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Hidden drifts in turbulence

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Abstract

The paper discusses the concept of hidden drifts in two-dimensional turbulence. They are ordered components of the trajectories that average to zero and do not produce direct transport. Their effects appear in the evolution of the turbulence as generation of a special type of fluxes, which consist of average motion of positive and negative fluctuations in opposite directions.

The transport by continuous movements is the generalization of the Brownian diffusion for smooth velocity fields that correspond to finite correlation lengths λ. The main characteristic of the trajectories determined by such fields consists of the existence of quasi-coherence that lasts for short time intervals of the order of the time of flight $\tau_{fl} = \lambda/V$, where V is the amplitude of the stochastic velocity. In special cases, as the two-dimensional incompressible turbulence, the coherence can last for much larger time essentially because the trajectories are trapped in the correlated zone.

The turbulence that is dominantly two dimensional has a self-organizing character [1]. It consists of the generation of quasi-coherent structure (vortices). This property appears at the basic level of particle trajectories. They are random sequences of trapping or eddying events and long jumps. The trapping process [2]-[4] strongly modifies the diffusion coefficients and leads to strong nonlinear effects. The long jumps are random while the motion during the trapping events has a high degree of order.

The ordered components of the motion are represented by quasi-coherent structures and flows, but also by more subtle effects: the hidden drifts (HDs). The paper discuss this special type of order and two important effects that the HDs determine on turbulence evolution.

We show that the HDs provide a mechanism of generation of zonal flow modes in plasma turbulence. These modes that are oscillations along the density and temperature gradients contribute to the saturation of turbulence amplitude and to a strong decrease of the transport [5], [6]. The physical explanation that is generally accepted is based on the Reynold stress produced by potential cell tilting. Zonal flow modes and their effects of improving the confinement is presently a very active research topic ([7]-[9]).

The relaxation of turbulent states in two dimensional ideal fluids is characterized by the separation of the vorticity according to its sign and by the inverse cascade that corresponds to energy flow towards large scales [10]-[12]. The physical bases of these processes is shown to be connected to the HDs.

1 Hidden drifts

Tracer trajectories in two-dimensional stochastic velocity fields are obtained from the equation

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}(\mathbf{x},t) + V_d \mathbf{e}_2, \quad \mathbf{v}(\mathbf{x},t) = -\nabla\phi(\mathbf{x},t) \times \mathbf{e}_3, \tag{1}$$

where $\mathbf{e}_1$, $\mathbf{e}_2$ are the unit vectors in the plane of the motion $\mathbf{x} = (x_1, x_2)$, $\mathbf{e}_3$ is perpendicular on this plane and V_d is a constant average velocity. The stochastic velocity field $\mathbf{v}(\mathbf{x},t)$, determined by the potential (or stream function) $\phi(\mathbf{x},t)$, has zero divergence. The Hamiltonian structure of Eq. (1) is the origin of the order that characterizes the two-dimensional incompressible turbulence. The velocity is tangent to the contour lines of the total potential $\phi_t(\mathbf{x},t) = \phi(\mathbf{x},t) + x_1 V_d$ at any moment, and, in the case of time independent potentials, the trajectories remain on these lines. They reflect the space structure of the potential.

The statistical characteristics of the trajectories are determined using the decorrelation trajectory method (DTM, [13], [4], [14]). This is a semi-analytic method that is in agreement with the statistical consequences of the invariance of the potential. The main idea of this approach is to determine the Lagrangian averages not on the whole set of trajectories but to group together trajectories that are similar, to average on them and then to perform averages of these averages. Similar trajectories are obtained by imposing supplementary initial conditions besides the necessary one $\mathbf{x}(\mathbf{0}) = \mathbf{0}$. The supplementary initial conditions are the values of the potential and of the velocity in the starting point of the trajectory

$$S : \phi^0 = \phi(\mathbf{0},0), \mathbf{v}^0 = \mathbf{v}(\mathbf{0},0). \tag{2}$$

