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Turbulence level effects on conventional reflectometry measurements observed with 2D full-wave simulations^{a)}

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Numerical simulations are critical in improving the capabilities of microwave diagnostics. In this work, the 2D finite-difference time-domain full-wave code REFNUM was applied to broadband turbulent plasmas using the conventional reflectometry set-up. Simulations were performed with O-mode waves, fixed frequency probing and I/Q detection. The plasma density, determining O-mode propagation, was modeled as the sum of a slab background plasma with a fluctuating component following a Kolgomorov-like amplitude k -spectrum. The density turbulence level $\delta n_e/n_e$ was scanned over several orders of magnitude for simulated plasma flows of constant plasma velocity in either the radial or the poloidal direction. Simulations show trends, such as spectral broadening of the complex $A(t)e^{i\varphi(t)}$ signals and increasing fluctuations in $A(t)$ and $\varphi(t)$ with increasing $\delta n_e/n_e$, that are similar for both plasma flow directions. These, together with possibilities to reconstruct poloidal wavenumber spectrum are discussed in view to extending the measuring capabilities. The onset of non-linear effects associated with phase runaway, as previously observed with other 1D and 2D codes, as well as radial Doppler effects are also observed and discussed.

I. INTRODUCTION

Interpreting reflectometry measurements and inferring turbulence characteristics from fusion plasmas probed with microwave beams remains a complex problem. Full-wave simulations have been instrumental in understanding reflectometry responses and different plasma-wave interactions, such as Bragg backscattering or forward scattering¹. In this work, the response of ordinary wave polarization (O-mode) reflectometry to analytically prescribed turbulent plasmas in simplified slab geometry is studied by performing 2D simulations in conventional set-up (i.e. perpendicular incidence) for both cases of poloidal and radial plasma flows while scanning the turbulence level. Possibilities for interpretation of reflectometry beyond linear regimes are explored, either through the spectral index of phase spectra or through combined information of phase and amplitude fluctuations.

II. BRAGG BACKSCATTERING

The most efficient scattering process of the probing wave is caused by density fluctuations that meet the Bragg resonant rule²⁻⁴. For conventional reflectometry, Bragg backscattering can occur when the local radial wavenumber of the injected wave $k(r)$ is resonant with the radial

wavenumber of a given Fourier component of the density fluctuations: $k_{\text{rad}} = 2k(r)$. The Bragg relation holds up to the limit wavenumber $k_{\text{rad}} = 2k_o$ set by the vacuum wavenumber of the probing wave k_o , near the plasma entrance. Moreover, only fluctuations with $k_{\text{rad}} > 2k_A$ can satisfy the Bragg condition, where k_A is the Airy wavenumber $k_A = 0.63(k_o^2 L_n^{-1})^{1/3}$ with L_n being the density gradient scale length. While the poloidal characteristics of turbulence should not be responsible for any backscattering, in 2D it is impossible to neglect the radiation pattern of the emitter which contains poloidal wavenumbers and could thus resonate with poloidal fluctuations. In the case of O-mode probing and small coherent perturbations in a linear radial profile, a linear relation between the phase perturbations of the reflectometer signal $\delta\varphi$ and the density perturbation amplitude δn has been shown^{5,6}:

$$\delta\varphi = \sqrt{\pi} k_o \sqrt{\frac{L_n}{k_f}} \frac{\delta n}{n_c} \quad (1)$$

where n_c is the cut-off density and k_f a given wavenumber component of the density fluctuation spectrum. However, additional non-linear contributions to phase variations may occur due to higher orders of Bragg backscattering⁷.

