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# Evidence about the hardness of hypothetical dense polymorphs of $C_3N_4$ relative to diamond

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## Abstract (600 characters with space!)

An empirical linear correlation of hardness and shear modulus was revealed for the spinel nitrides of the group 14 elements,  $\gamma$ - $M_3N_4$  where  $M=Si, Ge, Sn$ , which also holds for the hexagonal  $\alpha$ - and  $\beta$ -phases of  $Si_3N_4$ . The correlation, completed here by a careful measurement of the bulk modulus of  $\gamma$ - $Sn_3N_4$ , was used to predict the hardness of hypothetical dense phases of  $C_3N_4$  whose elastic moduli were calculated. Our calculation approach was supported by the agreement with the experimental data for  $\gamma$ - $M_3N_4$ . The hardness of all proposed dense polymorphs of  $C_3N_4$  are predicted to be below that of diamond.

**Keywords:** spinel tin nitride,  $C_3N_4$ , hardness, elastic moduli, first-principles calculations, powder XRD, high-pressure equation of state

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## I. INTRODUCTION

In the 1990's the excitement about materials harder than diamond was heated up after theoretical predictions of Liu and Cohen [1] that the hardness of the hypothetical carbon nitride  $\beta$ - $C_3N_4$  with the hexagonal structure, analogous to that of the well-known  $\beta$ - $Si_3N_4$  (Greek letters indicate in this work particular types of crystal structures described below), may be comparable with that of diamond. The Vickers hardness of  $H_V=100$  GPa is adapted in the literature for the latter well known dense phase of carbon forming in the deep Earth at high pressures and temperatures [2]. The basis of this speculation was the assumption that hardness of a covalent solid can be reliably predicted from its bulk modulus  $B_0$  at atmospheric pressure. The  $B_0$  value of  $\beta$ - $C_3N_4$  was estimated from an empirical dependence of the bulk moduli of covalent solids on the interatomic bond length as described in Ref. [1]. Later on, Teter and Hemley [3] theoretically examined other hypothetical polymorphs of  $C_3N_4$ , among which the cubic  $c$ - $C_3N_4$  (derived from the Willemite-II structure of  $Zn_2SiO_4$ ) was predicted to exhibit  $B_0=496$  GPa, which is significantly higher than that of diamond ( $B_0=442$  GPa [4]), and to be stable at high pressures. The proposition that  $c$ - $C_3N_4$ , if synthesized, could be harder than diamond heated up the search for dense forms of this hypothetical compound even further. For the next dense polymorphs such as  $\alpha$ - $C_3N_4$  (hexagonal crystal structure analogous to that of  $\alpha$ - $Si_3N_4$ ) and pseudo-cubic  $C_3N_4$  (derived from the zinc-blende structure)  $B_0$  above 400 GPa was predicted [3]. Similarly, further hypothetical polymorphs of  $C_3N_4$  examined theoretically in multiple publications, such as  $\gamma$ - $C_3N_4$  having the cubic spinel structure and  $zb$ - $C_3N_4$  having the cubic zinc-blende structure showed bulk moduli comparable with that of diamond [5]. Theoretical studies on  $C_3N_4$  were accompanied by multiple attempts to produce a dense form of this compound, but none of the reports about a successful synthesis could be reproduced so far e.g.[6,7]. Stabilization of the claimed  $C_3N_4$  products by hydrogen (usually present in the starting reactants) was considered only sporadically; e.g.[8,9]. With the research extension in this field, doubts about the reliability of  $B_0$  as a hardness predictor grew and empirical evidence was presented that the shear modulus  $G_0$  should be used instead [10]. The theoretical justification of this statement can be summarized as follows [11]: Hardness (or yield stress) is a value describing material resistance to plastic deformations which occur only when shear stress is applied. According to the Frenkel analysis, the strength of an ideal crystalline solid is proportional to its  $G_0$ . The strength of a real crystal is significantly smaller than that of an ideal crystal and is controlled by movement and multiplication of lattice defects such as dislocations. The shear stress needed for activation of these processes is also proportional to  $G_0$ . Even though a correlation of hardness and  $G_0$  is supported empirically, a significant scatter can be recognized if data for materials of different nature (e.g., ionic vs. covalent compounds) are compared. The attempts to establish empirical correlations for covalent compounds of one family (e.g. for the group 14 elements having diamond structure [12]) are rare because the number of such families, where  $G_0$  and  $H$  extend over a reasonable range, is strongly limited. Another source of significant uncertainties is the fact that experimental  $G_0$  and hardness of considered compounds are usually reported in distinct publications for samples of different origin with different or not described state of porosity, amount and nature of sintering additives *etc.* Up to very recently first-principles calculations provided rare and strongly deviating values of shear moduli of dense  $C_3N_4$  polymorphs [3,13,14]. Extended calculations of their single crystal elastic moduli  $C_{ij}$  and then  $G_0$  were presented in the last years [15,16], but without careful validation of the

calculation methods via comparison with experimental data for existing related compounds such as  $\alpha$ -,  $\beta$ - or  $\gamma$ - $\text{Si}_3\text{N}_4$ .

