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Transport theory of phase space zonal structures

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A set of equations that describes particle and energy transport in a thermonuclear plasma on the energy confinement timescale is derived. The equations thus derived allow to study collisional and turbulent transport self-consistently retaining the effect of magnetic field geometry without assuming any separation of scales between fluctuations and reference state. In a previous article [Phys. Plasmas 25, 032306 (2018)], transport equations holding on the reference state lengthscale have been derived using the moment approach introduced in the classical review work by Hinton and Hazeltine. Furthermore it has been shown how this approach is not suitable for the description of smaller length-scales; e.g., the mesoscales that are naturally formed due to equilibrium nonuniformity and/or fluctuation induced transport. In this work, this analysis is extended to micro- and meso-scales adopting the framework of phase space zonal structure theory. Previous results are recovered in the long wavelength limit and, in the general case, transport equations in the phase space for particles and energy are obtained that correctly take into account meso-scale structures.

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I. INTRODUCTION

Describing the evolution of macroscopic plasma profiles on long time scales i.e., of the order of the energy confinement time or even longer, requires to treat, on the same footing, transport processes induced by Coulomb collisions and by turbulent fluctuations which, in turn, are driven by the gradients of macroscopic profiles. The self-consistency of the adopted description is therefore of fundamental importance. A first principle approach is recommended and several theories based on asymptotic expansions of the kinetic equation have been proposed, see e.g. Refs. 1–6. The expansion order required to correctly describe transport processes, i.e., to calculate consistent fluxes in terms of fluctuating fields, is determined by the time scale of required validity of the theoretical description and by the derivation technique. Some of these works, i.e. Refs. 4–6, are based on a systematic separation of spatio-temporal scales between microscopic fluctuations and macroscopic profiles which allow to obtain the evolution of the latter by averaging turbulent transport fluxes over turbulence structures. This assumption is equivalent to conjecturing that microscopic fluctuations do not produce meso-scales in the plasma profiles i.e., spatiotemporal scales intermediate between the macroscopic and the microscopic ones. Although this approach is particularly convenient from the numerical perspective; i.e., it allows to simulate separately small sub-domains of the whole plasma[5,6], it is not suitable to describe particular phenomena observed in fusion plasmas, e.g. transport barriers[4], L-H transitions etc. Furthermore, we know[7–9] that fusion plasmas are characterized by the unique role of energetic particles as mediators of cross-scale couplings due to the fact that they can excite Alfvénic fluctuations at their gyroscales; i.e., meso-scales for the thermal plasma. Thus, a separation of scales cannot always be assumed and a novel approach must be developed[9]. Following this framework, in a previous article[10] we have derived a set of equations governing particle and energy transport on the energy confinement time using standard first order gyrokinetic theory[11,12]. The results of Ref. 10 10 are consistent with those of Refs. 4–6 and transport equations reduce to the ones found therein if we introduce proper spatio-temporal averages consistently with the assumed separation of scales. Furthermore, the formulation introduced in Ref. 10, which is valid point wise in space and time, naturally introduces the notion of spatiotemporal scales of equilibrium variations and of the corresponding (zonal) structures. In fact, starting from given plasma profiles, the spatiotemporal features of the corresponding dynamic evolution is given by collisional and fluctuation induced fluxes, self-consistently. Thus, we cannot conclude that the spatiotemporal scales of the considered plasma equilibrium will be preserved by the nonlinear evolution. What we typically know, from this analysis, is that our macroscopic transport equations are valid as long as the asymptotic expansion in the small drift parameter is consistent. The derivation of a theory to describe profile corrugations on the micro- and meso-scales self-consistently generated by nonlinear plasma behavior is the scope of this work. In particular we will show that the theory of phase space zonal structures[7,9] allows us to generalize the concept of plasma reference state[10] to self-consistently include intermediate spatiotemporal scales.