III. TURBULENCE MODEL

For O-mode propagation, the plasma cut-off of a given probing wavenumber k_o depends only on the plasma electron density n_e . Thus, a numerical plasma is sufficiently modeled by the definition of the electron density. In this

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work, the total density given by $n_e(\mathbf{r}, t) = n_{co}(\mathbf{r}, t) + \delta n_e(\mathbf{r}, t)$ accounts for a linear radial electron density profile $n_{co}(\mathbf{r}, t)$ and a broadband density turbulence term $\delta n_e(\mathbf{r}, t)$. While the background profile is simply set through a given density gradient, the turbulence is defined by a sum of modes with random phase^{8,9}:

$$\delta n_e = \sum_{i=i_m}^{i_M} \sum_{j=j_m}^{j_M} A(i, j) \cos[k_x(i)x + k_y(j)y + \varphi(i, j)]. \quad (2)$$

The coefficients $A(i, j)$ allow establishing a Kolmogorov-like amplitude k -spectrum, which is in line with several experimental observations in the edge fusion plasmas, while $\varphi(i, j)$ is a random generated phase that sets a particular turbulence *snapshot* with the given spectral conditions. The 2D k -spectrum is defined by constant amplitude at low k and up to a chosen wavenumber k_{knee} followed by a power law decay $k^{-\alpha}$ along both the x - and y -directions, corresponding to the radial and poloidal directions in a tokamak. The wavenumber *knee* is set to $k_{knee} = 400 \text{ rad/m}$, or $k_{knee} = 4 \text{ cm}^{-1}$ in usual (perhaps misleading) notation. The amplitude rolls off dropping to 1% of the flat top value at $k = 15 \text{ cm}^{-1}$, along with a spectral index $\alpha=3$.

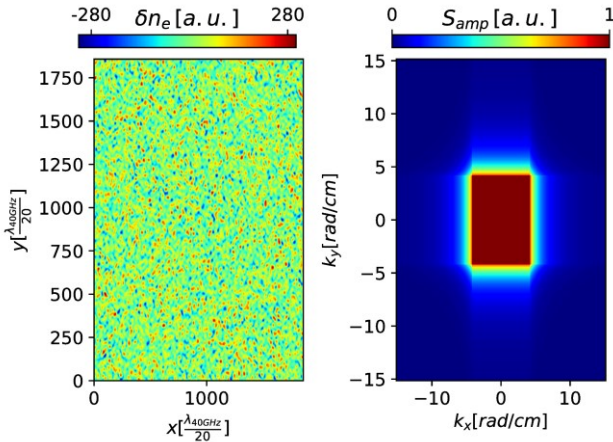


FIG. 1. Example of a 2D density turbulence realization (left) generated with a Kolgomorov-like model of given amplitude k -spectrum (right).

The Root Mean Square value $\delta n_{e,RMS}$ of the density fluctuations generated with the above model, can be easily modified and sets the turbulence level at the radial position r_c of the cut-off density in the slab linear profile:

$$\frac{\delta n_e}{n_e} \equiv \frac{\delta n_{e,RMS}}{n_e(r_c)} \times 100\%. \quad (3)$$

The total density, obtained by adding the linear profile and the turbulent density terms, results in a radial variation of the local turbulence level, which increases towards the periphery of the plasma as the background density decreases (see Fig.2). This radial trend is in general agreement with the experimental observations at the edge and SOL regions

of tokamaks. In this work, two different electron density gradient lengths L_n have been used for background profiles, corresponding to shallow (long $L_n = 0.17 \text{ m}$) and steep (short $L_n = 0.017 \text{ m}$) density profiles, for which the minimum Bragg wavenumbers ($2k_A$) are 1.8 cm^{-1} and 2.6 cm^{-1} , respectively. Simulations were carried out with the total density plasma matrix moving either in the poloidal direction or in the radial direction. In both cases, the Taylor “frozen turbulence” hypothesis is assumed, which considers that advection contribution by turbulent eddies is small and that therefore the advection past a fixed point can be taken to be entirely due to the mean flow. This hypothesis actually holds if the eddy mutation rate is much smaller than the mean plasma velocity.