In this work we unveil an empirical linear correlation of experimental values of  $G_0$  and  $H_n$  (hardness measured using the nanoindentation testing) for dense nitrides of the group 14 elements (to which carbon belongs also) having spinel structure  $\gamma\text{-M}_3\text{N}_4$  (where  $\text{M} = \text{Si}, \text{Ge}, \text{and Sn}$ ). In particular, we made a selection of reliable experimental data on  $G_0$  and  $H_n$  of the spinel nitrides and completed it by a thorough measurement of  $B_0$  of  $\gamma\text{-Sn}_3\text{N}_4$  permitting deriving of its  $G_0$  from the earlier nanoindentation data [17]. The family of spinel nitrides, discovered in 1999 in the course of the search for a superhard  $\text{C}_3\text{N}_4$  [18-22], is today well examined and measurements of elastic moduli and hardness for the same samples are available in the literature with one exception we remedy here. Spinel-nitrides are one of a few compound families having at least three members with a hardness spreading over a broad range. Obviously, three is the minimal number of data pairs needed to confirm empirically a linear relation between two physical values, e.g.  $H_n$  and  $G_0$ . Reliable data for  $G_0$  and nanohardness  $H_n$  were obtained for  $\gamma\text{-Si}_3\text{N}_4$  and  $\gamma\text{-Ge}_3\text{N}_4$  from measurements of equations of state (EOS) on compression,  $V(P)$ , of the reduced elastic modulus  $E_r$  and  $H_n$  in nanoindentation tests, and of sound velocities via Brillouin spectroscopy [17,23,24]. The values of  $E_r$  and  $H_n$  were measured also for densified samples of  $\gamma\text{-Sn}_3\text{N}_4$  [17] but the earlier experimental  $B_0$ , needed to derive  $G_0$  from  $E_r$  using the well established approach [25,26], is less reliable: In only one work,  $B_0$  of  $\gamma\text{-Sn}_3\text{N}_4$  was obtained from  $V(P)$  measured to relatively low pressures of 26 GPa and potentially systematically biased due to use of a methanol-ethanol-water pressure transmitting medium becoming non-hydrostatic above 11 GPa [27]. This shortcoming is remedied here:  $V(P)$  of  $\gamma\text{-Sn}_3\text{N}_4$  was accurately measured to much higher pressures under quasihydrostatic loading, and the  $G_0$  value derived from the  $B_0$  one that we have obtained, combined with the earlier reported  $E_r$ . The result permits us to confirm the reliability of a linear dependence of  $H_n$  on  $G_0$  for the three spinel nitrides. Furthermore, we have performed first-principles calculations of single crystal elastic moduli  $C_{ij}$ ,  $B_0$ , and  $G_0$  for the existing  $\gamma\text{-M}_3\text{N}_4$  in order to validate our calculation approach by comparison with the reliable experimental data. Then,  $C_{ij}$  values of all proposed dense polymorphs of  $\text{C}_3\text{N}_4$  were calculated and  $G_0$  values for the related isotropic polycrystalline bodies were derived. We used these values to estimate  $H_n$  of all proposed dense  $\text{C}_3\text{N}_4$  polymorphs using the established here empirical correlation  $H_n(G_0)$  for  $\gamma\text{-M}_3\text{N}_4$  and, finally, to make a conclusion about the hardness of dense  $\text{C}_3\text{N}_4$  materials.

## II. METHODS

The sample of  $\gamma\text{-Sn}_3\text{N}_4$  ( $\gamma$  indicates spinel structure described by the space group (S.G.)  $Fd\bar{3}m$  where 2/3 of cations are octahedrally coordinated and the remaining 1/3 is tetrahedrally coordinated by anions) used in this work was synthesized earlier [22]. It contained minor admixtures of  $\text{SnO}_2$  and  $\text{SnO}$  (Figure 1) which, however, did not bias our measurement of  $B_0$  and the first pressure derivative,  $B_0'$ . This is because X-ray diffraction (XRD) provides the unit cell volume (equivalent to the specific volume of a completely densified material) for each of the compounds independently. The unit cell parameter of  $\gamma\text{-Sn}_3\text{N}_4$  at ambient conditions was measured using a standard diffractometer (Cu  $\text{K}\alpha_1$  radiation).

The EOS of  $\gamma$ -Sn<sub>3</sub>N<sub>4</sub> was measured on compression in a diamond anvil cell (DAC) with anvils having culets of 350  $\mu\text{m}$  in diameter. A platelet of the powder sample of about 50  $\mu\text{m}$  in diameter and 10  $\mu\text{m}$  in thickness (obtained by a preliminary compaction between the diamond anvils) was placed in the 120  $\mu\text{m}$  hole in the pre-indented rhenium gasket of about 50  $\mu\text{m}$  in thickness and embedded in condensed argon, which solidifies above 1.2 GPa [28,29]. Use of the argon pressure medium provided *quasi*-hydrostatic pressure conditions up to 50 GPa because the uniaxial stress in solid argon does not exceed, similar to solid neon,  $\sim 1\%$  of the pressure value [30]. The pressure was determined from the earlier reported  $V(P)$  of crystalline argon [29]. The unit cell volumes of both  $\gamma$ -Sn<sub>3</sub>N<sub>4</sub> and argon were obtained from the angle-dispersive powder XRD patterns generated using a monochromatic X-ray synchrotron beam (wavelength 0.37380(4)  $\text{\AA}$ ) collimated to 20  $\mu\text{m}$  (beamline ID27, ESRF, Grenoble, France). The two-dimensional XRD patterns were collected with an image-plate detector MAR345 and converted to 1D-diffractograms (Figure 1) using the Fit2D software [31].

