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ExB mean flows in finite ion temperature plasmas

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The impact of ion pressure dynamics on $\boldsymbol{E} \times \boldsymbol{B}$ mean flows is investigated. Three stresses in addition to the Reynolds stress are shown to modify the $\boldsymbol{E} \times \boldsymbol{B}$ mean flow. These additional terms in the stress tensor all require ion pressure fluctuations. Quasi-linear analysis indicates that these additional stresses are as important as the standard Reynolds stress and hence must be taken into account in analysis of $\boldsymbol{E} \times \boldsymbol{B}$ mean flows.

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I. INTRODUCTION

Sheared mean flows are neccesary for the formation of transport barriers[38] in magnetically confined plasmas. Transport barriers are always accompanied by a sheared radial electric field E_r and an associated $\boldsymbol{E} \times \boldsymbol{B}$ mean flow[38], which in combination with flows along the magnetic field quench cross-field turbulent transport through decorrelation of turbulent eddies[3,7]. Several mechanisms capable of driving mean flows have been suggested[8], but it is unclear whether the observed mean flows are due to a single motive force or whether they are a result of an interplay between many mechanisms.

A particular mechanism for mean flow generation relies on the Reynolds stress tensor[29]. It couples fluctuations and mean flows and hence renders turbulence driven mean flows possible. In order to distinguish turbulence driven mean flows from equilibrium flows, turbulence driven mean flows are often called zonal flows. Both types of mean flows can suppress turbulence. In the fluid description the Reynolds stress originates from the advection non-linearity in the fluid momentum equation. By separating the velocity field into mean and fluctuating parts: $\boldsymbol{u} = \langle \boldsymbol{u} \rangle + \tilde{\boldsymbol{u}}$ and averaging the momentum equation one gets for an incompressible flow $\nabla \cdot \boldsymbol{u} = 0$:

$$\frac{\partial \langle \boldsymbol{u} \rangle}{\partial t} + \nabla \cdot \langle \tilde{\boldsymbol{u}} \tilde{\boldsymbol{u}} \rangle + \nabla \cdot (\langle \boldsymbol{u} \rangle \langle \boldsymbol{u} \rangle) = \mathcal{L}, \tag{1}$$

where $\mathcal{L}$ represents forces, sinks, and sources. The average operation $\langle \cdot \rangle$ is unspecified here but is usually either a time-average, a flux surface average, or both. The Reynolds stress tensor $\langle \tilde{\boldsymbol{u}} \tilde{\boldsymbol{u}} \rangle$ can inhibit as well as enhance mean flows, but in strongly magnetized plasmas the approximate two-dimensional character of turbulence implies that energy is preferably transfered from smaller to larger scales[10,13,33]. The energy transfer is between the kinetic energy of fluctuations and the kinetic energy of the mean flow. Reynolds stress driven mean flows cannot directly tap free energy but relies on conversion of free energy into fluctuating energy by other mechanisms[31]. On closed magnetic surfaces in strongly magnetized fusion plasmas, the mean convective term $\nabla \cdot (\langle \boldsymbol{u} \rangle \langle \boldsymbol{u} \rangle)$ is usually negligible because gradients of the mean flow are to a good approximation perpendicular to the mean flow itself.

When a plasma is subject to a strong confining magnetic field the dynamics is strongly anisotropic. Charged particles are approximately trapped on magnetic field lines along which they flow unhindered. When studying mean flows it is therefore convenient to apply models where this anisotropy is exploited a priori. The strong confining magnetic field implies that the magnetic dipole moment associated with the Larmor orbits of charged particles around magnetic field lines is an adiabatic invariant[1]. The invariance can be used in a dynamical reduction of the governing equations which lowers the computational costs by orders of magnitudes[5]. This is exploited in turbulence models which normally only consider dynamics on time scales longer than the inverse ion gyrofrequency[5,16,17]. In the resulting equations the strong anisotropy imposed by the strong magnetic field appears explicitly. Velocities are split into perpendicular and parallel parts. In the direction perpendicular to the magnetic field advection is in most cases dominated by the $\boldsymbol{E}\times\boldsymbol{B}$-drift: $\boldsymbol{u}_E = \boldsymbol{E}\times\boldsymbol{B}/B^2$. Other perpendicular fluxes associated with particle drifts such as the grad-B, curvature, and polarization drifts are inferior in comparison to the $\boldsymbol{E}\times\boldsymbol{B}$ flux, but are essential for the turbulence because the corresponding currents are of equal importance in the quasi-neutrality constraint $\nabla\cdot\boldsymbol{J} = 0$. In drift fluid models, which are used in this paper, the grad-B and curvature drifts and the magnetization current are contained in the diamagnetic drift $\boldsymbol{u}_D$[12]. As in gyrokinetic[5] and gyrofluid models[17], the diamagnetic and $\boldsymbol{E}\times\boldsymbol{B}$ drifts are assumed to be of the same order of magnitude. However, since advection of all fluid fields by the diamagnetic drift cancels in all moment equations[37], the diamagnetic flow is not responsible for transport over macroscopic distances. Therefore, it is only the mean $\boldsymbol{E}\times\boldsymbol{B}$ flow which is relevant in studies of decorrelation of turbulent eddies by perpendicular mean flows.

