEDURES

Approach	**Regularization Parameter μ**	**$\mathrm{S_N}$**	**$\mathrm{R_I}$ %**
TichR	$0.1^*\mu_{opt}$	7.98e+06	3.0
	μ_{opt} = 1.1e-10	4.68e+06	3.3
	$10^*\mu_{opt}$	3.92e+06	10
	$15^*\mu_{opt}$	3.40e+06	17
TSVD	14e-10	4.54e+06	7.2
	9.5e-10	4.60e+06	4.5
	4.7e-10	4.62e+06	4.0

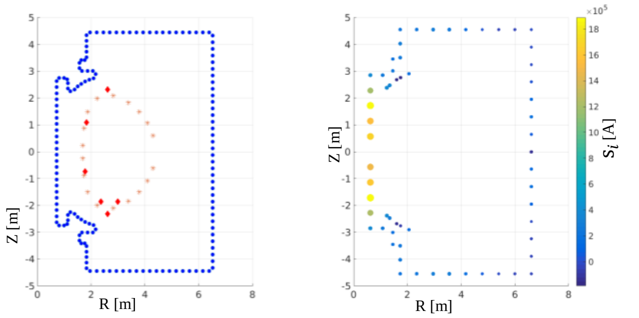

Fig.1. (left) Constraints on poloidal field map. Vanishing tangential field: circles; pickup coils: crosses. Flux loop: diamonds. (right) Evaluated equivalent currents by TichR approach with μ_{opt} = 1.1e-10.

Acknowledgment

This work has been partly supported by ENEA/Euratom/CREATE association. This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014-2018 under grant agreement No 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission. The authors are thankful to the fruitful discussions held with Dr. F. Rimini (EURATOM-CCFE Fusion Association) and Dr. A. Murari (EUROfusion Consortium, JET).

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