First-principles calculations of the stress-strain properties of the spinel nitrides and of a variety of dense C<sub>3</sub>N<sub>4</sub> polymorphs were performed using density-functional theory (DFT). We employed the Perdew-Burke-Ernzerhof (PBE) generalized gradient approximation (GGA) exchange-correlation functional [32], the improved Troullier-Martins norm-conserving pseudopotentials for core electrons and numerical atomic orbital basis set for valence electrons within the Kohn-Sham formalism of DFT [33]. Periodicity was enforced with a real-space mesh cut-off set to 200 Ry. Geometry optimizations were completed when the magnitude of forces on each atom is less than 0.008 eV/ $\text{\AA}$ . All calculations were performed using the SIESTA simulation package [34].

For calculations of the elastic stress tensors of cubic crystals, we introduced a uniaxial strain  $\varepsilon_{xx}$  from 0.5 % to 5 % in 0.5% increments.  $C_{11}$  and  $C_{12}$  were then calculated from the resulting stress tensor  $\sigma_{xx}$ . While introducing shear strain by shifting the lattice angle  $\varphi$  from  $-6^\circ$  to  $+6^\circ$ , a pure shear strain  $\varepsilon_{xy}$  was produced and  $C_{44}$  was calculated from the resulting stress tensor  $\sigma_{xy}$ . To derive shear moduli for isotropic polycrystalline samples of the spinel nitrides,  $\gamma$ -M<sub>3</sub>N<sub>4</sub> where M=Si, Ge, Sn, from our calculated  $C_{ij}$ , we used the Voigt-Reuss-Hill approach known to provide values (labeled as  $G_{0H}$ ) close to experiment. In the case of C<sub>3</sub>N<sub>4</sub> polymorphs we calculated shear moduli using the Voigt approach (labeled as  $G_{0V}$ ) which gives the upper limit for  $G_0$  and, thus, provides the upper limit for  $H_n$  when a  $H_n(G_0)$  systematics is applied.

### III. RESULTS AND DISCUSSION

The cubic lattice parameter of  $\gamma$ -Sn<sub>3</sub>N<sub>4</sub> at ambient conditions we measured to be  $a_0=9.033(6)$   $\text{\AA}$ , in agreement with the value  $a_0=9.037(3)$   $\text{\AA}$  reported in the original paper on its synthesis [22]. The latter value, measured with a higher precision, was used in our data treatment. XRD patterns of the sample compressed in a DAC up to 50 GPa showed ten or more diffraction peaks perfectly matching the spinel structure of  $\gamma$ -Sn<sub>3</sub>N<sub>4</sub> (Figure 1) and up to five peaks of crystalline argon. From these patterns we determined the unit cell volume of  $\gamma$ -Sn<sub>3</sub>N<sub>4</sub> as a function of pressure  $V(P)$  (Figure 2) which showed a smooth decrease with pressure. No signs of a phase transition could be recognized. The third-order Birch-Murnaghan EOS was fitted to the experimental data points  $V(P)/V_0$  and  $B_0 = 158(11)$  GPa and  $B' = 5.4(1.1)$  were obtained (Table I). A fit with  $B_0'$  fixed at 4 resulted in a slightly

higher  $B_0=174(3)$  GPa. The  $B_0$  values we obtained for  $\gamma$ - $\text{Sn}_3\text{N}_4$  are  $\sim 17\%$  higher than the only earlier reported experimental  $B_0=149$  GPa (with fixed  $B_0'=4$ ) [27] but still below the earlier predicted ones [17,35,36] (Table I). The significant discrepancy with the previous measurement we explain by use of a nonhydrostatic pressure medium and the short pressure range. Our experimental  $B_0=158$  GPa significantly surpasses the earlier theoretical values calculated both using the LDA [35] and GGA [27,36] DFT approaches. In contrast, it perfectly agrees with our calculated  $B_0=169$  GPa.

Applying our experimental  $B_0$  and the earlier reported  $E_r=167(36)$  from the nanoindentation tests of a densified sample [17], we derived  $G_0$  of  $\gamma$ - $\text{Sn}_3\text{N}_4$  to be  $67(20)$  GPa (if  $B_0=158$  GPa) or  $65(19)$  GPa (if  $B_0=174$  GPa) (Table I). The given error margins are mostly due to the uncertainty in  $E_r$  [17]. Surprisingly, a similar value of  $G_0=64$  GPa was derived in Ref. [17] using the same  $E_r$  and a higher theoretical value of  $B_0=186$  GPa. This indicates a weak influence of the approach on the bulk modulus uncertainty in contrast to uncertainties or errors in  $E_r$  [24]. Our experimental  $G_0=67(20)$  GPa agrees, within the experimental uncertainty, with our theoretical  $G_0=88$  GPa derived from the calculated  $C_{ij}$  (see Table S-I in *Supplementary information*) applying the Voigt-Reuss-Hill (VRH) approach (Table I). Because we are not aware of any other calculation of  $C_{ij}$  or  $G_0$  of  $\gamma$ - $\text{Sn}_3\text{N}_4$ , a direct comparison with other theoretical methods is not possible. However, we could recognise that the elastic anisotropy of  $\gamma$ - $\text{Sn}_3\text{N}_4$  of  $A=2 \cdot C_{44}/(C_{11}-C_{12})=2.1$  is very similar to  $A=1.9-2.0$  we have derived from the earlier calculated  $C_{ij}$  of  $\gamma$ - $\text{Ge}_3\text{N}_4$  and  $\gamma$ - $\text{Si}_3\text{N}_4$  [37,38].