Fluctuating fields in toroidal fusion plasmas can generate toroidal symmetric structures in the density and temperature profiles that are usually linearly stable and are characterized by a slow time variation[13] with respect to microscopic fluctuations. In particular, the poloidally symmetric response of these structures is unaffected by rapid collisionless dissipation[14,15], and may be regarded as radial corrugations of the "smooth" equilibrium profiles effectively redefining the characteristic spatiotemporal scales. These modifications are called zonal structures and they must satisfy $k_\parallel \equiv 0$ globally. Thus, in magnetically confined fusion plasmas, zonal structures correspond to long-lived or oscillating electromagnetic perturbations with predominant variations in the radial direction. As the zonal structures are nonlinearly excited (being linearly stable), they will scatter the primary driving instabilities to shorter-wavelength stable regime domain[16–18]. For this reason they can importantly regulate turbulence saturation level[13–15,19–27] and, eventually, turbulent transport; and, thus, they must be properly accounted for a self-consistent description of gyrokinetic transport. In addition to zonal structures, more general phase space zonal structures[7,9] can exist, which are also undamped by collisionless processes and characterize the phase space of the reference state. They represent a deviation of the plasma from the local thermodynamic equilibrium and their dynamics crucially determine the statistical properties of transport events such as intermittency, avalanches, bursting and/or non-local behaviors. They are particularly important when resonant wave-particle interactions are crucial for instability and transport processes, e.g. see Refs. 7–9. In this context and theoretical framework, zonal structures and phase space zonal structures are self-consistent counterparts of collisionless undamped (long-lived) nonlinear deviation of the plasma from the reference thermodynamic equilibrium state as a consequence of fluctuation-induced transport processes, due to emission and reabsorption of (toroidal equilibrium) symmetry-breaking perturbations[7,8]. They are eventually damped by collisions; but a realistic description of transport in collisionless plasmas must self-consistently take them into account.

As anticipated above, in this work we derive particle and energy transport equations based on the description of phase space zonal structures, and demonstrate that they reduce, as expected, to the equations obtained in Ref. 10, when only macroscopic spatial scale corrugations to the nonlinear evolving equilibrium are considered. As collisional transport manifests itself on long time and length scales only, and gyrokinetic theory is based on spatiotemporal scale separation between plasma reference state and fluctuation spectra[11,12], we discuss gyrokinetic transport equations in the collisionless, short wavelength limit. By doing so, we are able to isolate the linear polarization response[14,15], which can be considered of higher order in the usual macroscopic plasma transport analysis, and the fluctuation induced nonlinear fluxes, suitably modified at short scales. In general, we show that fast radial oscillations of the profiles are of crucial importance in the self-consistent description of the transport processes in a magnetically confined plasma.

This work is organized as follows. In Section II, we provide the definition of plasma reference state in terms of phase space zonal structures and we introduce the notation and the ordering of the physical quantities. In Section III, we derive the transport equations from the evolution of phase space zonal structures. In Section IV, we compare our results with Ref. 10 showing that the two approaches are consistent in the long wavelength limit. Final conclusions and discussions are given in Section V.

II. THEORETICAL FRAMEWORK AND ORDERING ASSUMPTIONS

In this work, we study transport processes in strongly magnetized plasmas and, therefore, for each species, the particle distribution function can be written as the sum of a reference distribution function F_0 and a small perturbation δf:

$$f = F_0 + \delta f \; , \tag{1}$$

where the characteristic (macroscopic) lengthscale of variation of F_0, i.e. L, is such that $\delta f/F \sim \rho/L \sim \delta \ll 1$ and ρ is the Larmor radius. Following Refs. 11,28, we assume that reference states are characterized by profiles (conventionally denoted as p_0) varying on macroscopic spatio-temporal scales, and satisfying the following time and scale ordering:

$$\omega^{-1}\frac{\partial}{\partial t}\ln p_0 = \omega^{-1}p_0^{-1}\frac{\partial p_0}{\partial t} \sim \mathcal{O}(\delta^2)\ , \tag{2}$$

.We further assume the so-called drift ordering:

$$\frac{cE}{B_0 v_{th}} \sim \mathcal{O}(\delta)\ , \tag{3}$$

where v_{th} is the particle thermal speed, and other symbols are standard. Following Ref. 11, we adopt the gyrokinetic ordering for fluctuating quantities:

$$\frac{|\partial/\partial t|}{|\Omega|} \sim \left|\frac{\delta B}{B_0}\right| \sim \frac{\boldsymbol{\nabla}_\parallel}{\boldsymbol{\nabla}_\perp} \sim \frac{\boldsymbol{k}_\parallel}{\boldsymbol{k}_\perp} \sim \mathcal{O}(\delta)\ , \tag{4}$$

where Ω is the particle cyclotron frequency in the reference state magnetic field. We assume axisymmetry of the reference state and, therefore, without loss of generality, the reference magnetic field can be written in the following form[29]:

$$\boldsymbol{B}_0 = F\boldsymbol{\nabla}\phi + \boldsymbol{\nabla}\phi\times\boldsymbol{\nabla}\psi. \tag{5}$$