In this paper we investigate how ion pressure dynamics influences $\boldsymbol{E}\times\boldsymbol{B}$ mean flows. Reynolds stress driven mean flows have been studied extensively[9] and studies including ion pressure dynamics are numerous[6,11,18,25,30,32,33]. A common feature of these studies is that they do not consider "pure" mean flows but rather mean flows with multiple components. In gyrokinetic and gyrofluid treatments[6,11,18,25,32], the results concern mean flows, actually mean gyro-center momentum densities, in gyro-center coordinate space. Gyro-center space is a mathematical construction which provides tractable equations describing the dynamics down to gyro-radius length scales. The use of gyro-center coordinates is motivated by the notorious tedious expressions[18,36] associated with gyro-radius length scale dynamics entering models expressed in standard coordinates. However, gyro-center coordinates are by construction not only functions of position and velocity but also of the electromagnetic potentials. To illustrate this point we express the zeroth order gyro-center moment, the gyro-center density N, in terms of physical quantities such as the particle density n, the ion scalar pressure p_i, and the electric potential ϕ. In a quasi-neutral plasma $n_i = n_e$ we get[21,22]

$$N_i = n_i - \nabla_\perp^2 \left(\frac{p_i}{2 m_i \Omega_i^2} \right) - \nabla \cdot \left(\frac{n_i}{B \Omega_i} \nabla_\perp \phi \right) \tag{2}$$

where only terms to second order in $k_\perp \rho_i$ are retained. Here, $k_\perp$ is a characteristic inverse gradient length scale,ρ_i is the ion gyro-radius, p_i is the ion pressure, and $\Omega_i = q_i B/m_i$ is the ion gyro frequency, where q_i and m_i are the ion charge and mass, respectively. The perpendicular projection of the gradient operator is defined as $\nabla_\perp = -\hat{\boldsymbol{b}} \times (\hat{\boldsymbol{b}} \times \nabla)$, where $\hat{\boldsymbol{b}} = \boldsymbol{B}/B$ is a unit vector parallel to the magnetic field $\boldsymbol{B}$. Results formulated in gyro-center coordinates are therefore only directly relevant for the dynamics of gyrocenters, which is of course highly relevant, but in order to translate these results to measureable quantities the results must be transformed to well-known physical variables, a process which is tedious[18,35]. In low-frequency fluid models[16] another but related issue appears. Here, the dominant perpendicular drifts are the fluid $\boldsymbol{E} \times \boldsymbol{B}$ and diamagnetic velocity fields. In previous works[25,27,30,33] only the momentum and mean flow equations for the combined $\boldsymbol{E} \times \boldsymbol{B}$ and ion diamagnetic flow were considered. This approach is problematic because the mean flow then includes the diamagnetic flow, which is not responsible for transport on the macroscopic length scale.

The main objective of this paper is to investigate the $\boldsymbol{E} \times \boldsymbol{B}$ mean flow and hence to disentangle the $\boldsymbol{E} \times \boldsymbol{B}$ and ion diamagnetic parts. Considering the pure $\boldsymbol{E} \times \boldsymbol{B}$ mean flow significantly complicates the governing equations. We have therefore deliberately chosen a paradigmatic, electrostatic drift fluid model in two-dimensional slab geometry, where dynamics along the magnetic field has been omitted. The model is presented in Sec. II. Even in this simplistic setup we show in Sec. III that the $\boldsymbol{E} \times \boldsymbol{B}$ mean flow can be modified by four terms: i) The pure $\boldsymbol{E} \times \boldsymbol{B}$ Reynolds stress $\langle \tilde{\boldsymbol{u}}_E \tilde{\boldsymbol{u}}_E \rangle$ and ii) a diamagnetic Reynolds stress[33] proportional to $\langle u_y \partial_y p_i \rangle$", where the u_y denotes the "azimuthal" component of the $\boldsymbol{E} \times \boldsymbol{B}$ drift. iii) We also show that $\boldsymbol{E} \times \boldsymbol{B}$ mean flows may be driven by a term proportional to $\langle \xi p_i u_x \rangle$ in the stress tensor which is only finite when the magnetic field is inhomogeneous $\xi = 1/R \neq 0$, where R is the major radius. iv) Lastly we demonstrate the existence of a component proportional to $2/3 \langle \xi p_i \partial_y p_e \rangle$ of the stress tensor, which does not require $\boldsymbol{E} \times \boldsymbol{B}$ drift fluctuations. The corresponding energy transfer terms, also commonly denoted production terms, are analyzed and conditions for enhancement and attenuation of $\boldsymbol{E} \times \boldsymbol{B}$ mean flows for the individual energy transfer channels are determined. Next, in Sec. IV we proceed with a quasi-linear analysis which reveals that that none of the four mean flow generation mechanisms are negligible. Lastly, our results are summarized and discussed in Sec. V.

II. MODEL

The fundamental assumptions in drift fluid models[15,16,20,30] are that the pressure force and the Lorentz force balance and that the dynamics evolve on a time-scale whose characteristic frequency is much smaller than the ion gyro-frequency $\omega/\Omega_i \ll 1$. These models are well-suited for studies of low-frequency turbulence in strongly magnetized plasmas particularly in the edge and scrape-off layer regions. An advantage of drift fluids, and all models which are based on the drift approximation, is that algebraic expressions for the perpendicular part of the odd fluid moment equations, e.g. momentum density, can be derived using an iterative procedure. Neglecting collisional effects the zeroth and first order perpendicular drifts, omitting species labels, are given as:

$$\boldsymbol{u}_{\perp,0} = \boldsymbol{u}_E + \boldsymbol{u}_D = \frac{\hat{\boldsymbol{b}} \times \nabla\phi}{B} + \frac{\hat{\boldsymbol{b}} \times \nabla p}{qnB}, \tag{3}$$

$$\boldsymbol{u}_{\perp,1} = \boldsymbol{u}_p + \boldsymbol{u}_\pi = \frac{1}{\Omega}\hat{\boldsymbol{b}} \times \frac{d}{dt}\boldsymbol{u} + \frac{\hat{\boldsymbol{b}} \times \nabla \cdot \pi}{qnB}, \tag{4}$$

respectively. Here, ϕ is the electrostatic potential and $\Omega = qB/m$ is the gyro-frequency, where q denotes charge, m is the species mass, and B is the magnetic field amplitude. The zeroth order drifts are the familiar $\boldsymbol{E} \times \boldsymbol{B}$-drift $\boldsymbol{u}_E$ and the diamagnetic drift $\boldsymbol{u}_D$. The first order drifts comprise of the polarization drift $\boldsymbol{u}_p$ and a gyroviscous drift $\boldsymbol{u}_\pi$ due to gyroviscosity entering the off-diagonal part of the pressure tensor π. The first order drifts are linear functions of mass and hence due to the electron ion mass ratio the first order electron drifts are neglected. The main effect of the gyroviscous drift is to cancel the advection of momentum by the diamagnetic drift. This cancellation is in the literature refered to as the *gyro-viscous cancellation*[2,4,16,36]. The polarization drift describes inertia. The first order drifts in $\boldsymbol{u}_{\perp,1}$ depend on the species mass, and hence only the ion drifts are retained. We are here concerned with studying the influence of a dynamically evolving ion pressure on the generation, sustainment, and damping of mean flows. For this purpose and for the convenience of exposition we neglect the time-evolution of the parallel momentum and consider only the drift fluid vorticity equation and the electron and ion pressure equations for an electrostatic, quasi-neutral ($n \equiv n_i = n_e$) plasma[20,24,28]:

$$\nabla \cdot (n\boldsymbol{u}_{p_i}) + \nabla \cdot (n\boldsymbol{u}_{\pi i}) + \nabla \cdot \big(n(\boldsymbol{u}_{Di} - \boldsymbol{u}_{De})\big) = \Lambda_w, \tag{5a}$$

$$\frac{3}{2}\frac{\partial}{\partial t}p_i + \frac{3}{2}\nabla \cdot \big(p_i[\boldsymbol{u}_E + \boldsymbol{u}_{Di} + \boldsymbol{u}_{p_i} + \boldsymbol{u}_\pi]\big) + p_i \nabla \cdot [\boldsymbol{u}_E + \boldsymbol{u}_{Di} + \boldsymbol{u}_{p_i} + \boldsymbol{u}_\pi] + \nabla_\perp \cdot \boldsymbol{q}_i^* = \Lambda_{p_i}, \tag{5b}$$

$$\frac{3}{2}\frac{\partial}{\partial t}p_e + \frac{3}{2}\nabla \cdot \big(p_e[\boldsymbol{u}_E + \boldsymbol{u}_{De}]\big) + p_e \nabla \cdot [\boldsymbol{u}_E + \boldsymbol{u}_{De}] + \nabla_\perp \cdot \boldsymbol{q}_e^* = \Lambda_{p_e}, \tag{5c}$$

where the diamagnetic heat flux is given as

$$\boldsymbol{q}_a^* = \frac{5}{2}p_a \frac{\hat{\boldsymbol{b}} \times \nabla T_a}{q_a B}. \tag{6}$$

The terms Λ_w, Λ_{p_i} and Λ_{p_e} on the right hand sides of Eqs. (5a)-(5c) represent, unspecified, parallel dynamics, collisional effects, and sources and sinks. The vorticity equation (5a) is derived from the quasi-neutrality constraint $\nabla \cdot \boldsymbol{J} = 0$. Notice that the polarization heat flux[26,36] is not kept in the pressure equation even though it enters with the same order as the polarization drift terms. However, we deliberately omit the polarization heat flux because it does not alter the governing equations for the mean flow and the corresponding kinetic energy. In the same spirit we reduce the model equations (5a)-(5c) further. The simplifications of the model equations aim at formulating an energy conserving, minimal model for investigations of the influence of ion pressure changes on the generation of mean flows. First, in the vorticity equation (5a) we invoke the thin-layer approximation which resembles the Boussinesq approximation[19] in neutral fluid dynamics. In this approximation the particle density entering the polarization and gyro-viscous fluxes is taken to be constant. Together with this assumption the first order ion drift $\boldsymbol{u}_{i\perp,1}$ must be simplified as

$$\nabla \cdot (n\boldsymbol{u}_{pi}) + \nabla \cdot (n\boldsymbol{u}_{\pi i}) \simeq -\nabla \cdot \left[\frac{n_0}{\Omega_0}\left(\frac{\partial}{\partial t} + \frac{B}{B_0}\boldsymbol{u}_E \cdot \nabla\right)\left(\frac{\nabla_\perp \phi}{B_0} + \frac{\nabla_\perp p_i}{q n_0 B_0}\right)\right] \tag{7}$$