They define a set of subensembles S of the realizations of the potential. Conditional averages lead to space-time dependent average potential $\Phi^S(\mathbf{x},t)$ and velocity $\mathbf{V}^S(\mathbf{x},t)$ in each subensemble S and to simple trajectories that represent average particle motion: the decorrelation trajectories (DTs) $\mathbf{X}(t;\phi^0,\mathbf{v}^0)$. The DTs are the main ingredient of DTM. They are smooth, simple trajectories determined from an equation with the same structure as Eq. (1)

$$\frac{d\mathbf{X}}{dt}=\mathbf{V}^S(\mathbf{X},t)+V_d\mathbf{e}_2,\quad \mathbf{V}^S(\mathbf{X},t)=-\nabla\Phi^S(\mathbf{X},t)\times\mathbf{e}_3, \tag{3}$$

but with the stochastic potential replaced by $\Phi^S(\mathbf{x},t)$. The average potential is determined by the Eulerian correlation (EC) of the stochastic fields, $E(\mathbf{x},t)$

$$\Phi^S(\mathbf{X},t)=\phi^0\frac{E(\mathbf{X},t)}{E(\mathbf{0},0)}-v_1^0\frac{E_2(\mathbf{X},t)}{E_{22}(\mathbf{0},0)}+v_2^0\frac{E_1(\mathbf{X},t)}{E_{11}(\mathbf{0},0)}, \tag{4}$$

where E_{ij} are space derivatives $E_i(\mathbf{x},t)=\partial E(\mathbf{x},t)/\partial x_i$. The DTs are much different from particle trajectories as they saturate after the decorrelation time. They represent the average evolution of the particles through the correlated zone of the potential and describe the decorrelation process.

The statistics of the trajectories is represented by weighted averages along the DTs. The weighting factors are the probabilities of the subensembles that correspond to the DTs, $P\left(\phi^0,v_1^0,v_2^0\right)$. The DTM essentially determines the correlations of the trajectories with the quantities that define the subensembles and the probability of the small scale displacements (until decorrelation).

The hidden drifts (HDs) are ordered displacements that average to zero and do not drive flows. They appear in the presence of an average velocity $\mathbf{V}_d$ and they are perpendicular on $\mathbf{V}_d$.

The HDs are found by analyzing the average displacements conditioned by the initial value of the potential $\langle x_i(t)\rangle_{\phi^0}$, which are evaluated in the frame of the DTM by

$$\langle x_i(t)\rangle_{\phi^0}=\int_{-\infty}^{\infty}dv_1^0dv_2^0X(t;\phi^0,v_i^0)P\left(v_1^0,v_2^0\right). \tag{5}$$

The conditional displacements are zero for $V_d=0$, but finite values of the component along x_1 axis, $\langle x_1(t)\rangle_{\phi^0}$, yield in the presence of an average velocity V_d directed along x_2 axis. A typical example is presented in Figure 1.a, which shows that $\langle x_1(0)\rangle_{\phi^0}=0$ at $t=0$ and that it increases and eventually saturates. Moreover, the sign of $\langle x_1(t)\rangle_{\phi^0}$ is the same as the sign of the initial potential.

The average displacements conditioned by the sign of the potential are defined by

$$\begin{aligned}\langle x_i(t)\rangle_+ &= \int_0^{\infty}d\phi^0\,\langle x_i(t)\rangle_{\phi^0}P(\phi^0), \\ \langle x_i(t)\rangle_- &= \int_{-\infty}^{0}d\phi^0\,\langle x_i(t)\rangle_{\phi^0}P(\phi^0)\end{aligned} \tag{6}$$

They are, as seen in Figure 1.b, time dependent functions that saturate. The sum $\langle x_i(t)\rangle_+ + \langle x_i(t)\rangle_- = 0$, which shows that there is no average motion.

Any process of decorrelation of the trajectories from the contour lines of the potential leads to conditional average velocities. The trajectories $\mathbf{x}(t)$ for $t >> \tau_d$ consists in this case of a time sequence of segments of duration τ_d that are statistically independent. Both the mean square displacement $\Delta^2(\tau) = \left\langle (\mathbf{x}(t_i+\tau) - \mathbf{x}(t_i))^2 \right\rangle$ and the average $\delta(\tau) = \langle \mathbf{x}(t_i+\tau) - \mathbf{x}(t_i) \rangle$ saturate for $\tau > \tau_d$ at values Δ and δ that represent the steps of the random and ordered walks, respectively. They lead at large time to diffusive and direct transport (or average velocity).

The displacements (6) saturate to δ_+ and δ_-, that represent ordered steps in opposite direction. They do not lead to direct transport (average velocity), but to a pair of conditional average velocities with opposite orientations

$$V_+ = \frac{\langle x_i(\tau_d)\rangle_+}{\tau_d} = \frac{\delta_+}{\tau_d}, \; V_- = \frac{\langle x_i(\tau_d)\rangle_-}{\tau_d} = \frac{\delta_-}{\tau_d}.$$

These are the hidden drifts (HDs).