Following Ref. 12, we express the particle distribution function in terms of the gyrocenter distribution function $\bar{F}$:

$$\begin{aligned} f =& e^{-\boldsymbol{\rho}\cdot\boldsymbol{\nabla}}\bar{F} - \tfrac{e}{m}e^{-\boldsymbol{\rho}\cdot\boldsymbol{\nabla}}\left\langle\delta\psi_{gc}\right\rangle\left(\tfrac{\partial\bar{F}}{\partial\mathcal{E}} + \tfrac{1}{B_0}\tfrac{\partial\bar{F}}{\partial\mu}\right) + \left[\tfrac{e}{m}\delta\phi\tfrac{\partial\bar{F}}{\partial\mathcal{E}}\right] + \\ &+ \left[\tfrac{e}{m}\left(\delta\phi - \tfrac{v_\parallel}{c}\delta A_\parallel\right)\tfrac{1}{B_0}\tfrac{\partial\bar{F}}{\partial\mu} + \delta\boldsymbol{A}_\perp\times\tfrac{\boldsymbol{b}}{B_0}\cdot\boldsymbol{\nabla}\bar{F}\right]\ , \end{aligned} \tag{6}$$

where ρ denotes the lowest-order gyroangle-dependent gyroradius vector, $\mathcal{E} = v^2/2$ is the energy per unit mass, μ is the magnetic moment adiabatic invariant $\mu = v_\perp^2/(2B_0) + \ldots$ and:

$$\delta\psi_{gc} = \delta\phi_{gc} - \frac{\boldsymbol{v}}{c}\cdot\delta\boldsymbol{A}_{gc} = e^{\boldsymbol{\rho}\cdot\boldsymbol{\nabla}}\left(\delta\phi - \frac{\boldsymbol{v}}{c}\cdot\delta\boldsymbol{A}\right) \equiv e^{\boldsymbol{\rho}\cdot\boldsymbol{\nabla}}\delta\psi. \tag{7}$$

Now, let us formally write the leading order plasma response to zonal structures; i.e., the component of the distribution function undamped by collisionless processes (see e.g. Ref. 7,9), in term of its adiabatic and non-adiabatic components:

$$\delta f_z = e^{-\boldsymbol{\rho}\cdot\boldsymbol{\nabla}}\delta\bar{G}_z + \frac{e}{m}\delta\phi_{0,0}\frac{\partial\bar{F}_0}{\partial\mathcal{E}}, \tag{8}$$

where, $0,0$ subscript to $\delta\phi$ indicates the $m = n = 0$ Fourier component, with m and n being, respectively, the poloidal and toroidal mode numbers of the fluctuation. We have also assumed that the equilibrium guiding center distribution is isotropic, that is $\partial_\mu \bar{F}_0 = 0$, and that the usual low-β tokamak ordering applies. Due to its slow dynamics, governed by nonlinear phenomena, δf_z will be adopted to effectively extend the notion of plasma reference state. The newly defined reference state will be $F_0 + \langle \delta f_z \rangle_\psi$, where $\langle \dots \rangle_\psi$ stand for flux surface average, and transport processes will be described uniquely in term of the evolution equation for δf_z. This definition allows us to describe self-consistently mesoscale corrugations of the reference state and is consistent with the conventional one, i.e. see Eq. (1), in the long-wavelength limit.

The (leading order) non-adiabatic gyrocenter plasma response to zonal structures, $\delta \bar{G}_z$, is obtained solving the first order nonlinear gyrokinetic equation[11,12]:

$$\left(\partial_t + v_\parallel \nabla_\parallel + \boldsymbol{v}_d \cdot \boldsymbol{\nabla}\right) \delta \bar{G}_z = -\frac{e}{m} \frac{\partial \bar{F}_0}{\partial \mathcal{E}} \frac{\partial}{\partial t} \left\langle \delta \psi_{gc} \right\rangle_z - \frac{c}{B_0} \boldsymbol{b} \times \boldsymbol{\nabla} \left\langle \delta \psi_{gc} \right\rangle \cdot \boldsymbol{\nabla} \delta \bar{G} \bigg|_z \,, \tag{9}$$

where:

$$\left\langle \delta \psi_{gc} \right\rangle_z = \hat{I}_0 \left(\delta \phi_{0,0} - \frac{v_\parallel}{c} \delta A_{\parallel 0,0} \right) + \frac{m}{e} \mu \hat{I}_1 \delta B_{\parallel 0,0}. \tag{10}$$