IV. SIMULATION SET-UP

The full-wave code REFMUL used in this work solves the Maxwell equations for O-mode¹⁰. Both electric and magnetic fields are coupled to the plasma electron density $n_e(\mathbf{r}, t)$ which is used as fusion plasma model, as mentioned in the previous section. Simulations were designed with a monostatic setup, i.e. one antenna for both emission and reception. A unidirectional transparent source was used for injection of the signal allowing separating the emitted probing wave from any returning waves. For directivity a monostatic 2D H-plane horn antenna with a half-power beam width $\leq 6.5 \text{ cm}$ at the plasma entry was employed. Probing waves with two different frequencies $f_p=33 \text{ GHz}$ and $f_p=49 \text{ GHz}$, corresponding to cutoff densities of $n_{c33} = 1.35 \times 10^{19} \text{ m}^{-3}$ and $n_{c49} = 2.99 \times 10^{19} \text{ m}^{-3}$, respectively, were used in separate runs. The synthetic reflectometry phase $\varphi(t)$ and amplitude $A(t)$ signals were obtained performing identical simulation runs with *sin* and *cos* excitation signals allowing in-phase and quadrature detection, which were then used to give the amplitude and phase signals through usual trigonometric combinations. The two slab plasma profiles with the radial cut-off locations of the two probing frequencies that were used are shown in Fig. 2. Also shown are the several turbulence level profiles that were employed in both radial and poloidal plasma flow simulations.

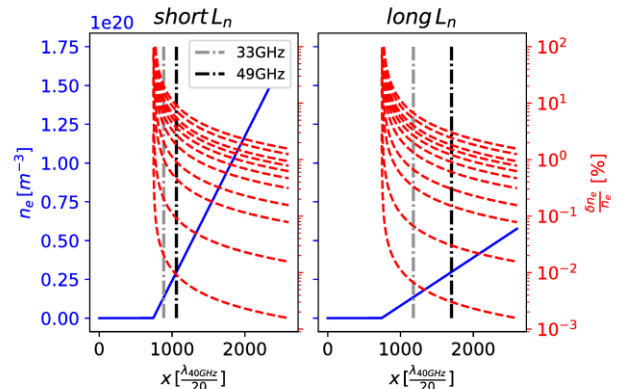


FIG. 2. Linear background density profiles (solid blue) and radial positions of the $f_p=33 \text{ GHz}$ and $f_p=49 \text{ GHz}$ cut-offs, together with the set of density turbulence level profiles (dashed red) employed for the numerical plasmas.

The simulation box comprised a rectangular grid while the plasma itself was quadrangular with side length $L=1859$ grid points, since a radial block of the simulation box was used for the antenna setup and a vacuum distance to the plasma. The spatial grid resolution was $\Delta x = \Delta y = \lambda_{40\text{GHz}}/20 = 3.75 \times 10^{-4}\text{m}$, where $\lambda_{40\text{GHz}}$ is the vacuum wavelength of a $f=40\text{GHz}$ wave frequency. The required time resolution of the simulations ($\Delta t = 6.25 \times 10^{-13}\text{s}$) impose that the duration of any simulated phenomena must be scaled down and any velocities involved scaled up. A rescaling is done such that $v_{\text{siml}} = R_t v_{\text{real}}$ where v_{real} is the characteristic velocity of a given plasma feature and v_{siml} the velocity in the simulation. To avoid relativistic effects, the scaled-up velocities v_{siml} are restricted to values of $v_{\text{siml}} < 0.1c$, where c is the speed of light. In this work, simulations of plasma propagation in both radial and poloidal directions have been performed with 28×10^4 time iterations, corresponding to $v_{\text{siml}} = 0.013c$.

V. SIMULATION RESULTS

A. Poloidal plasma flow

Independent simulations were performed for each given turbulence level, in a range of 0.02%-20%. The power spectra of the reflectometry complex amplitude signal $A(t)e^{i\varphi(t)}$ for the unperturbed profile case and a set of turbulence levels are shown in Fig. 3, for each density gradient length and probing frequency that were studied.