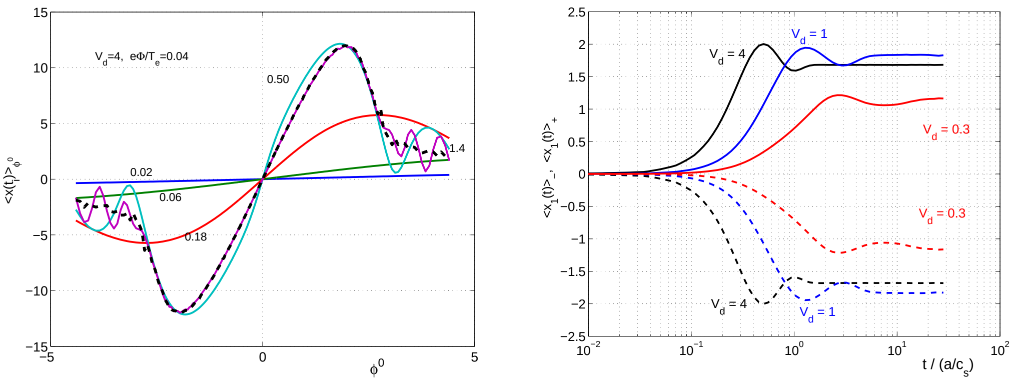

Figure 1: The average displacements that start from ϕ^0 () as functions of ϕ^0 for $V_d = 4$ (left panel) and the conditional average displacements $\langle x_i(t)\rangle_-$, $\langle x_i(t)\rangle_+$ as functions of time for the values of the average velocity V_d that label the curves. The other parameters are $e\Phi/T_e = 0.04$, $\lambda_x = 5$, $\lambda_y = 2$, $k_0 = 1$.

We note that in 2-dimensional velocity fields with $\nabla \cdot \overrightarrow{v} = 0$, the probability of the Lagrangian velocity is time independent (equal to the probability of the Eulerian velocity). In particular, the average Lagrangian velocity has to be equal to the Eulerian average velocity, which is zero along x_1 direction. This means that it is not possible to exist an average Lagrangian velocity along the x_1 axis. However, the ordered motion as symmetrical positive and negative flows that compensate is not forbidden by the zero-divergence condition.

The physical explanation of the HDs is based on two elements.

Firstly, the average velocity $V_d\mathbf{e}_2$ modifies the structure of the potential. The external contour lines that correspond to small values of the potential, situated in the side of the potential cells where the ExB velocity is parallel to $V_d\mathbf{e}_2$, are opened. The field lines with $\phi^0 < \phi_{\text{lim}}(V_d)$, where $\phi_{\text{lim}}(V_d)$ is an increasing function of V_d, form bands of open lines that extends along the average velocity, while oscillating in the perpendicular direction. The inner contour lines with $\phi^0 > \phi_{\text{lim}}(V_d)$ remain closed, but they are elongated towards the side where the ExB drift is antiparallel to the average velocity. The potential cells with opposite signs approach in this zone. They form pairs of positive and negative potential cells that are surrounded by bands of opened lines. The pair has the negative cell located at larger x_1 than the positive one. Thus, the average velocity $V_d\mathbf{e}_2$ reduces the size of the potential cells and produces a polarization effect.

Secondly, the modifications of the structure of the potential contour lines by V_d yields ordered perpendicular displacements and HDs. In the closed potential cells, the velocity along the contour lines is statistically non homogeneous due to $V_d\mathbf{e}_2$, which leads to total velocity V_t that is enhanced on half of the cells and reduced on the opposite half. The trajectories are concentrated on the zones with small V_t, which leads to an average displacement that has the sign of the potential. The opened trajectories also contribute to the conditional displacements (6), but due to a different reason. The invariance of the potential along the opened trajectories with initial ϕ^0, $\phi(\mathbf{x}(t))+x_1(t)V_d = \phi^0$, shows that the average displacement is $\langle x_1(t)\rangle = \phi^0/V_d$ because the average potential on these trajectories is zero. Thus, the opened lines determine an average velocity with the same sign as the potential. It is larger than in the case of closed contour lines.

