WPDC-CPR(17) 17067

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Preprint of Paper to be submitted for publication in Proceeding of
27th IEEE Symposium On Fusion Engineering (SOFE)

This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014-2018 under grant agreement No 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

A Plasma Boundary Reconstruction Method based on Reflectometric Measurements for Control Purposes

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***Abstract*—A purely geometric approach has been investigated to reconstruct the DEMO plasma boundary for control purposes. The whole plasma boundary is reconstructed by using a deformable template approach based on B-splines. The final curve shape is achieved by minimizing the distance between a limited number of estimated and measured (at present provided by an equilibrium code) plasma boundary points along the reflectometer lines of sight. This method is complemented by including the available plasma and poloidal field coil current measurements to refine the boundary reconstruction in the X-point region. The robustness with respect to a random measurements error and to a reduction in the number of measurements is discussed. The main equilibrium and shape geometric quantities (such as plasma cross section area, plasma centre position, elongation, triangularity) were computed and compared to the corresponding quantities of a DEMO reference equilibrium.**

***Index Terms*—Active contour methods, DEMO, plasma boundary reconstruction, plasma reflectometry.**

I. Introduction

IN the roadmap to the achievement of fusion energy the construction of the demonstration fusion power reactor DEMO is envisaged by EUROfusion (the European Consortium for the development of fusion energy) as the step where electricity will be finally supplied to the grid. To implement this strategy a plant conceptual design is under study through a series of work packages in the framework of the Power Plant Physics and Technology Work Programme. In particular, the Work Package on Diagnostics and Control (WPDC) aims at delivering a feasible, integrated conceptual design of the DEMO diagnostics and control systems, capable of meeting the operation and maintenance requirements in the presence of extreme environmental conditions due to the high neutron flux and fluence and the pulse duration. For these reasons the actual availability of in-vessel magnetic sensors is still uncertain and presently under study, so alternative strategies based on in-vessel non-magnetic sensors (like reflectometry antennas) and ex-vessel sensors should be pursued, finding the optimum trade-off between the redundancy in the measurements needed for safe operations and the physical constraints that the machine design will impose.

Preliminary experimental results [1] suggest that the reflectometry system could represent a possible alternative to provide the signals needed to meet the requirements of the plasma position and shape control. A crucial question to be answered is the minimum number of measurements and their poloidal position for a reliable and effective plasma shape control. Another aspect to be considered is the likely lack of reflectometric measurements in some poloidal positions (top and divertor regions) and during transition phases (ramp-up, ramp-down) [2]. A plasma boundary reconstruction method based only on a number of plasma-wall distances and the measures of plasma and poloidal field coil currents is presented in this paper with the aim of contributing to answer these questions.

The paper is organized as follows: in section II the use of active contour methods in fusion research is briefly reminded, the algorithm description and its application to DEMO are given in section III along with the results of some sensitivity analyses to test its robustness. In section IV a solution is proposed to detect the X-point and strike point positions by exploiting the available current measurements. In section V the plasma equilibrium and shape geometric quantities provided by the algorithm and by a reference equilibrium code are compared. Some conclusions and future outlook are finally presented.

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II. Active Contour Methods

Active contour methods used in computer vision have been proposed for the detection of the plasma boundary represented as a continuous curve instead of a set of discrete quantities such as gaps or flux control points [3]. This approach was tested with JET experimental data using the flux map provided by XLOC code with satisfactory results [4]. One of the possible advantages of the continuous curve representation is a more reliable and accurate evaluation of global shape parameters such as plasma section, elongation, triangularity, which are used in confinement scaling relations [5] [6]. An estimate of the plasma boundary curvature could also allow an improved coupling with the RF heating antennas. In a typical application, the plasma boundary is reconstructed as the edge of an object, whose image is represented by the modified flux map $\tilde{\psi} = |\psi(R,Z) - \psi_b|$ where ψ(R, Z) is the flux value at (R, Z) location and ψ_b is the boundary flux. Among the parametric contour models obtained as functional minimizing splines ("snakes"), the B-spline approach is convenient for its compactness and because it implicitly forces the curve smoothness. The basic underlying assumption is that the plasma boundary, determined by MHD equilibrium equations, can be represented by a regular curve. On the other hand, examples of description of the plasma boundary by compact analytic functional forms can be found in the literature [7] [8]. The B-spline is a piecewise polynomial curve and the resulting contour shape can be modified by acting on a finite number of control points related to the curve points through a time invariant matrix. The whole curve is subdivided in sections, each of them mainly, but not exclusively, affected by the movement of a single control point. In previous applications of the method the number of control points was kept close to the number of actuators and a control on the entire plasma shape could be envisaged, in principle, producing a mapping between the actuator currents and the control point positions, at least when a biunique correspondence exists between the actuator current and its position.