in order to conserve energy[20]. This approximation is routinely invoked in reduced models to reduce the computational cost of solving the model equations. The electric potential and the ion pressure entering Eq. (7) represent the ion inertia when the $\boldsymbol{E} \times \boldsymbol{B}$ and diamagnetic drifts change in the comoving frame of reference. The vorticity equation is a current compression balance equation, but it is termed the vorticity equation because the compression of the ion polarization flux in equation (7) contains the magnetic field aligned $\boldsymbol{E}\times\boldsymbol{B}$ and ion diagmagnetic vorticities $\hat{\boldsymbol{b}}\cdot\nabla\times\boldsymbol{u}_{\perp,0i}$. The first order drifts $\boldsymbol{u}_{\perp,1i}$ entering the vorticity equation (5a) originate from the ion particle density equation. Equivalently the first order drifts must be retained in the ion pressure equation in order to conserve energy and eventually because they modify $\boldsymbol{E}\times\boldsymbol{B}$ mean flows. Also, it has been demonstrated that the correspondence between the drift fluid and gyrofluid ion pressure equations requires that the first order drifts are retained in the ion pressure equation[35]. Consistency requires that the thin-layer approximation and the subsequent simplification of the first order drifts in the vorticity equation must be invoked in ion pressure in exactly the same manner[20]. In the ion pressure equation we neglect the compression of the first order fluxes $\frac{3}{2}\nabla\cdot(p_i\boldsymbol{u}_{\perp,1i})$ because they do not alter the energy theorem and do not directly enter the equation governing the time evolution of the mean flow. In the pressure equations (5b) and (5c), advection by the diamagnetic drifts vanishes as a consequence of the diamagnetic cancellation

$$\frac{3}{2}\nabla\cdot(p\boldsymbol{u}_{Da})+p_a\nabla\cdot(\boldsymbol{u}_{Da})+\nabla\cdot\boldsymbol{q}^*_{\perp a}=\nabla\times\big(\frac{\hat{\boldsymbol{b}}}{q_aB}\big)\cdot\nabla(p_aT_a). \tag{8}$$

The resulting curvature terms are neglected in the pressure equations because they neither alter the mean flow energy equation nor the mean flow time-evolution equations. We take the magnetic field entering the $\boldsymbol{E}\times\boldsymbol{B}$ drift to a constant, and hence only retain the energy exchange term $p\nabla\cdot\boldsymbol{u}_E$ in the pressure equations (5b)-(5c). Lastly, we restrict the model to a 2D slab geometry (x,y,z) at the outboard midplane with the unit vector $\hat{\boldsymbol{z}}$ aligned with the inhomogeneous magnetic field $\boldsymbol{B}=B(x)\hat{\boldsymbol{z}}$. Periodic boundary conditions are invoked in the y-direction. It is convenient to express the model in Gyro-Bohm normalized units

$$\Omega_{i0}t\to t,\quad \frac{x}{\rho_s}\to x,\quad \frac{p_{e,i}}{p_{e0}}\to p_{e,i},\quad \frac{e\phi}{T_{e0}}\to\phi, \tag{9}$$

where $\Omega_{i0}=q_iB_0/m_i$ is the characteristic ion gyro frequency, $\rho_s=\sqrt{\frac{T_{e0}}{m_i}}$ is the hybrid thermal gyro-radius, n_0 and T_{e0} are characteristic particle density and electron temperature values. The normalized, minimal three-field model is given as:

$$\nabla\cdot\big(\frac{d}{dt}\nabla_\perp\phi^*\big)+\xi\frac{\partial}{\partial y}(p_e+p_i)=\Lambda_w, \tag{10a}$$

$$\frac{3}{2}\frac{d}{dt}p_i-p_i\xi\frac{\partial\phi}{\partial y}+p_i\xi\frac{\partial}{\partial y}(p_e+p_i)=\Lambda_{p_i}, \tag{10b}$$

$$\frac{3}{2}\frac{d}{dt}p_e-p_e\xi\frac{\partial\phi}{\partial y}=\Lambda_{p_e}, \tag{10c}$$

where $\xi = \frac{\rho_s}{R}$ is the curvature constant and R denotes the major radius. The advective derivatives are defined as

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \{\phi, \cdot\}, \tag{11}$$

where the $\boldsymbol{E} \times \boldsymbol{B}$-advection is written in terms of the anti-symmetric bracket

$$\{f, g\} = \frac{\partial f}{\partial x}\frac{\partial g}{\partial y} - \frac{\partial f}{\partial y}\frac{\partial g}{\partial x}, \tag{12}$$

and the modified potential is defined by

$$\phi^* = \phi + p_i. \tag{13}$$

A. Energy theorem

The conserved energy is derived in two steps. First, the electron and ion pressure equations (10c)-(10b) are integrated neglecting surface terms. Next the vorticity equation (10a) is multiplied by "$-\phi$" and integrated again neglecting surface terms. Adding the results we get

$$\frac{d}{dt}\int d\boldsymbol{x}\,\mathcal{E} = \int d\boldsymbol{x}\,S_\parallel, \tag{14}$$

where the energy density is given by

$$\mathcal{E} = \mathcal{E}_i + \mathcal{E}_e + \mathcal{E}^* = \frac{3}{2}[p_i + p_e] + \frac{|\nabla_\perp \phi^*|^2}{2}, \tag{15}$$

and

$$\mathcal{S}_\parallel = \Lambda_{p_i} + \Lambda_{p_e} - \phi^* \Lambda_w. \tag{16}$$