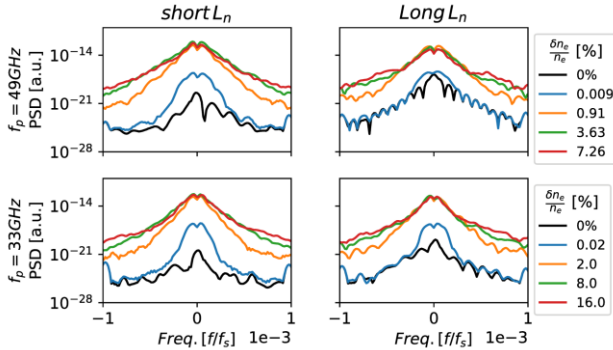


FIG. 3. Power spectra of complex amplitude signals $A(t)e^{i\varphi(t)}$ for turbulence level scans, in two different density gradient lengths L_n and two probing frequencies f_p . Turbulence was generated using a Kolgomorov-like model and set to flow in the poloidal direction.

In general, spectral broadening is observed with increasing turbulence level. The usual increase of central peak amplitudes and integrated spectral powers (see also Fig. 4) with increasing turbulence level are also observed. Similar results have been obtained previously with simple physical optics model⁸. It is known that fluctuations moving in the poloidal direction, acting as a phase grating, may result in frequency shifted sidebands at the receiver antenna. The amplitudes of those sidebands are expected to be equal in conventional reflectometry, at least for geometrically symmetric perturbations and typical propagation velocities. On the other hand, the poloidal propagation of density structures results in radial displacements of the cut-off layer as the 2D structures project different radial profiles along

time in a given perpendicular line of sight. Thus, any radial movements of the reflecting layer should lead to a phase modulation of the reflected signal, resulting in a symmetric broadening of the central line in the frequency spectrum. However, Doppler effects in both directions can take place with different efficiency depending on the Bragg resonant position (less efficiently at the plasma edge and more efficiently in the vicinity of the cut-off) which could shape the spectra. After rescaling the frequency axis, according to the ratio of velocities, assuming $v_{\text{real}}=5\text{km/s}$ and taking into account that the frequency of the density fluctuations is related to the poloidal wavenumber (or equivalently in this case, to the perpendicular wavenumber) $v_{\text{pol}}k_{\text{pol}}=2\pi f$, the wavenumber spectra can be reconstructed. This is shown, for example, in Fig. 4 for one probing frequency and a more comprehensive set of turbulence level simulations.

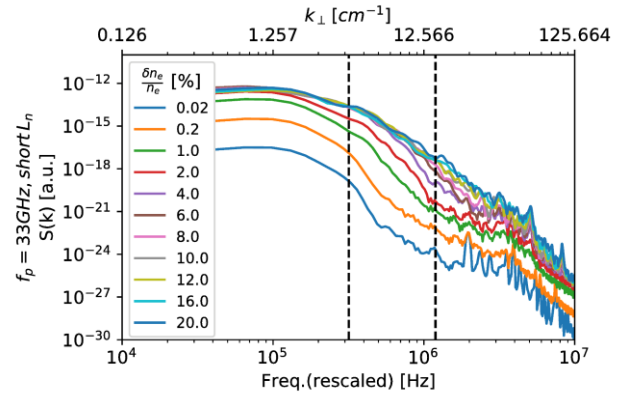


FIG. 4. Power spectra of $A(t)e^{i\varphi(t)}$ signals on re-scaled single-sided frequency axis and corresponding k_{\perp} -axis, for a turbulence scan with probing frequency $f_p=33\text{GHz}$ and short density gradient scale length L_n .

The spectral shape of the input Kolgomorov-like amplitude k -spectrum is fairly well recovered. A flat amplitude region at low- k followed by a strong roll-off above a given k region is obtained – the $k_{\text{knee}} = 4\text{cm}^{-1}$ and $k = 15\text{cm}^{-1}$ (1% of the flat top value) values of the original model are shown in Fig. 4 by vertical dashed lines. Applying the same spectral analysis and re-scaling to the reconstructed reflectometry phase $\varphi(t)$ and amplitude $A(t)$ signals, estimates of the spectral index are obtained by linear fitting the data in the roll-off region, which is shown in Fig. 5. The finite poloidal resolution of the reflectometer, acting as a low-pass filter in k -space, may explain the observed under-estimated values of the spectral index. However, for $\delta n_e/n_e > 2\text{-}4\%$ the phase spectra do allow to recover the spectral index value (-3) of the 2D model, suggesting that the sensitivity to high k fluctuations may increase at higher turbulence levels or that simply 2D effects farther from the cut-off become increasingly more important. On the other hand, the evolution of the integrated spectral power of the $A(t)e^{i\varphi(t)}$ signals with the turbulence level, displayed in Fig. 6, shows a linear relation between these two quantities only up to $\delta n_e/n_e \approx 2\text{-}4\%$ followed by non-linear behavior at higher turbulence levels.