The last term of Eq. (9) depends quadratically on the fluctuation strength, and is due to fluctuations with opposite toroidal mode number, $\hat{I}_n(x) \equiv (2/x)^n J_n(x)$[30], $J_n(x)$ are the Bessel functions, $\lambda^2 \equiv 2(\mu B_0/\Omega^2) k_\perp^2$ and the definition of $\hat{I}_n$ acting on a generic function $g(\boldsymbol{r}) = \int \hat{g}(\boldsymbol{k}) \exp(i\boldsymbol{k} \cdot \boldsymbol{r}) d\boldsymbol{k}$ is:

$$\hat{I}_n g(\boldsymbol{r}) \equiv \int d\boldsymbol{k} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \hat{I}_n(\lambda) \hat{g}(\boldsymbol{k}). \tag{11}$$

This equation states that zonal structures are driven by zonal fields with $n = m = 0$ and by the nonlinear interplay between the gyro-center response and the perpendicular gradient of fluctuating fields that generate terms with the same property. Note that the gyro-center zonal structure response, $\delta \bar{G}_z$, must be axisymmetric in order to avoid collisionless damping processes. The exact expression for $\delta \bar{G}_z$ which annihilates linear collisionless dissipation will be derived in the next sections. Furthermore, we will show that leading order particle and energy cross-field fluxes that are valid up to the energy confinement time scale can be derived by means of first order gyrokinetic theory. This generalizes to an arbitrary characteristic lengthscale of radial profiles the procedure adopted in Ref. 10, where the first order push-forward representation of particle moments[12] is exploited for a compact derivation of transport equations valid on the energy confinement time. We show that the two approaches are consistent and one reduces to the other as limiting case, when the plasma response to zonal structures is macroscopic; i.e., is described by the characteristic lengthscale of reference fields.

III. TRANSPORT EQUATIONS

In this section, as anticipated in Sec. I, we derive particle and energy transport on the energy confinement time from Eq. 9. Particle drift velocity due to reference fields can be written in the following form:

$$\boldsymbol{v}_d = \frac{v_\parallel}{\Omega}\boldsymbol{\nabla}\times(\boldsymbol{b}v_\parallel). \tag{12}$$

Using toroidal flux coordinates, along with the equilibrium magnetic field representation of Eq. (5), and exploiting the axisymmetry of $\delta\bar{G}_z$, we obtain:

$$\boldsymbol{v}_d\cdot\boldsymbol{\nabla} = \frac{v_\parallel}{J\Omega}\left[\frac{\partial}{\partial\theta}\left(\frac{Fv_\parallel}{B_0}\right)\frac{\partial}{\partial\psi} - \frac{\partial}{\partial\psi}\left(\frac{Fv_\parallel}{B_0}\right)\frac{\partial}{\partial\theta}\right]. \tag{13}$$

Furthermore, the following relation holds:

$$\boldsymbol{\nabla}_\parallel = \frac{1}{JB}\frac{\partial}{\partial\theta}\ ; \tag{14}$$

and, therefore, we can rewrite the linear part of the free streaming operator in Eq. (9) as:

$$\partial_t + v_\parallel\left[1-\frac{\partial}{\partial\psi}\left(\frac{Fv_\parallel}{\Omega}\right)\right]\nabla_\parallel + v_\parallel\nabla_\parallel\left(\frac{Fv_\parallel}{\Omega}\right)\frac{\partial}{\partial\psi}. \tag{15}$$

Introducing the toroidal angular momentum $P_\phi = (e/c)(Fv_\parallel/\Omega-\psi)$, Eq. (15) can be rewritten as:

$$\partial_t - \frac{v_\parallel c}{e}\left[\frac{\partial P_\phi}{\partial\psi}\nabla_\parallel - \nabla_\parallel P_\phi\frac{\partial}{\partial\psi}\right] = \partial_t - \frac{v_\parallel c}{e}\left[\frac{\partial P_\phi}{\partial\psi}\right]_\theta \nabla_\parallel\big|_{P_\phi}\,, \tag{16}$$

which explicitly shows that particles belonging to the phase space zonal structure move along surfaces of constant P_ϕ. Up to the leading order in δ, the linear part of the particle free streaming operator can therefore be written as:

$$\partial_t + v_\parallel\nabla_\parallel + v_\parallel\nabla_\parallel\left(\frac{Fv_\parallel}{\Omega}\right)\frac{\partial}{\partial\psi}\ . \tag{17}$$