The energy density consists of the ion and electron thermal energy densities $\mathcal{E}_i$ and $\mathcal{E}_e$, respectively, and the "drift energy" density $\mathcal{E}^*$. The absence of the particle density n and the magnetic field in the drift energy is a consequence of the thin-layer approximation invoked in the vorticity equation (10a). The drift energy is a function of the modified potential ϕ^* and can be understood as the energy associated with the $\boldsymbol{E} \times \boldsymbol{B}$ and diamagnetic drifts, or alternatively as describing the finite Larmor radius (FLR) corrected $\boldsymbol{E} \times \boldsymbol{B}$ kinetic energy and FLR corrections to the ion thermal energy[23,35,40]. The time-evolutions of the individual parts of the integrated energy densities are given as

$$\frac{d}{dt}E^* = \frac{d}{dt}\int d\boldsymbol{x}\,\mathcal{E}^* = \int d\boldsymbol{x}\; -\xi[p_i + p_e]\frac{\partial\phi}{\partial y} + \xi p_i\frac{\partial p_e}{\partial y} - \phi^*\Lambda_w, \tag{17}$$

$$\frac{d}{dt}E_i = \frac{d}{dt}\int d\boldsymbol{x}\,\mathcal{E}_i = \int d\boldsymbol{x}\,\xi p_i\frac{\partial\phi}{\partial y} - \xi p_i\frac{\partial p_e}{\partial y} + \Lambda_{p_i}, \tag{18}$$

$$\frac{d}{dt}E_e = \frac{d}{dt}\int d\boldsymbol{x}\,\mathcal{E}_e = \int d\boldsymbol{x}\,\xi p_e\frac{\partial\phi}{\partial y} + \Lambda_{p_e}. \tag{19}$$

There are two types of energy transfer channels: i) the finite compression of the $\boldsymbol{E}\times\boldsymbol{B}$ drift[34], represented by the $\xi p_i\partial_y\phi$ and $\xi p_e\partial_y\phi$ terms, allow an *interchange* of thermal energy and kinetic energy. ii) The finite compression of the first order drifts are responsible for the second type of energy transfer channel. This effect is represented by the $\xi p_i\partial_y p_e$ terms.

III. MEAN FLOWS

In this section we analyze how ion pressure dynamics influences $\boldsymbol{E}\times\boldsymbol{B}$ mean flows in our two-dimensional interchange turbulence model presented in Sec. II. The analysis encompasses a derivation of a $\boldsymbol{E}\times\boldsymbol{B}$ mean flow equation and an analysis of energy transport between free (thermal) energy, fluctuations and mean quantities.

In this paper the averaging operation defining mean quantities is a spatial average in the periodic y-direction direction

$$\langle f\rangle = \frac{1}{L_y}\int_0^{L_y} dy\, f. \tag{20}$$

Here, f is an arbitrary function and L_y is the domain length in the y-direction. The fluctuating part is defined accordingly $\tilde{f} = f - \langle f\rangle$. Using the vorticity equation (10a) the time evolution[30,33] of the mean and fluctuating parts of the drift energy is obtained

$$\frac{d}{dt}E_0^* = \frac{d}{dt}\int d\boldsymbol{x}\,\frac{1}{2}|\frac{\partial\langle\phi^*\rangle}{\partial x}|^2 = \int d\boldsymbol{x}\; -\frac{\partial^2\langle\phi^*\rangle}{\partial x^2}\langle\frac{\partial\tilde{\phi}}{\partial y}\frac{\partial\tilde{\phi}^*}{\partial x}\rangle - \langle\phi^*\rangle\langle\Lambda_w\rangle, \tag{21}$$

$$\frac{d}{dt}\tilde{E}^* = \frac{d}{dt}\int d\boldsymbol{x}\,\frac{1}{2}|\nabla_\perp\tilde{\phi}^*|^2 = \int d\boldsymbol{x}\,\frac{\partial^2\langle\phi^*\rangle}{\partial x^2}\langle\frac{\partial\tilde{\phi}}{\partial y}\frac{\partial\tilde{\phi}^*}{\partial x}\rangle + \xi(p_e+p_i)\frac{\partial\phi^*}{\partial y} - \tilde{\phi}^*\Lambda_w. \tag{22}$$

The time evolutions of the energy integrals given in Eqs. (22),(18), and (19) reveal an energy transfer between $\tilde{E}^*$ and the ion and electron thermal energy densities E_i and E_e by the term: $\xi(p_e+p_i)\frac{\partial\phi^*}{\partial y}$. The first term on the right hand sides of both equations, the modified Reynolds stress production terms, yield a energy transfer between the mean and the fluctuating drift energies. This term includes the standard $\boldsymbol{E}\times\boldsymbol{B}$ Reynolds stress production term $u_0'\langle u_x u_y\rangle$, where $u_x = -\partial_y\tilde{\phi}$ and $u_y = \partial_x\tilde{\phi}$ denote the x and y components of the fluctuating $\boldsymbol{E}\times\boldsymbol{B}$ drift, respectively, and $u_0' = \partial_x u_0$ is the shear of the mean $\boldsymbol{E}\times\boldsymbol{B}$ flow

$$u_0 = \frac{\partial\langle\phi\rangle}{\partial x}. \tag{23}$$

The Reynolds stress production term describes an energy transfer due to fluctuating radial transport of azimuthal momentum in the presence of a sheared mean flow. However, due to the presence of the modified potential ϕ^* in the modified production term, it is also a function of the mean and fluctuating parts of the ion diamagnetic drift. Since no fields are advected by the diamagnetic drift, these extra terms lack an obvious interpretation. Furthermore, the interpretation of the drift energy density $\mathcal{E}^*$ itself is not immediately obvious. Since the particle density is advected by the $\boldsymbol{E}\times\boldsymbol{B}$ drift, it is more informative to consider the time evolution of the integrated $\boldsymbol{E}\times\boldsymbol{B}$ mean flow energy, the integrated fluctuating $\boldsymbol{E}\times\boldsymbol{B}$ energy, and the residual drift energy defined as:

$$E_0 = \int d\boldsymbol{x}\,\frac{u_0^2}{2},\quad \tilde{E} = \int d\boldsymbol{x}\,\langle\frac{|\nabla_\perp\tilde{\phi}|^2}{2}\rangle,\quad E_\times = \int d\boldsymbol{x}\,\langle\frac{|\nabla_\perp p_i|^2}{2}\rangle + \langle\nabla\phi\cdot\nabla_\perp p_i\rangle, \tag{24}$$

respectively. The residual drift energy can be shown to describe an FLR correction to the ion thermal energy density. The time-evolution of these energy integrals are derived from the vorticity equation (10a) and the ion pressure equation(10b)

$$\frac{d}{dt}E_0 = \int d\boldsymbol{x}\left[\underset{\mathbf{A}}{\langle u_y u_x\rangle} - \underset{\mathbf{B}}{\langle u_y\frac{\partial p_i}{\partial y}\rangle} - \underset{\mathbf{C}}{\frac{2}{3}\xi\langle p_i\frac{\partial p_e}{\partial y}\rangle} - \underset{\mathbf{D}}{\frac{2}{3}\xi\langle p_i u_x\rangle}\right]u_0' - \langle\phi\rangle\left[\langle\Lambda_w\rangle - \underset{\mathbf{E}}{\frac{2}{3}\frac{\partial^2}{\partial x^2}}\langle\Lambda_{p_i}\rangle\right], \tag{25}$$

$$\begin{aligned}\frac{d}{dt}\tilde{E} = \int d\boldsymbol{x}&\left[-\underset{\mathbf{A}}{\langle u_y u_x\rangle} + \underset{\mathbf{B}}{\langle u_y\frac{\partial p_i}{\partial y}\rangle}\right]u_0' + \xi\langle(p_e \underset{\mathbf{F}}{+} p_i)u_x\rangle - \frac{2}{3}\xi\langle p_i\nabla_\perp^2\tilde{\phi}\underset{\mathbf{G}}{\frac{\partial}{\partial y}}(p_i + p_e - \phi)\rangle\\ &- \langle\tilde{\phi}\left[\Lambda_w - \underset{\mathbf{E}}{\frac{2}{3}}\frac{\partial^2}{\partial x^2}\Lambda_{p_i}\right]\rangle,\end{aligned} \tag{26}$$

$$\begin{aligned}\frac{d}{dt}E_\times = \int d\boldsymbol{x}\,&\xi\langle p_i\underset{\mathbf{H}}{\frac{\partial p_e}{\partial y}}\rangle + \left[\frac{2}{3}\xi\langle p_i\underset{\mathbf{C}}{\frac{\partial p_e}{\partial y}}\rangle + \underset{\mathbf{D}}{\frac{2}{3}\xi\langle p_i u_x\rangle}\right]u_0' + \frac{2}{3}\xi\langle p_i\nabla_\perp^2\tilde{\phi}\underset{\mathbf{G}}{\frac{\partial}{\partial y}}(p_e + p_i - \phi)\rangle\\ &- \langle p_i\Lambda_w\rangle - \underset{\mathbf{E}}{\frac{2}{3}}\langle\phi\frac{\partial^2}{\partial x^2}\Lambda_{p_i}\rangle.\end{aligned} \tag{27}$$

The energy integrals are accompanied by an equation for the mean $\boldsymbol{E}\times\boldsymbol{B}$ flow, which is obtained by averaging the vorticity equation (10a) over the periodic y-direction making use of the ion pressure equation (10b)

$$\frac{\partial u_0}{\partial t} + \underset{\mathbf{a}}{\frac{\partial}{\partial x}\langle u_x u_y\rangle} - \underset{\mathbf{b}}{\frac{\partial}{\partial x}\langle u_y \frac{\partial p_i}{\partial y}\rangle} - \underset{\mathbf{c}}{\frac{2}{3}\xi \frac{\partial}{\partial x}\langle p_i \frac{\partial}{\partial y} p_e\rangle} - \underset{\mathbf{d}}{\frac{2}{3}\xi \frac{\partial}{\partial x}\langle p_i u_x\rangle} = -\frac{2}{3}\frac{\partial}{\partial x}\langle \Lambda_{p_i}\rangle + \underset{\mathbf{e}}{\int_0^x dx\, \langle \Lambda_w\rangle}, \tag{28}$$

where boundary terms were neglected. Integrating the mean flow equation in the x-direction shows that no mean flow is generated without external sources. The time-evolution of the energy integrals and the mean flow equation are principal results of this paper.