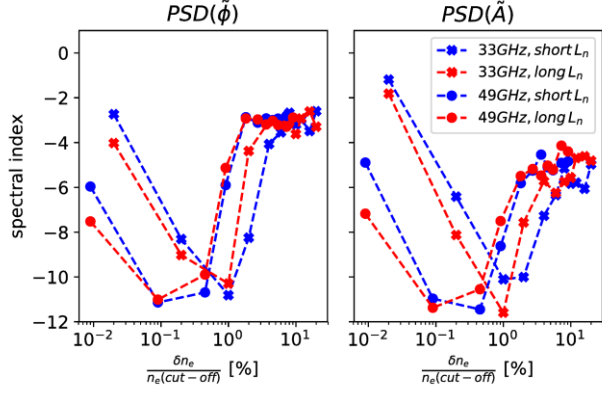


FIG. 5. Spectral index obtained from the fluctuating components of $A(t)$ and $\varphi(t)$ signals in turbulence levels scans with 2 probing frequencies f_p and 2 density gradient scale lengths L_n .

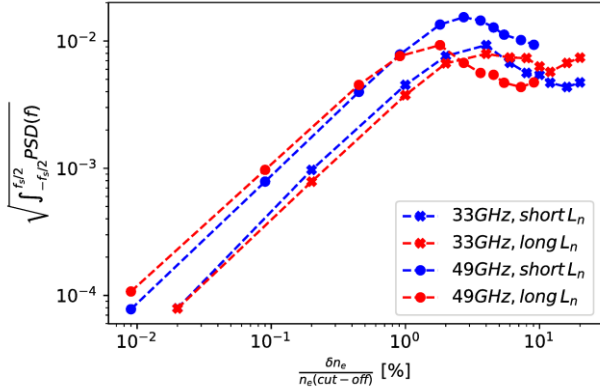


FIG. 6. Root Mean Square (total integrated power spectra) of $A(t)e^{i\varphi(t)}$ signals for turbulence level scans with 2 probing frequencies f_p and 2 density gradient scale lengths L_n .

A similar relation is also observed between the phase fluctuations and the turbulence level, as shown in Fig. 7. In this case, and again only for low turbulence levels, the fluctuating component of the reconstructed phase signals also scales with both k_0 and $\sqrt{L_n}$ as prescribed by Eq. (2). Starting at moderate turbulence levels, the phase time traces (not shown here) display phase drifting and phase runaway effects that become ubiquitous at higher turbulence levels.

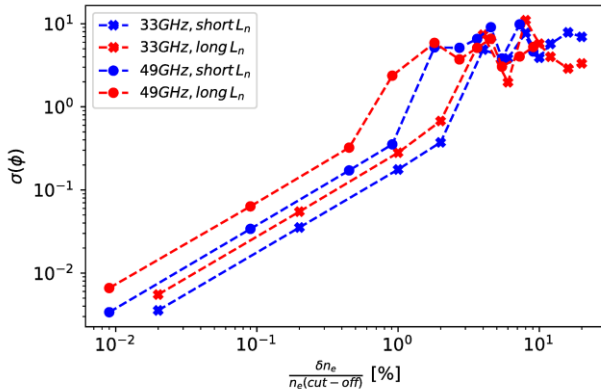


FIG. 7. Standard deviation of reflectometry $\varphi(t)$ signals for turbulence level scans with 2 probing frequencies f_p and 2 density gradient scale lengths L_n .