It is interesting to note that HDs are not related to the quasi-coherent structures that can be generated in turbulence. HDs exists even when the structures are absent. They are the effects of the space correlation of the total potential that includes the average velocity V_d. The HDs reflect the order of the contour lines of the potential. The existing condition for the HDs involves the decorrelation time τ_d. It has to be long enough such that the trajectories stick on the contour lines for lengths of the order of the correlation length.

HDs do not yield a direct influence on transport because the contribution of the ordered steps δ is implicitly included in the mean square displacement that defines the step Δ of the random motion. However, they represent a reservoir for direct transport. Perturbations produced by other components of the motion that determine a weak compressibility can perturb the equilibrium of the HDs leading to an average velocity of the test particles.

The main effect of the HDs consists of the change of turbulence parameters, which eventually determines the modification of the transport. We discuss here the effects of the HDs in two cases: drift turbulence in collisionless (hot) plasmas confined in strong magnetic fields and the relaxation of turbulent states in two-dimensional ideal (inviscid) fluid turbulence. These physical systems are described by similar evolution equations that represent the advection along characteristics obtained from Eq. (1). They are formally linear, but there are in both cases nonlinear constraints that strongly influence the evolution. The nature of the nonlinearity and the physical significance of the advected fields are completely different in the two systems. We show that these differences lead to completely different effects of the hidden drifts, although they are the same in the two systems.

2 Hidden drifts in turbulent plasmas

We consider a plasma confined by an uniform magnetic field B taken along the $\mathbf{e}_3$ axis of a rectangular system of coordinates. A density gradient (along $\mathbf{e}_1$ axis, with characteristic length L_n) makes plasma unstable (see e.g. [15], [16]). Drift wave instability that is produced by electron kinetic effect and ion polarization drift velocity is analyzed here using the test mode approach.

The main idea of the studies of test modes on turbulent plasmas is to separate the distribution function into an approximate equilibrium f_0 and the response h to the small perturbation (with wave number components k_i and frequency ω), $\delta\phi\exp(ik_ix_i - i\omega t)$, that adds to the background potential $\phi_b(\mathbf{x}, t)$. The function $f_0(\mathbf{x}, t)$ is solution of the approximate evolution equation obtained by neglecting the small terms. The latter are not important at small time, but only at large times when the small effects accumulate. The function h can be linearized in the small perturbation $\delta\phi$. The solution of the dispersion relation yields the frequencies $\omega(\mathbf{k})$ that can be supported by the system and the tendency of amplification or damping given by the growth rate $\gamma(\mathbf{k})$, the imaginary part of $\omega(\mathbf{k})$. These quantities, which include the effects of the small terms neglected in the evolution of f_0, provide the short time change of the spectrum of the background turbulence.

Drift modes have small parallel wave numbers $k_3 \ll k_1, k_2$ and small frequency such that $v_{Ti} \ll \omega/|k_3| \ll v_{Te}$, where v_{Ti}, v_{Te} are the thermal velocities of the ions and electrons, respectively. The fast parallel motion of the electrons leads to the adiabatic approximate equilibrium of the electrons and to the response to the perturbation $\delta\phi$

$$\delta n^e = n_0(x)\frac{e\delta\phi}{T_e}\left(1 + i\sqrt{\frac{\pi}{2}}\frac{\omega - k_y V_*}{|k_z|\, v_{Te}}\right) \tag{7}$$

that does not depend on the background turbulence [15]. $V_* = \rho_s c_s / L_n$ is the electron diamagnetic velocity (where $\rho_s = c_s/\Omega_i$, $c_s = \sqrt{T_e/m_i}$ is the sound velocity and $\Omega_i = eB/m_i$ is the ion cyclotron frequency).

The drift kinetic equation for the ions is

$$\partial_t f^i - \frac{\nabla\phi_b \times \mathbf{b}}{B} \cdot \nabla f^i + f^i \nabla \cdot \mathbf{u}_p = 0, \tag{8}$$

where the parallel motion is neglected due to their small velocity. The ion polarization drift $\mathbf{u}_p$

$$\mathbf{u}_p = \frac{1}{B\Omega_i}\partial_t \mathbf{E}_\perp. \tag{9}$$

is also small, but it was maintained in Eq. (8) because it determines a compressibility effect that makes drift waves unstable. The last term is neglected when determining the approximate equilibrium solution in turbulent plasma. Using the constraint of neutrality one obtains

$$f_0^i = n_0(x) F_M^i \exp\left(\frac{e\phi_b(\mathbf{x} - \mathbf{V}_* t)}{T_e}\right). \tag{10}$$

The short time equilibrium solution shows that the background potential determines density fluctuations that are proportional with the potential ($\delta n/n \cong e\phi_b/T_e$, since the potential energy is small $e\phi_b \ll T_e$). This means that the average velocity $V_d \mathbf{e}_2$ produces the polarization of both potential and density cells. In the presence of a decorrelation process (finite τ_d), HDs of density appear, which move the density fluctuations in opposite directions according to their sign.