We introduce the (drift/banana center) decomposition[31,32] $\delta\bar{G}_z = e^{-iQ_z}\delta\bar{g}_z$ and impose:

$$i\nabla_\parallel Q_z = i\nabla_\parallel\left(\frac{Fv_\parallel}{\Omega}\right)\frac{k_z}{d\psi/dr}\ , n \tag{18}$$

with $k_z \equiv (-i\partial_r)$, that simplifies the nonlinear gyrokinetic equation. Integrating Eq. (18), we obtain the following expression:

$$Q_z = F(\psi)\left[\frac{v_\parallel}{\Omega} - \overline{\left(\frac{v_\parallel}{\Omega}\right)}\right]\frac{k_z}{d\psi/dr}\ , \tag{19}$$

where we have introduced the average along unperturbed particle orbits:

$$\overline{[\dots]} \equiv \tau_b^{-1}\oint \frac{d\ell}{v_\parallel}[\dots]\ ; \tag{20}$$

and τ_b is the time required for particles to complete an (integrable) close poloidal orbit in the reference magnetic field. Note that introducing:

$$\rho_{drift} \equiv \frac{F(\psi)}{d\psi/dr}\left[\frac{v_\parallel}{\Omega} - \overline{\left(\frac{v_\parallel}{\Omega}\right)}\right], \tag{21}$$

we can write the pullback operator as $e^{i\rho_{drift}k_z}$, which is the same formal expression used for the guiding center pullback operator. Also the physical meaning is the same: it allows a simplified description of the plasma in terms of "moving drift and/or banana orbits" 33,34. Using these results, we can rewrite Eq. (9) as

$$\left(\partial_t + v_\parallel\nabla_\parallel\right)\delta\bar{g}_z = e^{iQ_z}\left(-\frac{e}{m}\frac{\partial\bar{F}_0}{\partial\mathcal{E}}\frac{\partial}{\partial t}\left\langle\delta\psi_{gc}\right\rangle_z - \frac{c}{B_0}\boldsymbol{b}\times\boldsymbol{\nabla}\left\langle\delta\psi_{gc}\right\rangle\cdot\boldsymbol{\nabla}\delta\bar{G}\right)\ . \tag{22}$$

The requirement for the phase space zonal structure to be long lived; i.e., that it annihilates the linear part of the free streaming operator, imposes that $\nabla_\parallel\delta\bar{g}_z = 0$. The pullback operator does not depend on the ϕ coordinate and, therefore, we obtain that $\delta\bar{g}_z$ must be toroidally symmetric and characterized by $m = 0$. We note that $\delta\bar{G}_z = e^{-iQ_z}\delta\bar{g}_z$, instead, is not necessarily poloidally symmetric. The equation governing the evolution of $\delta\bar{g}_z$ is:

$$\partial_t\delta\bar{g}_z = \overline{\left[e^{iQ_z}\left(-\frac{e}{m}\frac{\partial\bar{F}_0}{\partial\mathcal{E}}\frac{\partial}{\partial t}\left\langle\delta\psi_{gc}\right\rangle_z - \frac{c}{B_0}\boldsymbol{b}\times\boldsymbol{\nabla}\left\langle\delta\psi_{gc}\right\rangle\cdot\boldsymbol{\nabla}\delta\bar{G}\right)\right]}. \tag{23}$$

Next, let us recall the relationship between lowest order bounce averaging and the flux surface average of a velocity space integral in order to conveniently rewriting governing equations for the moments of δf_z. Consider the definitions of flux surface average and velocity space integration,

$$\begin{aligned}\left\langle\dots\right\rangle_v &= 2\pi B_0\sum_{v_\parallel/|v_\parallel|=\pm}\int\frac{d\mu d\mathcal{E}}{|v_\parallel|}(\dots)\ ,\\ \left\langle\dots\right\rangle_\psi &= \frac{1}{V'}\oint Jd\theta d\phi(\dots) = \frac{1}{V'}\oint\frac{d\theta d\phi}{\boldsymbol{B}\cdot\boldsymbol{\nabla}\theta}(\dots),\end{aligned}$$

where J is the Jacobian of the toroidal flux coordinates and $V' = \oint d\theta d\phi / \boldsymbol{B} \cdot \boldsymbol{\nabla}\theta$ is the flux surface volume element. Noting that particle motion is along $\boldsymbol{B}_0$ at the lowest order in the drift parameter expansion; i.e.:

$$\int \frac{d\theta}{\boldsymbol{B} \cdot \boldsymbol{\nabla}\theta} = \int \frac{d\ell}{B_0} \,, \tag{24}$$

we can derive, for any velocity space function f, the following expression:

$$\langle \langle f \rangle_v \rangle_\psi = \frac{4\pi^2}{V'} \sum_{v_\parallel / |v_\parallel| = \pm} \int d\mu d\mathcal{E} \, \tau_b \overline{f_{n=0}}. \tag{25}$$