In order to further validate our theoretical approach, we calculated  $C_{ij}$  moduli of  $\gamma$ - $\text{Si}_3\text{N}_4$  and  $\gamma$ - $\text{Ge}_3\text{N}_4$ , as well as their  $B_0$  and  $G_0$  using the VRH approach and compared with the existing reliable measurements (Table I). In the case of  $\gamma$ - $\text{Ge}_3\text{N}_4$  only one experimental  $B_0=295(5)$  from  $V(P)$  was reported [19] and used in the unique nanoindentation work to derive  $G_0=124$  GPa applying  $E_r=275$  GPa [17,24]. This experimental  $B_0$  is remarkably higher than our theoretical  $B_0=227$  GPa (by  $\sim 25\%$ ) as well as the earlier one of  $B_0=242$  GPa calculated using the LDA approach [38] (Table I). In contrast, our calculated  $G_0=168$  GPa and the earlier LDA result [38] outperform (also by  $\sim 25\%$ ) the experimental  $G_0=124$  GPa [17,24]. On the other hand, applying our calculated  $B_0$  and  $G_0$  for  $\gamma$ - $\text{Ge}_3\text{N}_4$  we obtain a theoretical value of  $E_r=308$  GPa which is only  $\sim 10\%$  above the measured one. This indicates a need of an independent measurement of  $B_0$  and  $G_0$  of  $\gamma$ - $\text{Ge}_3\text{N}_4$  and verification of a low porosity of samples used in the nanoindentation tests. It should be mentioned here that we did not consider for  $\gamma$ - $\text{Ge}_3\text{N}_4$ , and below for  $\gamma$ - $\text{Si}_3\text{N}_4$ , multiple first-principles calculations where only bulk moduli were reported because our work is focused on  $G_0$  values needed to predict the hardness of  $\gamma$ - $\text{M}_3\text{N}_4$  and of hypothetical dense polymorphs of  $\text{C}_3\text{N}_4$ .

Despite a large number of measurements of  $B_0$  for  $\gamma$ - $\text{Si}_3\text{N}_4$ , only one very recent work reports both hardness and shear modulus (Tables I and II) for one and the same isotropic densified polycrystalline sample [23]. In this work both  $G_0=248(1)$  GPa and  $B_0=303(4)$  GPa were obtained using Brillouin light spectroscopy (BLS). This  $B_0$  is slightly higher than  $B_0=290(5)$  GPa determined from the high-pressure EOS measurements [25]. In contrast, in the earlier nanoindentation tests  $E_r$  was measured and  $G_{0p}=148(16)$  derived for a sample with an unknown porosity [25]. Even though this  $G_{0p}$  is significantly lower than that of the dense sample, it was also considered here as an example of a porous sample. This could be done because above we have recognized that  $G_0$  derived from  $E_r$  depends only weakly on

the bulk modulus.  $B_0$  and  $G_0$  of  $\gamma$ - $\text{Si}_3\text{N}_4$  obtained from the BLS measurements [23] agree very well with our present first-principles calculations which give  $B_0=285$  GPa and  $G_{0H}=237$  GPa (Table I). This agreement supports further the reliability of our calculation approach thus permitting its extension to  $\text{C}_3\text{N}_4$  polymorphs. Previous theoretical works where all  $C_{ij}$  or the pairs  $B_0$  and  $G_0$  of  $\gamma$ - $\text{Si}_3\text{N}_4$  were calculated [37-39] show also a reasonable agreement with the most recent measurements [23] (Table I). However, these calculations were not validated for  $\gamma$ - $\text{Sn}_3\text{N}_4$  or  $\gamma$ - $\text{Ge}_3\text{N}_4$  and were not extended to dense  $\text{C}_3\text{N}_4$  polymorphs.

In order to predict the hardness of dense  $\text{C}_3\text{N}_4$  phases proposed in the literature, we first verified the empirical linear relation between  $G_0$  (Table 1) and the hardness of the above considered dense spinel phases  $\gamma$ - $\text{M}_3\text{N}_4$  where  $\text{M}=\text{Si}$ ,  $\text{Ge}$ , or  $\text{Sn}$  (Figure 3). In particular, we compared hardness values obtained in nanoindentation tests,  $H_n$ , (Table II) for the same samples for which elastic moduli, especially  $E_r$  or  $G_0$ , were measured simultaneously using the same technique [17,25] or BLS [23] (Figure 3). Moreover, we selected, if available,  $H_n$  values obtained for similar loads in order to exclude the biasing due to the indentation size effect e.g.[23]: If we consider only the  $H_n$  measured for  $\gamma$ - $\text{M}_3\text{N}_4$  with small indentation loads of 3-5 mN (Table II) then the linear relation between  $G_0$  and  $H_n$  is described by the empirical equation  $H_n(G_0)=0.240(26)\cdot G_0$  (Figure 3). Taking into account of the  $H_n$  for a transparent  $\gamma$ - $\text{Si}_3\text{N}_4$  sample measured with a higher load of  $\sim 200$  mN insignificantly changes the empirical relation (due to the indentation size effect) to  $H_n(G_0)=0.212(8)\cdot G_0$  (Figure 3). Interestingly, the result for one and the same sample of hexagonal  $\alpha$ - $\text{Si}_3\text{N}_4$  (S.G.  $P3_1/c$ ) reporting  $G_0 = 144(7)$  GPa (derived from the measured  $E_r = 289(7)$  GPa using the earlier reported  $B_0=248(10)$  GPa [40]) and  $H_n=34.5(2.0)$  GPa, obtained for a high indentation load of 700 mN [41], is located between these two lines. A good agreement with our steeper  $H_n(G_0)$  empirical equation follows from the nanoindentation testing of a polycrystalline sample of  $\beta$ - $\text{Si}_3\text{N}_4$  (S.G.  $P6_3/m$ ) where  $H_n = 26(4)$  GPa was measured for the indentation load of 7.9 mN [42] and  $G_0 = 101(21)$  GPa could be derived from  $E_r$  obtained for the same sample and the same load using the earlier reported  $B_0 = 259$  GPa [43].