The linearized equation in the perturbation $\delta\phi$ and ion response h is

$$\partial_t h - \frac{\nabla\phi_b \times \mathbf{b}}{B} \cdot \nabla h = -i n_0(x) F_M^i \frac{e\delta\phi}{T_e}\left(k_y V_* - \omega \rho_s^2 k_\perp^2\right), \tag{11}$$

where the compressibility term $h\nabla \cdot \mathbf{u}_p$ was neglected for simplicity (see an analysis of its effects in [17], [14]). The formal solution is

$$h(\mathbf{x}, v, t) = -n_0(x) F_M^i \frac{e\delta\phi}{T_e}\left(k_y V_* - \omega \rho_s^2 k_\perp^2\right) \overline{\Pi}^i, \tag{12}$$

where the propagator is

$$\overline{\Pi}^{i} = i\int_{-\infty}^{t} d\tau \; M(\tau) \; \exp\left[-i\omega\left(\tau - t\right)\right], \tag{13}$$

$$M(\tau) \equiv \left\langle \exp\left[\frac{e\phi_b(\mathbf{x}(\tau))}{T_e} + i\mathbf{k}\cdot\left(\mathbf{x}(\tau) - \mathbf{x}\right)\right]\right\rangle, \tag{14}$$

and the average $\langle\rangle$ is on the trajectories obtained from Eq. (1) in the background potential ϕ_b.

The HDs lead to the correlation of the potential and the displacements in the propagator (14). Introducing the formal expression of the displacement as the integral of the velocity and using the stationarity of the background turbulence and the fact that the "initial" condition for a trajectory (1) can be any of its points, one obtains

$$\langle\phi_b(\mathbf{x}(\tau)) \; x_1(\tau)\rangle = \langle\phi_b(\mathbf{0}) \; x_1(\tau)\rangle \tag{15}$$

The decorrelation of the trajectories from the contour lines of the potential leads to an average velocity determined by HDs

$$V_{HD} = \frac{e}{T_e}\frac{\langle\phi_b(\mathbf{0}) \; x_1(\tau_d)\rangle}{\tau_d} \tag{16}$$

This average velocity is due to a sequence of ordered steps produced by the polarization of the potential cells. They are symmetrical and have zero average, but their sign is the same as the sign of the density fluctuations, which means that the peaks of the density move in the positive direction while the holes move in the opposite direction. Both contribute to a positive flux of density.

The normalized average velocity determined by the HDs V_{HD}/V_{*0} is shown in Figure 2 as function of the decorrelation time τ_d for several values of the background turbulence amplitude. $V_{*0} = \rho_s c_s/a$ and a is plasma size. V_{HD} is small at small τ_d, it has a maximum and eventually decays as $1/\tau_d$ because the average in Eq. (16) saturates.

The average in Eq. (14) that is over the background potential and the stochastic trajectories is calculated taking into account the trajectory structures as in [17]. One obtains for the case of weak turbulence that corresponds to small fraction of trapped ions

$$M(\tau) \equiv A\left\langle \exp\left[ik_1 V_{HD}(\tau - t) - \frac{1}{2}k_i^2 D_i(\tau - t)\right]\right\rangle, \tag{17}$$

$$A = \exp\left(\frac{1}{2}\left(\frac{e\Phi_b}{T_e}\right)^2 - \frac{1}{2}k_i^2 s_i^2\right), \tag{18}$$