This result shows that the flux surface average of a velocity integral depends only on the bounce averaged response of the $n = 0$ toroidal Fourier harmonic at the leading order in the asymptotic expansion. This is clearly connected with phase space zonal structures. In fact, in the presence of fluctuations in the gyro-center particle distribution, the drift/banana-center non-adiabatic particle response yields the following form for the phase space zonal structure[7,9]:

$$\overline{\langle \delta f_z \rangle} = \overline{\left(e^{-iQ_z} \hat{I}_0 \right)} \delta \bar{g}_z + \frac{e}{m} \delta\phi_{0,0} \frac{\partial \bar{F}_0}{\partial \mathcal{E}} \,, \tag{26}$$

where the gyrophase average is indicated by $\langle \dots \rangle$. Acting on this expression by ∂_t, substituting Eq. (23) and integrating in velocity space, we obtain:

$$\begin{aligned} \partial_t \langle \langle \delta f_z \rangle_v \rangle_\psi = {} & \frac{e}{m} \partial_t \delta\phi_{0,0} \left\langle \frac{\partial \bar{F}_0}{\partial \mathcal{E}} \right\rangle_v + \frac{4\pi^2}{V'} \sum_{v_\parallel / |v_\parallel| = \pm} \int \tau_b d\mu d\mathcal{E} \\ & \times \overline{\left(e^{-iQ_z} \hat{I}_0 \right)} \left[- \frac{e}{m} \frac{\partial \bar{F}_0}{\partial \mathcal{E}} \overline{\left(e^{iQ_z} \hat{I}_0 \right)} \partial_t \delta\phi_{0,0} \right. \\ & \left. - \overline{e^{iQ_z} \left(\frac{c}{B_0} \boldsymbol{b} \times \boldsymbol{\nabla} \langle \delta\psi_{gc} \rangle \cdot \boldsymbol{\nabla} \delta \bar{G} \right)} \right] \,, \end{aligned} \tag{27}$$

where we have used Eq. (25) to remove the bounce average from the LHS of this expression.

Equation (27) is the gyrokinetic extension of Eq. (15) in Ref. 10, and it is valid for corrugations of the reference state characterized by a lengthscale up to the particle Larmor radius. As anticipated above, collisional transport is suppressed here but could be readily restored. If weighted by $mv^2/2$, Eq. (27) would give the gyrokinetic extension of Eq. (20) in Ref. 10, and shows that fluctuation induced particle and energy transport are obtained from the same "formal" expressions; i.e., the moments of the newly defined reference state. In particular in the next section, we will explicitly show that, considering the long wavelength component of zonal structures; i.e., $k_z L < \delta^{-1/2}$, we obtain a density transport equation that is identical to the fluctuation induced part of Eq. (15) in Ref. 10.

IV. COMPARISON WITH PREVIOUS TREATMENTS

In order to make a comparison with the transport equations derived in 10, it is convenient to cast Eq. (27) in a different form. We can re-write the last term of its RHS:

$$\boldsymbol{b}\times\boldsymbol{\nabla}\langle\delta\psi_{gc}\rangle\cdot\boldsymbol{\nabla}\delta\bar{G}=\boldsymbol{b}\cdot(\boldsymbol{\nabla}r\times\boldsymbol{\nabla}\theta)\left(\frac{\partial\langle\delta\psi_{gc}\rangle}{\partial r}\frac{\partial\delta\bar{G}}{\partial\theta}-\frac{\partial\langle\delta\psi_{gc}\rangle}{\partial\theta}\frac{\partial\delta\bar{G}}{\partial r}\right)+ \tag{28}$$
$$+\boldsymbol{b}\cdot(\boldsymbol{\nabla}r\times\boldsymbol{\nabla}\phi)\left(\frac{\partial\langle\delta\psi_{gc}\rangle}{\partial r}\frac{\partial\delta\bar{G}}{\partial\phi}-\frac{\partial\langle\delta\psi_{gc}\rangle}{\partial\phi}\frac{\partial\delta\bar{G}}{\partial r}\right)+$$
$$+\boldsymbol{b}\cdot(\boldsymbol{\nabla}\phi\times\boldsymbol{\nabla}\theta)\left(\frac{\partial\langle\delta\psi_{gc}\rangle}{\partial\phi}\frac{\partial\delta\bar{G}}{\partial\theta}-\frac{\partial\langle\delta\psi_{gc}\rangle}{\partial\theta}\frac{\partial\delta\bar{G}}{\partial\phi}\right).$$