III. Application To The DEMO Case

A. *Description of the algorithm*

In DEMO we assumed not to have a flux map available and we tried to reconstruct the plasma boundary by deforming the curve so as to minimize the distance between the "real" boundary positions provided by the reflectometric measures and the currently estimated boundary positions. A cubic B-spline representation of the contour with 8 control points (CP) was chosen (6 associated to the PF coils and 2 to the CS coils). To reduce the dimension of the problem, again taking into account the experience of previous applications, each control point was allowed to slide only along the line joining the machine centre and the coil centre (see Fig. 1). Thus it can be represented by the position vector $\mathbf{P}_{CP}(i)=\mathbf{P0}_{CP}(i)+\alpha(i)\mathbf{t}_{CP}(i)$, where $\mathbf{P0}_{CP}(i)$ indicates the i-th control point location on the First Wall and the vector $\mathbf{t}_{CP}(i)$ components are the direction cosines of the corresponding allowed line of movement. This implies only 8 degrees of freedom, which are the components of the vector $\boldsymbol{\alpha}$. Moreover, a total number of 10 points was used for each section of the curve with a total number of 80 points, which was assessed sufficient for its full description. An initial estimate of the boundary can be obtained either by arbitrarily positioning the control points or from the flux map of an equivalent filamentary current model. In Fig. 1 the initial estimated boundary is shown along with the plasma boundary evaluated by an equilibrium code. The extremes of the allowed stroke of the control points are also represented and correspond to the variation interval $-0.5 \leq \alpha(i) \leq 0.5$. The initial estimate of the boundary of Fig. 1 (green curve) was obtained by arbitrarily positioning the control points at the intermediate position between first wall and coil centre ($\alpha(i)=0.5$).

The antenna positions and the corresponding lines of sight as in the present stage of the reflectometric diagnostics design were considered (Fig. 2). Different sets of antennas were analysed to carry out sensitivity studies and to obtain a first assessment of the minimum number and the best suited locations.

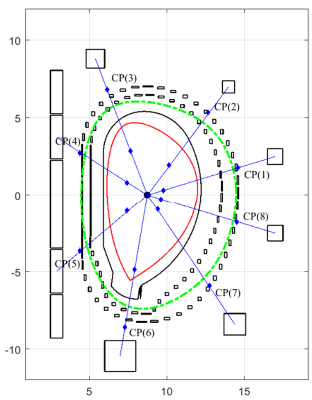

Figure 1. Initially estimated (green) and equilibrium code boundary (red). Extremes of the control point stroke (blue diamonds)

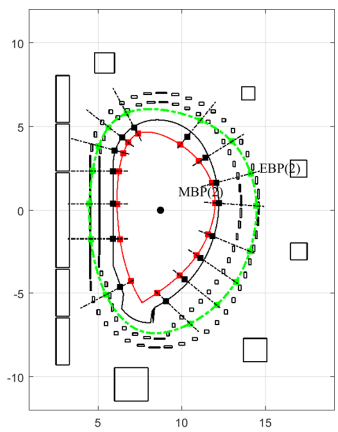

Figure 2. Antenna positions (black squares) and lines of sight (black dashed segments). Measured and initially estimated plasma boundary positions along the lines of sight (red and green squares, respectively).

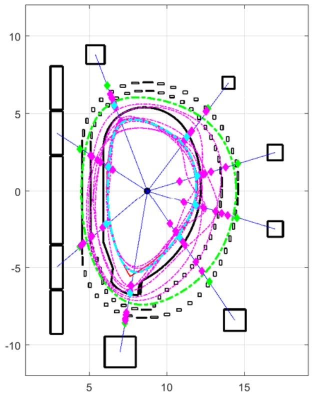

Figure 3. Initial (green), current (magenta) and final (cyan) estimated boundary and control point positions. Equilibrium code boundary (red).

The reconstruction of the boundary is accomplished by performing an iterative procedure, subdivided in a few steps. Before its description, it is worthwhile reminding that the "measured" boundary points, provided by the reflectometric diagnostics, are assumed as exactly belonging to the plasma boundary unless modifications are brought, for example as explicitly declared in the study of noise effect. In this study we could take advantage of the plasma boundaries evaluated in different instants of the pulse by the CREATE-NL equilibrium code [9] and available as EQDSK files in the IDM DEMO data base. The plasma boundary consists of a sufficiently high number of points (526) to allow finding directly the intersection point with the line of sight of each measurement by seeking the point with minimum alignment error. On the contrary, additional points were created by a first order interpolation between the closest couple of points in the case of the estimated boundary. The intersection was then found again by seeking the point with minimum alignment error. The need of performing an interpolation to assure the maximum accuracy in finding the intersection point may suggest reviewing the choice of limiting the total number of points of the estimated boundary to 80. A trade off should be found by comparing the relative computation time between the generation of the curve by the B-spline method and the interpolation procedure to produce the additional points.