Finally, we performed calculations of  $C_{ij}$  of the five hypothetical dense carbon nitrides proposed in the literature [1,3,14]. In particular, we examined  $\alpha$ - $\text{C}_3\text{N}_4$ ,  $\beta$ - $\text{C}_3\text{N}_4$  and  $\gamma$ - $\text{C}_3\text{N}_4$  having the same structures as the three well-known phases of  $\text{Si}_3\text{N}_4$ , namely, hexagonal low-pressure  $\alpha$ - and  $\beta$ -phases (see above) and the  $\gamma$ -phase having the cubic spinel structure (S.G.  $Fd\bar{3}m$ ). We also considered two other cubic structures, namely the defect zink-blende  $zb$ - $\text{C}_3\text{N}_4$  (S.G.  $P43m$ ) and the Willemite-II type  $c$ - $\text{C}_3\text{N}_4$  (S.G.  $I\bar{4}3d$ ). The calculated  $C_{ij}$  moduli of these five phases of  $\text{C}_3\text{N}_4$  are given in Table S-I of the *Supplementary Information*, and compared with earlier predictions. From our  $C_{ij}$  we calculated  $G_0$  moduli of the  $\text{C}_3\text{N}_4$  polymorphs using the Voigt approach,  $G_{0V}$  (Table I), and applied to them the empirical systematics  $H_n(G_0)$  developed above for the spinel nitrides  $\gamma$ - $\text{M}_3\text{N}_4$  (Figure 3). The vertical bars in Figure 3 thus indicate our predicted upper limits for  $H_n$  for the considered here dense  $\text{C}_3\text{N}_4$  phases (Table II). Furthermore, because  $H_n$  obtained for low loads in nanoindentation tests provide, in general, upper bounds for the standardized (micro)hardness values, such as  $H_V$ , due to the indentation size effect, our empirical  $H_n$  values for the  $\text{C}_3\text{N}_4$  phases are well above their (micro)hardness. The highest shear modulus of  $G_{0V} = 386$  GPa was calculated for  $zb$ - $\text{C}_3\text{N}_4$  (Table I). This value surpasses only insignificantly that of  $c$ - $\text{C}_3\text{N}_4$  calculated to be  $G_{0V} = 380$  GPa. From our empirical



$H_n(G_0)$  systematics (Figure 3), we predict  $H_n \leq 93(10)$  GPa for  $zb\text{-C}_3\text{N}_4$ , and thus for any other  $\text{C}_3\text{N}_4$  phases considered here. This nanoindentation hardness value is well below the Vickers hardness of 100 GPa [2] adopted in the literature for diamond which shear modulus was measured to be  $G_0=536$  GPa [44]. If we apply our systematics to the  $G_0$  value of diamond then the nanoindentation hardness of diamond should be  $H_n=122(7)$  GPa, consistent with the concept of the indentation size effect. Therefore, we can conclude that all hypothetical dense  $\text{C}_3\text{N}_4$  proposed in the literature will exhibit a hardness below that of diamond.

#### IV. CONCLUSIONS

We performed accurate measurements of  $B_0$  and  $B'$  of  $\gamma\text{-Sn}_3\text{N}_4$  via quasi-hydrostatic compression of a polycrystalline sample in a DAC up to 50 GPa. Combining earlier nanoindentation data for a densified sample with our  $B_0$  value we determined the shear modulus of  $\gamma\text{-Sn}_3\text{N}_4$ . Selecting and analyzing the existing reliable data for the hardness and shear moduli of all known spinel nitrides,  $\gamma\text{-M}_3\text{N}_4$ , we revealed a linear correlation of their nanohardness and shear moduli. We performed calculations of elastic moduli of the same compounds and found good agreement with the trustworthiest experimental data. The confirmed reliability of our calculation approach permitted its extension to the proposed in the literature dense polymorphs of hypothetical  $\text{C}_3\text{N}_4$ . Applying the developed above correlation of hardness and shear moduli of  $\gamma\text{-M}_3\text{N}_4$  we predicted the hardness of several dense  $\text{C}_3\text{N}_4$  phases and found that all of them are well below the hardness of diamond. Although the prediction of a superhard  $\text{C}_3\text{N}_4$  was disproved here via combined experimental and computational approaches, full experimental verification of the formation of a dense polymorph of this compound remains an exciting task for the field of solid-state chemistry.