First, we note that the energy integrals and the mean flow equation reduce to the well-known system of equations in two-dimensional interchange driven convection[14] in the limit of constant ion pressure. Specifically, all ion pressure dependent terms vanish, $E_\times = 0$, and the time-evolution of the mean flow is governed by two effects: the divergence of the Reynolds stress tensor marked "**a**", which describes radial transport of azimuthal momentum, and collisional viscous damping marked "**e**". These two effects are accompanied by corresponding energy transfer terms in the mean flow energy equation 25. Collisional dissipation damps the mean flow energy through the term "**E**". The Reynolds stress production terms marked "**A**" in equations (25) and (26) yield a energy transfer between the mean and fluctuating $\boldsymbol{E} \times \boldsymbol{B}$ kinetic energies. From the energy integrals it is evident that the mean flow energy E_0 is only altered by the Reynolds stress when the mean flow is sheared $u_0' \neq 0$. The condition of a sheared mean flow is necessary but not sufficient. By expanding the electric potential into an infinite Fourier series in the periodic y-direction, the $x-y$ component of the Reynolds stress tensor can be written as

$$\langle u_x u_y\rangle = -2 \sum_{k_y=1}^{\infty} k_y |\phi_{k_y}|^2 \delta_\phi', \tag{29}$$

where $|\phi_{k_y}(x,t)|$ and $\delta_\phi(x,t)$ denote the radially varying amplitude and phase, respectively, and $\delta_\phi' = \partial_x \delta_\phi$. The mean flow energy is therefore only altered if the mean flow is sheared *and* if the phase of the electrostatic potential varies radially. The thermal and fluctuating energies are coupled through the term marked "**F**" whose origin is magnetic field inhomogeneity. This energy transfer describes fluctuating radial transport of thermal energy. The spectral representation of this *interchange drive term* is

$$\xi \langle p_e u_x\rangle = \xi \sum_{k_y=1}^{\infty} 2k_y |\phi_{k_y}||p_{ek_y}| \sin(\delta_\phi - \delta_{p_e}) \tag{30}$$

demonstrating that the direction of the energy flux is determined by the phase difference between electric potential and electron pressure fluctuations. Note that there is no direct energy transfer between the integral of the electron thermal energy E_e and the mean flow energy E_0; the only path for thermal energy to the mean flow energy goes through the fluctuating energy $\tilde{E}$.

When the assumption of constant ion pressure is relaxed, additional mean flow sources emerge. First, the Reynolds stress in the mean flow equation (28), marked "**a**", is accompanied by a diamagnetic Reynolds-stress-like term, marked "**b**" and corresponding production terms marked "**B**" in the mean and fluctuating energy integrals equations 25 and 26. Like the Reynolds stress production term, a finite energy transfer by the diamagnetic Reynolds energy transfer term requires a sheared mean flow $u_0' \neq 0$. The spectral representation in the y-direction

$$\langle u_y \frac{\partial p_i}{\partial y} \rangle = \sum_{k_y > 0} 2k_y \left[\sin(\delta_\phi - \delta_{p_i}) |p_{ik_y}| |\phi_{k_y}|' + \cos(\delta_\phi - \delta_{p_i}) |p_{ik_y}| |\phi_{k_y}| \delta_\phi' \right] \tag{31}$$

shows that the diamagnetic Reynolds stress and the corresponding production term may modify the mean flow both when ϕ and p_i are in and out of phase. Furthermore, the ability of the diamagnetic Reynolds stress production term to modify the mean flow does not require that the phase of the electric potential is radially inhomogeneous as is required for the standard Reynolds stress. We also note that if $\phi = -p_i + \text{const.}$, which is an approximate steady state solution to the vorticity equation 10a, then the Reynolds and the diamagnetic Reynolds stresses cancel.

In addition to the diamagnetic Reynolds stress, two transfer terms marked "**c**" and "**d**" enter the mean flow equation (28) when the ion pressure is non-constant. These transfer terms differ from the standard and diamagnetic Reynolds stresses because of their ability to modify the mean flow rely on an inhomogeneous magnetic field $\xi \neq 0$. The corresponding energy transfer terms, marked "**C**" and "**D**" in equations (25) and (27), couple the mean flow energy E_0 and the residual energy $E_\times$. In the constant ion pressure limit, the fluctuating kinetic energy and therefore also instabilities can only grow because the fluctuations can feed on the thermal energy through the interchange drive term marked "**F**". When the ion pressure is not constant, an additional energy transfer emerges. The term marked "**H**" in the residual energy integral equation (27) allows energy exchange between the residual energy and the ion thermal energy. In many respects the generation of mean flows in interchange driven turbulence is therefore potentially fundamentally different when ion temperature dynamics is taken into account. The energy transfer channels are schematically depicted in figure 1. We note that the appearance of the terms "**C**","**D**", and "**H**" in the energy integral

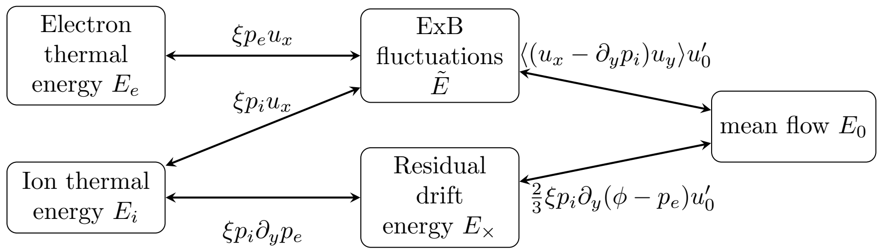

Figure 1. Diagram illustrating the energy transfer channels between the five energy integrals in equations (18)-(19) and (25)-(27). Energy transfer channels are shown as uni-directional arrows; the corresponding energy transfer terms label the arrows.

equations and the terms "**c**" and "**d**" in the mean flow equation is a direct consequence of consistently keeping the first order drifts in the ion density and in the ion pressure equations. The terms in equations (26) and (27) marked "**G**" yield a energy transfer between the fluctuating $\boldsymbol{E} \times \boldsymbol{B}$ energy and the residual drift energy. We do not analyze these terms further in this paper. A detailed analysis most likely requires that the residual drift energy is split into mean and fluctuating components. We leave this analysis for future work.