The evolution of the average and fluctuating components of the reflectometry amplitude $A(t)$ is shown in Fig. 8. The average amplitude of the reflected signal decreases with increasing turbulence level. And the average amplitude is lower, at each turbulence level, in the case of higher L_n . These results must be interpreted in the light of 2D effects since 1D backscattering efficiency is proportional to $\sqrt{L_n}$ however, the beam spreading associated to a longer path into the turbulent plasma should provide a competing geometrical effect¹¹. A rebound in the average amplitude is also observed at the highest turbulence levels. This could be explained by a transition to a secondary cut-off, from the slab plasma to a high amplitude density front. While the low turbulence range of linear reflectometry phase response has been previously predicted with 1D analytical reasoning¹² or 2D physical optics models¹³ these results suggest a possible extension to measuring capabilities, for instance through proper modelling of the amplitude signals.

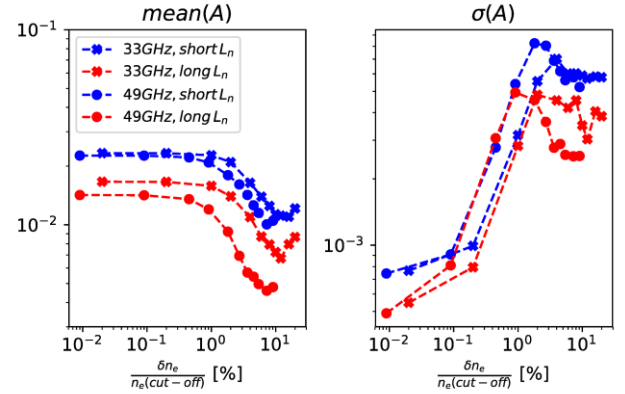


FIG. 8. Average and standard deviation of reflectometry amplitude $A(t)$ signals for turbulence level scans with 2 probing frequencies f_p and 2 density gradient scale lengths L_n .

B. Radial plasma flow

The turbulent numerical plasmas employed in the previous section were also used to run simulations with the plasma flowing in both inward \uparrow and outward \downarrow radial directions. The double-sided Fourier spectra of the complex amplitude signals obtained with the $f_p=33\text{GHz}$ cases are presented in Fig. 9.

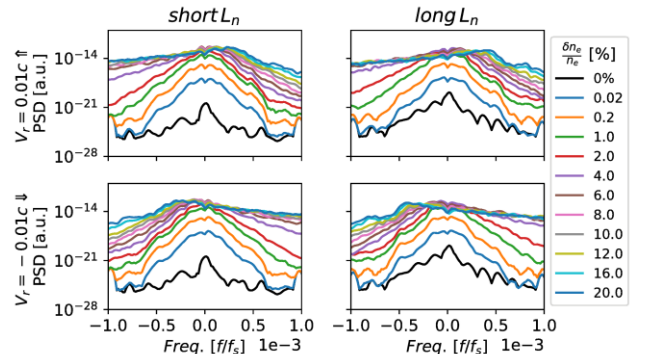


FIG. 9. Power spectra of reflectometry complex amplitude signals $A(t)e^{i\varphi(t)}$ for turbulence level scans in both radial plasma flow directions, with $f_p=33\text{GHz}$ probing frequency, and two different density gradient lengths L_n of background slab plasmas.

The effect of spectral broadening with increasing turbulence levels is observed again. This is more intense than in poloidal flow cases, observed previously in Fig. 3. The spectra become increasingly asymmetric with increasing turbulence levels. The radial flow of turbulent plasmas can induce Doppler effects at both the cut-off and at the Bragg backscattering locations. A single sideband in the frequency spectrum can thus be produced by a density fluctuation with $k_{\text{rad}} = 2k(r)$ propagating radially. This is an additional mechanism leading to phase runaway, which becomes persistent as seen in Fig. 10.

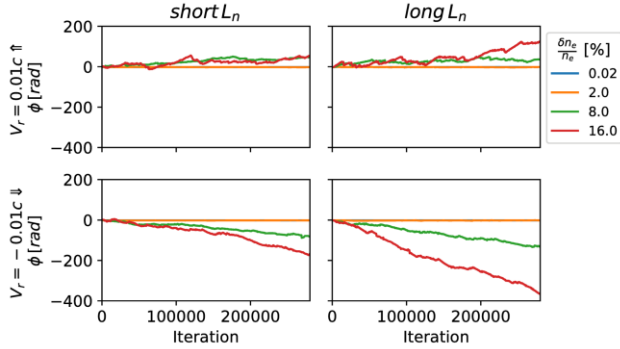


FIG. 10. Time traces of reflectometry $\phi(t)$ signals for turbulence level scans with $f_p=33\text{GHz}$ probing frequency and 2 density gradient scales L_n for both cases of inward \uparrow and outward \downarrow radial plasma flow.