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**TABLE I.** Parameters of the equation of state ( $B_0, B_0'$ ) and  $G_0$  of  $\gamma$ - $\text{Sn}_3\text{N}_4$  measured here in comparison with the experimental data (Exp) for all existing  $\gamma$ - $\text{M}_3\text{N}_4$  (where M = Si, Ge, Sn) and calculated moduli (Calc) of  $\gamma$ - $\text{M}_3\text{N}_4$  and of the five dense polymorphs of  $\text{C}_3\text{N}_4$  obtained using either the Generalized Gradient Approximation (GGA), the Local Density Approximation (LDA), or the Molecular Simulation Force Field (MSFF) approach. **Our results are highlighted by bold fonts.**

Material S.G.		$B_0$ (GPa)	$B_0'$	$G_0$ (GPa)	Comment/Ref
$\gamma$ - $\text{Sn}_3\text{N}_4$ $Fd\bar{3}m$	Exp	<b>158(11)</b> <b>174(3)</b> 149(1.2)	<b>5.4(1.1)</b> <b>4 (fixed)</b> 4 (fixed)	<b>67(20)<sup>a</sup></b> <b>65(19)<sup>a</sup></b>	[27]
	Calc	<b>169</b> 187 186 204	4.34 4.53 4.98	<b>88<sup>b</sup></b> 64 <sup>c</sup>	<b>GGA</b> GGA [36] LDA [17] LDA [35]
$\gamma$ - $\text{Ge}_3\text{N}_4$ $Fd\bar{3}m$	Exp	295(5)	3.8(2)	124(17)	[19] & [17,24]
	Calc	<b>227</b> 242		<b>168<sup>b</sup></b> 176 <sup>b</sup>	<b>GGA</b> LDA [38]
$\gamma$ - $\text{Si}_3\text{N}_4$ $Fd\bar{3}m$	Exp	290(5) 303(4)	4.9(6)	148(16) <sup>d</sup> 248(1)	[25] [23]
	Calc	<b>285</b> 273 284 311 305 312		<b>237<sup>b</sup></b> 252 <sup>b</sup> 265 <sup>b</sup> 258 <sup>b</sup> 261	<b>GGA</b> GGA [37] GGA [38] LDA [37] LDA [38] [39]
$\alpha$ - $\text{C}_3\text{N}_4$ $P3_1/c$	Calc	<b>378</b> 388 191		<b>359<sup>e</sup></b> 342 <sup>e</sup> 300 <sup>e</sup>	<b>GGA</b> GGA [16] MSFF [13]
$\beta$ - $\text{C}_3\text{N}_4$ $P6_3/m$	Calc	<b>411</b> 408 252 433		<b>303<sup>e</sup></b> 322 <sup>e</sup> 268 <sup>e</sup> 326 <sup>e</sup>	<b>GGA</b> GGA [16] MSFF [13] LDA [14]
$\gamma$ - $\text{C}_3\text{N}_4$ $Fd\bar{3}m$	Calc	<b>433</b> 442		<b>351<sup>e</sup></b> 352 <sup>e</sup>	<b>GGA</b> LDA [15]
$c$ - $\text{C}_3\text{N}_4$ $I\bar{4}3d$	Calc	<b>441</b> 441 487 496		<b>380<sup>e</sup></b> 400 <sup>e</sup> 393 <sup>e</sup> 319 <sup>e</sup>	<b>GGA</b> GGA [16] LDA [15] LDA [3]
$zb$ - $\text{C}_3\text{N}_4$ $P\bar{4}3m$ ( $P\bar{4}2m$ )	Calc	<b>395</b>		<b>386<sup>e</sup></b>	<b>GGA</b>
		436 422		409 <sup>e</sup> 397 <sup>e</sup>	LDA [15] LDA [14]
		390		388 <sup>e</sup>	GGA [16]

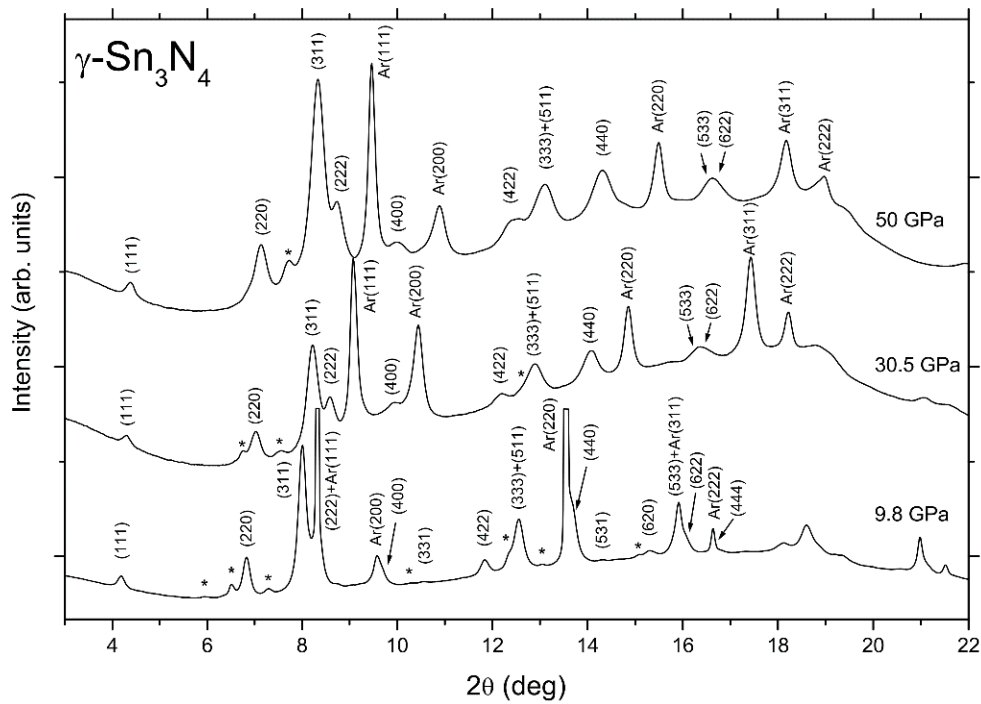
<sup>a</sup> derived using our  $B_0$  and  $E_r$  measured for a densified sample in Ref. [17]; <sup>b</sup> shear modulus derived from theoretical  $C_{ij}$  using the Voigt-Reuss-