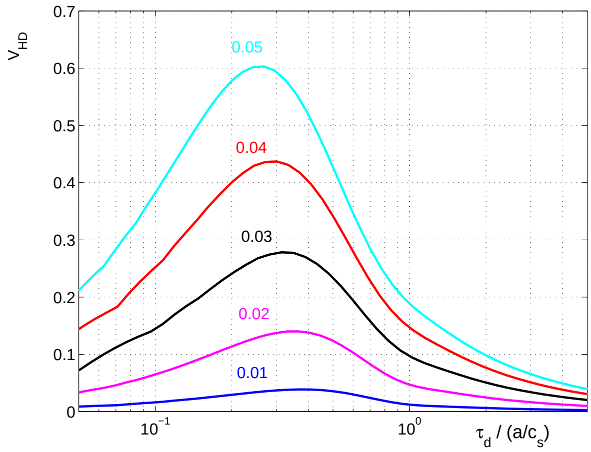

Figure 2: The radial velocity V_{HD} determined by the HDs as function of the decorrelation time for the values of the amplitude that label the curves and $V_d = 4$. The other parameters are as in Fig.1.

where $s_i,\ i = 1, 2$ are the average sizes of the trajectory structure along the x_1 and x_2 axis. The HDs determine a new term in Eq. (17) through the correlation of the displacements with the background potential (15).

The dispersion relation for test modes in turbulent plasma obtained from the quasineutrality condition $\delta n^e = \delta n^i$,

$$-\left(k_y V_{*e} - \omega \rho_s^2 k_\perp^2\right) \overline{\Pi}^i = 1 + i\sqrt{\frac{\pi}{2}} \frac{\omega - k_y V_{*e}}{|k_z|\, v_{Te}}, \tag{19}$$

has the solution

$$\omega = \frac{A k_2 V_* - k_1 V_{HD}}{1 + A k_\perp^2} \tag{20}$$

$$\gamma = cA \frac{\left(k_2 V_* + k_1 V_{HD}\right)\left(k_2 V_* (1 + A k_\perp^2 - A) + k_1 V_{HD} k_\perp^2\right)}{\left(1 + A k_\perp^2\right)^3} - k_i^2 D_i, \tag{21}$$

where $c = \sqrt{\pi/2}/\, |k_3|\, v_{T_e}$.

The velocity determined by the HDs modifies both the frequency and the growth rate. It destabilizes a new type of modes that have $k_2 = 0$, the zonal flow modes. Their growth rate

$$\gamma_{zfm} = \sqrt{\frac{\pi}{2}} \frac{A}{|k_3|\, v_{T_e}} \frac{k_1^4 V_{HD}^2}{\left(1 + A k_1^2\right)^3}$$

that is determined by V_{HD} (16) increases with the increase of the amplitude Φ of the background turbulence.

We have found here a different mechanism for the generation of the zonal flow modes. It is associated to the HDs, which combined with the adiabatic fluctuations of the density determine an average radial velocity that drive potential oscillations along x_1 axis.

3 Hidden drifts in fluid turbulence

Turbulence relaxation in ideal fluids is described by the Euler equation with stochastic initial condition

$$\partial_t \omega + \mathbf{v} \cdot \nabla \omega = 0, \tag{22}$$

$$\mathbf{v} = -\nabla \phi \times \mathbf{e}_z + V_d \mathbf{e}_2, \quad \omega = \triangle \phi, \tag{23}$$

where ω is the vorticity, ϕ is the stream function and $\mathbf{v}$ is the fluid velocity. Thus vorticity elements are advected by the velocity field and the vorticity is conserved along the trajectories (1). The nonlinearity of the process is determined by the relations between $\mathbf{v}$ and ω, which show that vorticity is an active field.

The motion of vorticity elements is determined by Eq. (1), which leads to HDs. Fluxes of vorticity perpendicular to the average velocity $V_d \mathbf{e}_2$ are expected to appear because ϕ and ω are correlated

$$\langle \phi(\mathbf{x},t) \omega(\mathbf{x},t) \rangle = \langle \phi(\mathbf{x},t) \triangle \phi(\mathbf{x},t) \rangle = \triangle E(\mathbf{0}, 0), \tag{24}$$

and it has negative values.