Substituting the following expressions into Eq. (28):

$$\boldsymbol{b}\cdot(\boldsymbol{\nabla}r\times\boldsymbol{\nabla}\theta)=\frac{F}{(d\psi/dr)}\frac{1}{JB}\,;\qquad \boldsymbol{b}\cdot(\boldsymbol{\nabla}r\times\boldsymbol{\nabla}\phi)=\frac{F}{(d\psi/dr)}\left(\frac{q}{JB}-\frac{B}{F}\right) \tag{29}$$

and noting that:

$$\mathcal{O}\left(\frac{1}{qR_0}\right)\sim k_\parallel=-\frac{i}{JB}\left(\frac{\partial}{\partial\theta}+q(r)\frac{\partial}{\partial\phi}\right), \tag{30}$$

that $(JB)^{-1}\sim 1/qR_0$ and that $\partial_\theta\sim Lk_\perp\sim\delta^{-1}k_\parallel$ when acting on plasma turbulence thus:

$$\frac{\partial}{\partial\theta}\simeq -q(r)\frac{\partial}{\partial\phi}+\mathcal{O}(\delta). \tag{31}$$

We obtain, at the leading order, the following expression:

$$\frac{c}{B}\boldsymbol{b}\times\boldsymbol{\nabla}\langle\delta\psi_{gc}\rangle\cdot\boldsymbol{\nabla}\delta\bar{G}\Big|_z=\frac{c}{(d\psi/dr)}\left(\frac{\partial\langle\delta\psi_{gc}\rangle}{\partial\phi}\frac{\partial\delta\bar{G}}{\partial r}-\frac{\partial\langle\delta\psi_{gc}\rangle}{\partial r}\frac{\partial\delta\bar{G}}{\partial\phi}\right)_z+\mathcal{O}(\delta). \tag{32}$$

Therefore we can re-write the LHS of Eq. (32):

$$\frac{c}{B}\boldsymbol{b}\times\boldsymbol{\nabla}\langle\delta\psi_{gc}\rangle\cdot\boldsymbol{\nabla}\delta\bar{G}|_z=\frac{c}{(d\psi/dr)}\left(\frac{\partial\langle\delta\psi_{gc}\rangle}{\partial\phi}\frac{\partial\delta\bar{G}}{\partial r}-\frac{\partial\langle\delta\psi_{gc}\rangle}{\partial r}\frac{\partial\delta\bar{G}}{\partial\phi}\right)_z+\mathcal{O}(\delta)\simeq \tag{33}$$
$$\simeq\frac{c}{(d\psi/dr)}\left(in\langle\delta\psi_{gc}\rangle\frac{\partial\delta\bar{G}}{\partial r}+in\frac{\partial\langle\delta\psi_{gc}\rangle}{\partial r}\delta\bar{G}\right)+\mathcal{O}(\delta)\simeq$$
$$\simeq\frac{inc}{(d\psi/dr)}\frac{\partial}{\partial r}\left(\delta\bar{G}\langle\delta\psi_{gc}\rangle\right)+\mathcal{O}(\delta)\simeq$$
$$\simeq c\frac{\partial}{\partial\psi}\left(R^2\boldsymbol{\nabla}\phi\cdot\boldsymbol{\nabla}\langle\delta\psi_{gc}\rangle\delta\bar{G}\right)+\mathcal{O}(\delta).$$

Here, we have noted that zonal structures must have $n=0$, which is obtained only if the toroidal mode number of $\langle\delta\psi_{gc}\rangle$ is opposite to the toroidal mode number of $\delta\bar{G}$. Taking into account Eq. (33), the equation governing zonal structures dynamics reads:

$$\partial_t\langle\langle\delta f_z\rangle_v\rangle_\psi=\frac{e}{m}\partial_t\delta\phi_{0,0}\left\langle\left[1-\left(e^{-iQ_z}\hat{I}_0\right)\overline{\left(e^{iQ_z}\hat{I}_0\right)}\right]\frac{\partial\bar{F}_0}{\partial\mathcal{E}}\right\rangle_v$$
$$-\frac{1}{V'}\frac{\partial}{\partial\psi}\left\langle\left\langle V'\left(e^{-iQ_z}\hat{I}_0\right)\overline{\left[ce^{iQ_z}R^2\boldsymbol{\nabla}\phi\cdot\boldsymbol{\nabla}\langle\delta\psi_{gc}\rangle\delta\bar{G}\right]}\right\rangle_v\right\rangle_\psi. \tag{34}$$