Once the (R, Z) coordinates of each couple of measured boundary point (MBP(i)) and estimated boundary point (EBP(i)) along the i-th measurement line of sight are available, the corresponding distance *d(i)* can be calculated. In Fig. 2 the algorithm start condition is shown. If an objective acceptable error $d_o(i)$ is set for each distance, the cost function can be defined as:

$$F(\boldsymbol{d}) = \sqrt{\sum_{i=1}^{N_m}(d(i) - d_o(i))^2} \qquad (1)$$

where $\boldsymbol{d}$=[d(1), d(2),…d(Nm)] is the distance array, N_m is the number of measurements and a constant value (2.e-3 m in the test) is assumed for all $d_o(i)$ (i=1, 2, …N_m). The problem to be solved is the minimization of the cost function F($\boldsymbol{d}$(α)), where α is the above mentioned 8-element parameter vector which rules the stroke of the control points. It was formulated as an unconstrained optimization problem with bounds on the independent variable components and solved by a simulated annealing algorithm. Simulated annealing is a random search method which simulates the cooling process of a physical system towards its thermal equilibrium and it can find a global minimum of the function. To speed up the convergence the parameter vector elements α(i) were bounded between -0.5 and 0.5. First, the full set of N_m=15 available measurements was tested. In Fig. 3 different curves representing the currently estimated boundary produced by the iterative procedure are plotted with a sampling rate of one out of ten, the initial curve and the final one are also included. The agreement is fairly good, even if significant deviation can be noticed in the X-point region, where the regularity of the B-spline does not allow a reliable reconstruction. A number of 150 "temperature" reductions was adopted along with a maximum number of 200 iterations for each "temperature" reduction.

B. Sensitivity analyses

In order to assess the algorithm robustness and to provide a first indication on the minimum number of measurements needed to get a satisfactory boundary reconstruction and to meet the DEMO control requirements, tests have been performed by gradually reducing the subset of measurements. Possibly due to the aleatory behaviour of the minimization algorithm, a subset with only 10 measurements resulted not always adequate to reconstruct the boundary within the desired accuracy. The sensitivity of the reconstruction to the poloidal position of the measurements was also studied. In fact, situations are expected in which not all the measurements will be available. This could be due to an intrinsic difficulty to perform measurements when the reflectometric beam is not exactly perpendicular to the plasma flux surfaces (basically top and divertor regions) and/or to the plasma transition phases [2]. Thus different subsets of measurements (same number but different positions) have been compared. In order to highlight the effect of the lack of one single measurement on the whole reconstruction, two different sets of 10 measurements were chosen.

The scheme with 15 measurements is more robust and, thus, the effect would have been less evident. The algorithm proved to be more robust with respect to the lack of the measurements in the top and divertor region than the ones located around the equatorial plane. This consideration is particularly important, since top and divertor measurements in a real D-shaped Tokamak are the most difficult to be obtained. The reasons behind this result likely pertain to the purely geometric B-spline approach and the way in which it is implemented for plasma boundary reconstruction. However, an extended statistical study is expected to allow a thorough characterization of the algorithm behavior and, possibly, an improvement of its robustness with respect to the lack of one or more equatorial measurements.

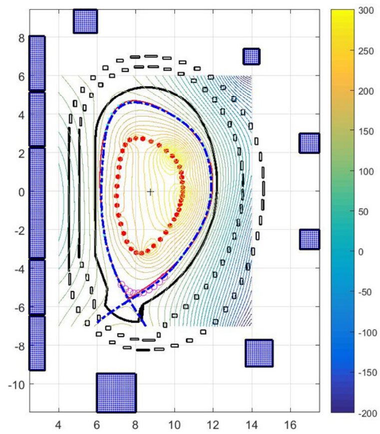

Figure 4. Refined boundary (dashed blue line), plasma equivalent current PEC (red dots), removed points of the B-spline estimated boundary (magenta circles), equilibrium code boundary (red line).

Finally, the role of the measurement error has been investigated. In addition to the angle of the beam with respect to the plasma surface, the measurement accuracy will depend on the plasma shape, on the level of fluctuations and on the electron cyclotron emission (ECE) that can reduce the signal-to-noise ratio. This can be partially counteracted by the use, for instance, of multiple receiving antennae at the same location. Some preliminary results show that the error can be as high as 10 cm for top launchers [2]. For the sake of simplicity (and in a conservative way) in the reconstruction algorithm an up to 10 cm random uncertainty has been "added" to the exact boundary position, "measured" by the reflectometers along their corresponding line of sight. The effect of the error is almost negligible in these cases. A study showed that the algorithm is by far more robust to the presence of a 10 cm error in the measurements than to the above mentioned measurement reductions. Thus 15 reflectometric measurements appear the advisable lower threshold for a reliable and robust plasma shape reconstruction on DEMO based on the proposed algorithm.