Hill- or related approximation,  $G_{0H}$  [37,38]; <sup>c</sup> derived from experimental  $E_r$  using the theoretical  $B_0$  reported in the same work; <sup>d</sup> derived using the  $E_r$  measured for a porous sample; <sup>e</sup> shear modulus derived from theoretical  $C_{ij}$  using the Voigt approximation,  $G_{0H}$ .

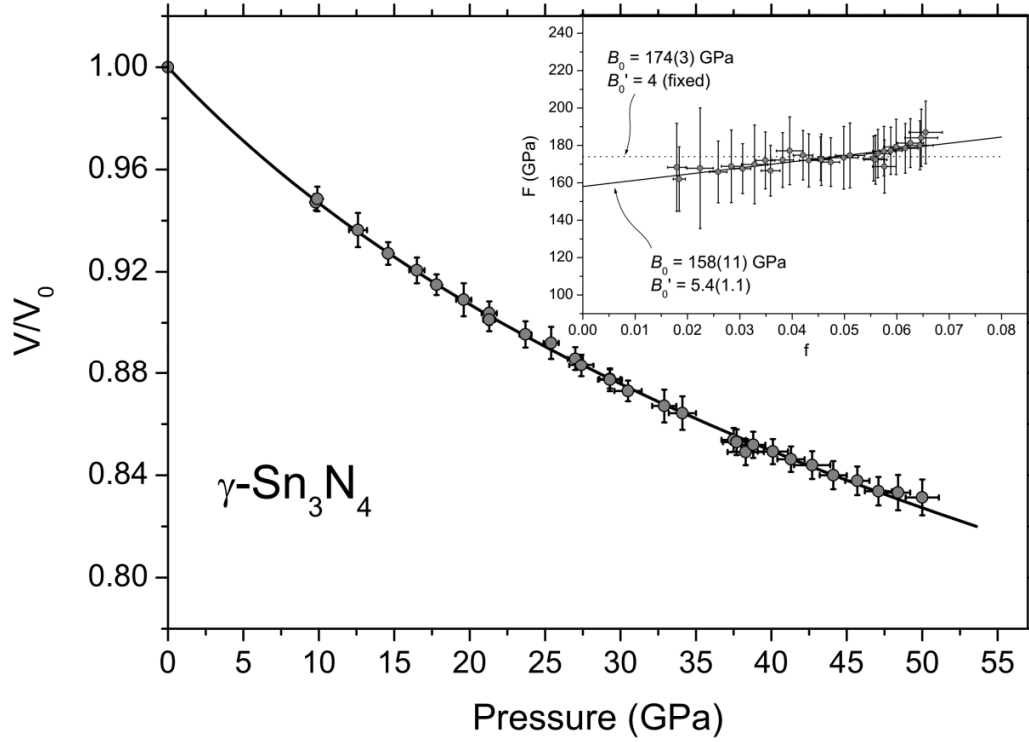
**TABLE II.** Nanoindentation hardness ( $H_n$ ) of the dense  $C_3N_4$  phases predicted in the present work using the experimental data for  $\gamma$ - $M_3N_4$  (where  $M = Si, Ge, Sn$ ). The earlier measured and predicted  $H_V$  values of  $\gamma$ - $M_3N_4$  and of the dense  $C_3N_4$  phases, respectively, are also shown for comparison.

Material S.G.	$H_V$ GPa	Ref.	$H_n$ GPa	Load mN	Ref.
$\gamma$ - $Sn_3N_4$ $Fd\bar{3}m$	11(2)	[17]	13(5)	3	[17]
$\gamma$ - $Ge_3N_4$ $Fd\bar{3}m$	28(5)	[17]	31(6)	5	[17]
$\gamma$ - $Si_3N_4$ $Fd\bar{3}m$	35(2) 34.9(7)	[45] [23]	36(5) <sup>a</sup> 52(2)	5 200	[25] [23]
$\alpha$ - $C_3N_4$ $P3_1/c$	84 <sup>b</sup>	[5]	76-86 <sup>b</sup>	3- 200	<i>This work</i>
$\beta$ - $C_3N_4$ $P6_3/m$	85 <sup>b</sup>		64-73 <sup>b</sup>	3- 200	
$\gamma$ - $C_3N_4$ $Fd\bar{3}m$	59 <sup>b</sup>		74-84 <sup>b</sup>	3- 200	
$c$ - $C_3N_4$ $I\bar{4}3d$	91 <sup>b</sup>		81-91 <sup>b</sup>	3- 200	
$zb$ - $C_3N_4$ $P\bar{4}3m$	89 <sup>b</sup>		82-93 <sup>b</sup>	3- 200	