Reflectometry phase changes sign with opposite directions of propagation. Despite the change on phase runaway direction, the phase fluctuations behave similarly for the inward and outward directions at low turbulence levels, becoming somewhat distinct at high $\delta n_e/n_e$. Possibly, trapped waves responding differently when being *pushed* out or into the probing wave could cause this effect. As in the case of poloidal plasma flows, the phase fluctuations scale linearly with the turbulence level for the low $\delta n_e/n_e$ regime, as shown in Fig. 11.

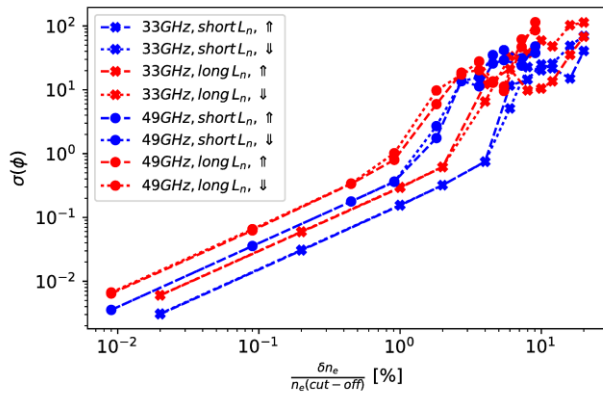


FIG. 11. Standard deviation of reflectometry $\phi(t)$ signals for turbulence level scans with 2 probing frequencies f_p and 2 density gradient scale lengths L_n , for both cases of inward \uparrow and outward \downarrow radial plasma flow.

In fact, standard deviation values $\sigma\phi(t)$ largely exceeding 2π are perhaps a good proxy for the phase drift slope rather than the phase fluctuations. Nevertheless, this quantity seems to be useful in tracking (quasi-linearly) the turbulence level. The reduction in the average amplitude with both increasing

turbulence levels and density gradient length is also observed again, as displayed in Fig. 12 for the case of outward plasma propagation.

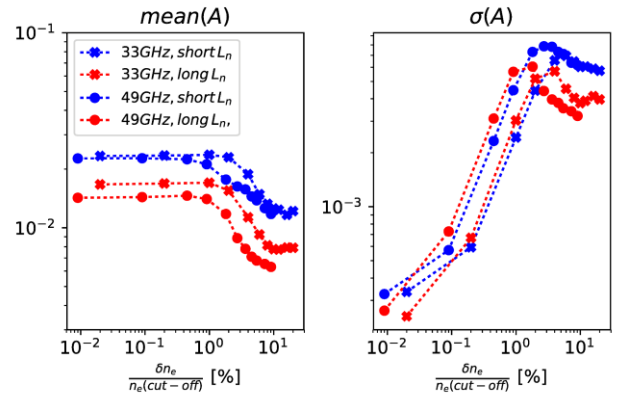


FIG. 12. Average and standard deviation of the reflectometry amplitude $A(t)$ signals for turbulence level scans in the case of outward radial plasma flow for 2 probing frequencies f_p and 2 density gradient scale lengths L_n .

Nevertheless, it is arguable for both $A(t)$ and $\phi(t)$ fluctuations (and in radial and poloidal plasma flow cases for that matter) if non-linear or saturation regimes are reached at high $\delta n_e/n_e$. Further simulations, extending the turbulence level range should clarify this behavior. Also, simulations with plasmas obtained from the output of gyrofluid or gyro-kinetic turbulence codes are envisaged, to have more realistic plasma models, which can then be benchmarked against the results presented here.

VI. ACKNOWLEDGMENTS

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