IV. Estimate Of The X-Point Position

The regularity properties of the B-splines intrinsically prevent from a correct representation of the boundary in the X-point region. In order to overcome this problem a different approach was followed. The Plasma current was approximated by a set of Equivalent filamentary Currents (PECs) radially distributed from the estimated plasma section center [10]. Similarly, the Poloidal Field Coils (PFCs) were discretized so as to approximate a uniform current density across the section. An inverse problem whose unknowns are the PECs could then be formulated by a two-step procedure. First, the subset of points belonging to the stretches of the curve before and after the lowermost available measure was ruled out (magenta circles in Fig. 4). They correspond to the two most critical intervals, where the largest deviation with respect to the equilibrium code boundary is observed. Then the Green matrices providing the magnetic flux produced by both plasma and PFC unit currents on the remaining N_p boundary points were calculated by numerical integration of filamentary contributions. The N_p-1 flux differences between adjacent points were set at zero, which is equivalent to force that part of the estimated boundary to be a flux surface. Furthermore, the PEC total sum was set to be equal to the plasma current value. A regularization method was necessary to get around the ill-conditioned nature of the problem: either SVD truncation of the plasma matrix or Fourier polynomial representation of PECs yielded equivalent results. Once the PECs were determined, a new flux map could be directly calculated and the new boundary was obtained by sorting out the flux value at the zero gradient point (X-point). A different curve shape and a definitely much better agreement with the equilibrium code boundary was observed as shown in Fig. 4, where the flux map has obviously no physical meaning near and inside the PEC domain, so justifying the fact that PECs are kept far from the actual boundary. On the contrary, the method allows estimating the strike point positions on the divertor plates, as well.

TABLE I

	1	2	3
A (plasma area, m^2)	43.52	43.23 (-0.7)	43.04 (-1.1)
Rpmajor (m)	9.07	9.06 (-0.1)	9.07 (0.)
a (m)	2.93	2.85 (-2.7)	2.84 (-3.1)
Zpmax (m)	4.69	4.63 (-1.3)	4.61 (-1.7)
Zpmin (m)	-5.58	-5.30 (-5.0)	-5.65 (+1.2)
Elongation 1	1.62	1.69 (+4.3)	1.69 (+4.3)
Elongation 2	1.75	1.74 (-0.6)	1.80 (+2.8)
Upper Triangul.	0.41	0.40 (-2.4)	0.38 (-7.3)
Lower Triangul.	0.50	0.32 (-36)	0.50 (0.)
Triangularity	0.45	0.36 (-20)	0.44 (-2.2)

Plasma equilibrium quantities: Reference equilibrium (1); B-spline reconstructed boundary (2); Refined boundary with X-point position estimate (3). Elongation 1=A/(πa^2); Elongation 2=(Zpmax-Zpmin)/(Rpmax-Rpmin); percentage deviations in brackets.

V. Comparison Of Equilibrium Geometric Quantities

In Table I the most significant equilibrium geometric quantities provided by the equilibrium code for the reference case are compared to the same quantities computed with the full set of 15 measurements before and after the refinement in the X-point region. The percentage deviations (between brackets in the table) refer to the reported quantities. It can be appreciated that all the percentage deviations but one are below 5% in the refined boundary case. Even if a limited variation around these values is expected on the basis of the performed tests, statistical analyses will allow to accomplish a comprehensive quantitative assessment of the achievable accuracy.

VI. Conclusion

In order to evaluate the quantities needed for DEMO plasma control independently of the actual availability of in-vessel magnetic sensors, a purely geometric approach has been investigated to reconstruct the plasma boundary by means of the reflectometric measurements. The plasma boundary could be reconstructed by using a deformable template approach based on B-splines and minimizing the distance between a limited number of estimated and measured plasma boundary points along the design lines of sight. The method proved to be more robust with respect to a synthetic random measurement error (≤10cm) than to a reduction in the measurement number and, in this case, to the measurement direction.

The present design number of 15 measurements seems to be adequate for a reliable reconstruction except for the X-point region. A substantial refinement of the boundary reconstruction in this region, including the location of the X-point itself, was achieved by taking advantage of the knowledge of plasma and PFC currents.

The main equilibrium parameters were computed and compared to the corresponding quantities of a DEMO reference equilibrium. A fairly good agreement was observed in the case with full set of measurements and boundary refinement in the X-point region.

Further statistical analyses on the method accuracy and the assessments on the availability of the reflectometric measurements might allow delivering a preliminary requirement in terms of channel redundancy.

ACKNOWLEDGMENT

The authors would like to thank R. Ambrosino and R. Wenninger for providing the DEMO equilibrium data. This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014-2018 under grant agreement No. 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

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