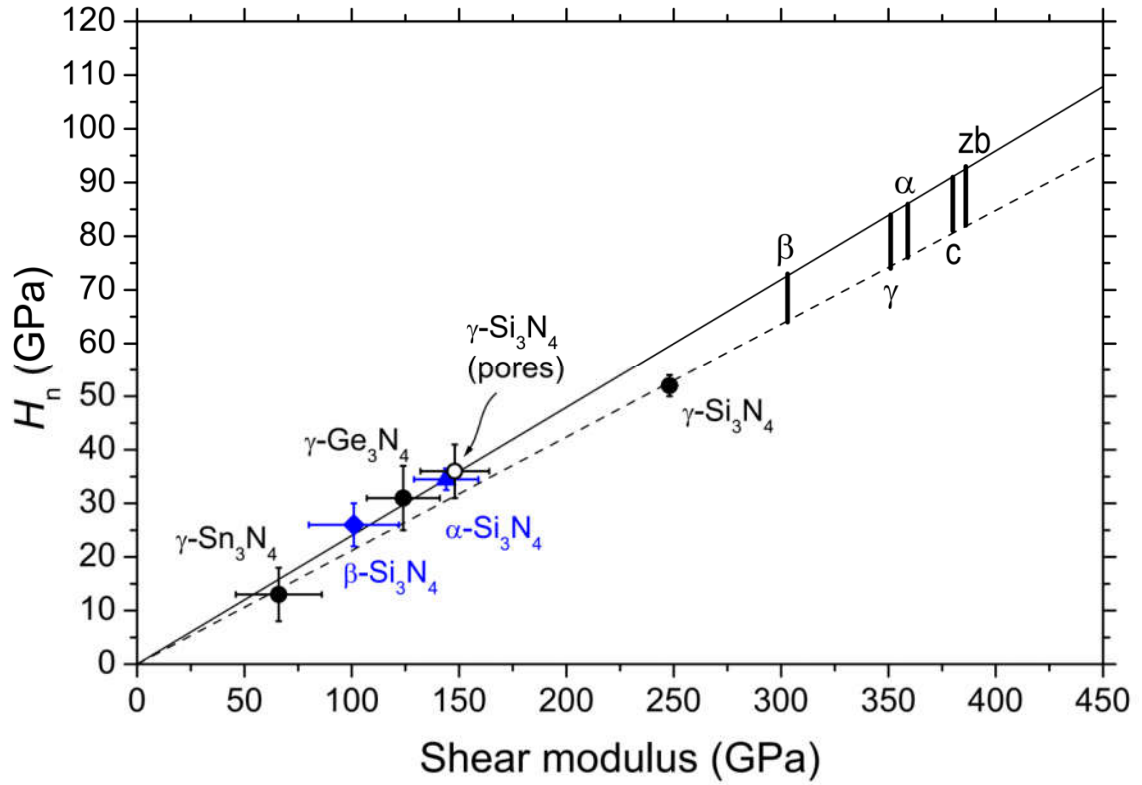
<sup>a</sup> porous sample, <sup>b</sup> predicted



**FIG. 1.** Examples of powder diffraction patterns of  $\gamma\text{-Sn}_3\text{N}_4$  embedded in a quasi-hydrostatic argon pressure medium, which is similarly soft as solid neon [30], and compressed in a DAC to high pressures. Asterisks indicate minor admixtures of  $\text{SnO}_2$  and  $\text{SnO}$  present in the starting material [22].



**FIG. 2.** Relative volume,  $V/V_0$ , of  $\gamma\text{-Sn}_3\text{N}_4$  as a function of pressure: Solid symbols represent the experimental data-points while the solid line is the least-squares fit of the third order Birch-Murnaghan EOS to the data. **Insert:** compression data of  $\gamma\text{-Sn}_3\text{N}_4$  in terms of normalized pressure  $F = P/[3f(1+2f)^{5/2}]$  and Eulerian strain  $f = [(V/V_0)^{-2/3} - 1]/2$ . The solid and dotted lines represent the least-squares fits of the third- and second-order Birch-Murnaghan EOS, respectively.



**FIG. 3.** Empirical relation between the nanohardness  $H_n$  and shear modulus  $G_0$  of  $\gamma$ - $M_3N_4$  (where  $M = \text{Si}, \text{Ge}, \text{Sn}$ ) where only experimental data-pairs obtained for one and the same sample were considered. Solid and open symbols indicate experimental  $H_n$ - $G_0$  data measured for densified and porous samples, respectively (Tables I and II). Error bars for the experimental  $G_0$  of  $\gamma$ - $\text{Ge}_3\text{N}_4$  and  $\gamma$ - $\text{Si}_3\text{N}_4$  were calculated here using the literature data. The solid line indicates a linear fit to the data obtained for  $\gamma$ - $M_3N_4$  with low loads of 3-5 mN and the dashed one to all available data for  $\gamma$ - $M_3N_4$  (Table II). The  $H_n$ - $G_0$  data for  $\alpha$ - $\text{Si}_3\text{N}_4$  and  $\beta$ - $\text{Si}_3\text{N}_4$  (blue solid triangle and rhombus, respectively) are shown for comparison. Vertical solid bars span the possible  $H_n$  values of dense  $\text{C}_3\text{N}_4$  phases we predict applying our calculated  $G_0$  values and the empirical linear relation  $H_n(G_0)$  for  $\gamma$ - $M_3N_4$ .

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