In order to determine the displacements $x_1(t)$ conditioned by the initial vorticity $\langle x_1(t) \rangle_{\omega^0}$, the DTM is extended by introducing a supplementary condition $\omega(\mathbf{0}, 0) = \omega^0$ in the definition (2) of the subensembles S. This modifies the probability of the subensembles that becomes

$$P(\phi^0, \mathbf{v}^0, \omega^0) = P(\mathbf{v}^0) P(\omega^0) \sqrt{\frac{2\pi}{D_{\phi\omega}}} \exp\left(-\frac{(\phi^0 - \Phi^\omega)^2}{2 D_{\phi\omega}} \right), \tag{25}$$

$$\Phi^\omega = \omega^0 \triangle E(\mathbf{X}, t) / \triangle^2 E(\mathbf{0}, 0), \; D_{\phi\omega} = E(0) - (\triangle E(\mathbf{0}, 0))^2 / \triangle^2 E(\mathbf{0}, 0), \tag{26}$$

where Φ^ω is the average potential conditioned by the vorticity and $D_{\phi\omega}$ is the dispersion of the potential in the subensemble. The potential and the vorticity are not independent in a point $\mathbf{x}$ (as are the velocity components). The existence of the correlation (24) determines the displacements of the probability of the potential at Φ^ω that has the sign opposite that of ω^0, and

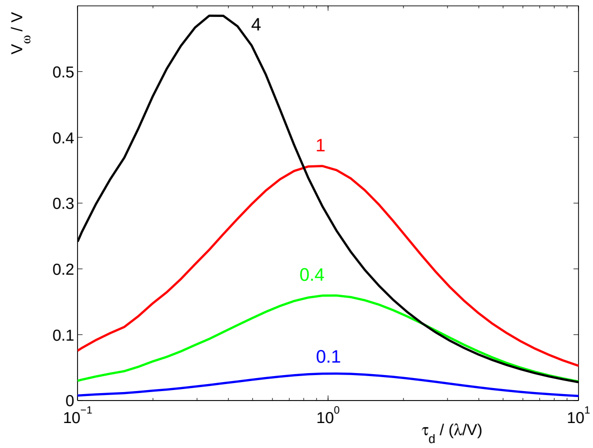

Figure 3: The velocity of separation of the positive and negative vorticity for the values of V_d/V that label the curves.

the decrease of its width. the integration over the initial vorticity has to be introduced in the calculation of the statistical quantities, which leads to the strong increase of the number of DTs (3) and of the typical run time of the code (up to 2-3 hours on a PC).

The correlation (24) also modifies the subensemble average potential Φ^S in Eq. (4) by adding the term Φ^ω, which strongly changes the shapes of the DTs. However, the diffusion coefficient is not much modified.

We have found that the sign of the average displacements $\langle x_1(t)\rangle_{\omega^0}$ is opposite to the sign of ω^0 and that symmetrical HDs appear as in the case of the initial potential. The negative vorticity yields a positive average velocity $V_- > 0$. These HDs conditioned by the vorticity sign directly lead to a negative vorticity flux. The positive and the negative vorticity fluctuations separate with the average velocity $V_\omega = V_- - V_+$. This velocity is represented in Figure 3 as function of the normalized decorrelation time for several values of V_d/V, where $V = \Phi/\lambda$ is the amplitude of the stochastic velocity of the fluid. The EC of the stream function is $E(\mathbf{x}, t) = \Phi^2 \exp\left(-(\mathbf{x}/\lambda)^2/2 - t/\tau_d\right)$.

The average velocity $V_d\mathbf{e}_2$ can be produced by a large scale vortex. The flux of the vorticity fluctuations determined be the HDs correspond to the approach of the large vortex by the small vortices with the same sign and to the departure of those with opposite sign. This process contributes to the inverse cascade of energy that characterizes two-dimensional fluid turbulence.

4 Conclusions

The hidden drifts (HDs) are found in the statistics of the trajectories (1) as organized components of the motion oriented perpendicular to the average velocity $V_d\mathbf{e}_3$. They compensate exactly and do not yield radial average displacements of the trajectories. However, the HDs can drive fluxes of density and vorticity fluctuations of a special type.

The approximate adiabatic response that characterizes drift type turbulence in confined plasmas determines the correlation of the sign of the density fluctuations with the sign of the HDs. Positive density fluctuations move in a direction while the negative fluctuations move in the opposite direction, determining a net flux that is two times larger that produced by the conditional average velocity V_+. This radial flux determine a mechanism of generation of zonal flow modes, which are very important for turbulence saturation and for the decrease of the transport.

In turbulent fluids, the correlation of the vorticity ω with the stream function ϕ determines a similar flux of vorticity that consists of displacements of the positive and negative fluctuations in opposite directions. The process determines the separation of the vorticity according to its sign and the attraction of the small scale vortices by a large scale vortex of the same sign, which corresponds to the inverse cascade of energy. These are fundamental processes in the relaxation of turbulent states in two-dimensional ideal fluid.

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