The first two terms on the RHS represent the linear polarization response of the plasma while the third term is the long-lived effect (not related to collisionless processes) of turbulent transport. Mesoscales in the density profile are spontaneously produced by this term. In order to show the consistency of the long wavelength limit of this equation, i.e. $\left(e^{iQ_z}\hat{I}_0\right) \to 1$, with Eq. (15) of Ref. 10, we note that the first term on the RHS of Eq: (34) reads:

$$\frac{e}{m}\partial_t \delta\phi_{0,0} \left\langle \left[1 - \left(e^{-iQ_z}\hat{I}_0\right)\overline{\left(e^{iQ_z}\hat{I}_0\right)}\right] \frac{\partial \bar{F}_0}{\partial \mathcal{E}} \right\rangle_v \sim (Q_z^2 + \lambda^2)\frac{e\delta\phi}{T} n_0 \sim (k_z \rho_{drift})^2 \delta\omega n_0 \tag{35}$$

and, therefore, we can neglect this term in the study of the effect of structures with long wavelength, i.e. $k_z L < \delta^{-1/2}$, on the energy confinement time. Analogously, the second term on the RHS reads:

$$\begin{aligned} &-\frac{1}{V'}\frac{\partial}{\partial\psi}\left\langle\left\langle V'\left(e^{-iQ_z}\hat{I}_0\right)\overline{\left[ce^{iQ_z}R^2\boldsymbol{\nabla}\phi\cdot\boldsymbol{\nabla}\left\langle\delta\psi_{gc}\right\rangle\delta\bar{G}\right]}\right\rangle_v\right\rangle_\psi \sim \\ &-\frac{1}{V'}\frac{\partial}{\partial\psi}\left\langle\left\langle V'\overline{\left[cR^2\boldsymbol{\nabla}\phi\cdot\boldsymbol{\nabla}\left\langle\delta\psi_{gc}\right\rangle\delta\bar{G}\right]}\right\rangle_v\right\rangle_\psi \end{aligned} \tag{36}$$

which is the fluctuation-induced term of Eq. (15) in Ref. 10.

V. CONCLUSIONS & FUTURE PERSPECTIVES

In this work, we have derived transport equations valid up to the energy confinement time in the framework of phase spaces zonal structures theory[7,9]. The governing equations allow describing multiple spatiotemporal scales generated nonlinearly, eventually invalidating the hypothesis of scale separation[4–6] between reference state and fluctuations. Furthermore, we have shown that the equations in the long wavelength limit yields fluctuation induced fluxes consistent with Ref. 10. Meanwhile, the resulting fluxes have the same formal expression for both particle and energy transport, suggesting, thus, and illuminating further, that the fundamental structures governing turbulent transport processes are phase space zonal structures, which can be viewed as the lifting of transport processes to the phase space. These results allow to extend the concept of plasma reference state to self-consistently include spatiotemporal meso-scales.

In general, it is possible to obtain both collisional and fluctuation induced fluxes by studying phase space zonal structures equations including an appropriate collisional term. This requires the introduction of a gyrokinetic collision integral[35–37]. Gyrokinetics codes, based on Lagrangian particle-in-cell approaches[20,38–44] as well as Eulerian descriptions[45–53] could be used to calculate fluxes, obtaining thus, a complete description of the transport processes in magnetized plasmas on the energy confinement time. Obviously, transport is already accounted for by the gyrokinetic description adopted in numerical simulations. However, the advantage of using explicit expressions for turbulent and collisional fluxes to evolve plasma profiles has been demonstrated in practical applications[4–6] and may yield an efficient way to approach the simulation of global plasma transport on long time scales . Furthermore, explicit expressions of turbulent and collisional fluxes allow for higher order accuracy in the evolution of plasma equilibrium profiles, once the accuracy of the plasma response to symmetry breaking perturbations is fixed[10,28]. The present work, adds to this framework the possibility of self-consistently accounting of mesoscale structures that are naturally formed in collisionless fusion plasmas in the form of phase space zonal structures.These results allow to extend the concept of plasma reference state to self-consistently include spatiotemporal meso-scales.

In this work, the relative ordering of temporal and spatial scales, as well as fluctuation amplitudes, has been assumed consistent with gyrokinetic field theory of the core region of magnetized thermonuclear plasmas. As the edge plasma region is approached, where equilibrium magnetic field is modified from closed to open field lines, the relative ordering of spatiotemporal scale of turbulent fluctuation spectra and transport phenomena is also modified and not so well separated as in the plasma core. These aspects have been recently addressed by Abel and coworkers in Ref.[54]. The development of a gyrokinetic field theory that encompasses these different ordering within a unified framework is of main importance. Progress in this research field will be crucial in order to describe transport processes on long time scales. The present approach poses an important issue concerning collisional transport and its synergistic interplay with fluctuation induced transport, since collisions eventually damp phase space zonal structures and tend to bring the reference state closer to local thermodynamic equilibrium. Because of its implications (cf., e.g., the recent review by Sugama[55], this problem will be addressed in a separate work.

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