The term marked "**d**" in the mean flow equation (28) originates from the finite compression of the $\boldsymbol{E} \times \boldsymbol{B}$ drift in the ion pressure equation 10b. The spectral decomposition

$$\frac{2}{3}\xi \langle p_i u_x \rangle = \frac{4}{3}\xi \sum_{k_y=1}^{\infty} k_y |\phi_{k_y}||p_{ik_y}| \sin(\delta_\phi - \delta_{p_i}) \tag{32}$$

shows that a finite phase difference between the potential and ion pressure fluctuations is required for modification of the mean flow. It is interesting that this term apart from a factor "2/3" shares the same functional form as the interchange drive term "**F**" in the energy integral equation (27), and hence they are always simultaneously active. The direction of the energy flux by the corresponding energy transfer terms marked "**D**" in Eqs. (25) and (27) is determined by the phase shift and the mean flow shear.

Finally, we analyze the transfer mechanisms described by the terms "**C**" and "**H**" in the energy integral equations (25) and (27) and the corresponding term "**c**" in the mean flow equation (28). A remarkable feature of these terms is that they are independent of the fluctuating part of the $\boldsymbol{E} \times \boldsymbol{B}$ drift, and hence may alter the mean flow when $\boldsymbol{E} \times \boldsymbol{B}$-drift fluctuations vanish $u_x = u_y = 0$. As illustrated in Fig. 1, ion thermal energy $\mathcal{E}_i$ can be transferred to the mean flow energy $\mathcal{E}_0$ via the residual energy $\mathcal{E}_\times$ by these transfer channels. Common to all these terms is the appearance of

$$\xi \langle p_i \frac{\partial p_e}{\partial y} \rangle = -\xi \sum_{k_y=1}^{\infty} 2k_y |p_{ek_y}||p_{ik_y}| \sin(\delta_{p_e} - \delta_{p_i}), \tag{33}$$

showing that they are only active if the phase shift between electron and ion pressure fluctuations is finite. It is important to keep in mind that these terms vanish in the isothermal limit; electron or ion temperature fluctuations are required. The direction of the energy flux through the transfer channel "**H**" between the ion thermal energy $\mathcal{E}_i$ and the residual drift energy $\mathcal{E}_\times$ is solely determined by the phase-shift $\delta_{p_e} - \delta_{p_i}$. Specifically, energy is transported from the ion thermal energy to the residual drift energy when $\sin(\delta_{p_e} - \delta_{p_i}) < 0$, and is maximal when $\delta_{p_e} - \delta_{p_i} = -\pi/2$. For the residual drift energy to flow simultaneously from the residual drift energy $\mathcal{E}_\times$ to the mean flow energy $\mathcal{E}_0$, the shearing rate u_0', entering the transfer term $2/3\xi u_0' \langle p_i \partial_y p_e \rangle$ marked "**C**" in equations (25) and (27), must be negative $u_0' < 0$.

IV. LINEAR ANALYSIS

In this section we investigate the additional terms, beyond the Reynolds stress and associated production term, in the mean flow and energy integral equations which arise when ion temperature dynamics is taken into account. The analysis is carried out by means of linear and quasi-linear analysis. This approach allows us to estimate under which conditions these additional terms are active and to some extend to estimate their magnitude and whether they act as to inhibit or enhance mean flows

Neglecting dissipative effects assuming a local plane wave solution $\exp(i\boldsymbol{k} \cdot \boldsymbol{x} - i\omega t)$ to the model equations (10a)-(10c), the linearized equations are

$$\omega k_\perp^2 (\phi_{\boldsymbol{k}} + p_{i\boldsymbol{k}}) + \xi k_y (p_{e\boldsymbol{k}} + p_{i\boldsymbol{k}}) = 0, \tag{34}$$

$$-\frac{3}{2}\omega p_{i\boldsymbol{k}} + \phi_{\boldsymbol{k}} k_y (\frac{3}{2}\kappa_i - \xi) + \xi k_y (p_{e\boldsymbol{k}} + p_{i\boldsymbol{k}}) = 0, \tag{35}$$

$$-\frac{3}{2}\omega p_{e\boldsymbol{k}} + \phi_{\boldsymbol{k}} k_y (\frac{3}{2}\kappa_e - \xi) = 0, \tag{36}$$

with the dispersion relation

$$\lambda\left[\lambda^2+\lambda(\bar{\kappa}_i-\frac{4}{3})+(\frac{2}{3}\bar{\kappa}_e-\frac{4}{9})+\frac{1}{k_\perp^2}(\bar{\kappa}_e+\bar{\kappa}_i-\frac{4}{3})\right]=0, \tag{37}$$

where $\lambda=\frac{\omega}{\xi k_y}$, $\bar{\kappa}_i=\kappa_i/\xi$, $\bar{\kappa}_e=\kappa_e/\xi$, and κ_i and κ_e denote the ion and electron inverse profile gradient length scales, respectively. Besides the trivial solution $\lambda=0$, the dispersion relation has the solutions

$$\lambda=\frac{\frac{4}{3}-\bar{\kappa}_i\pm\sqrt{(\bar{\kappa}_i-\frac{4}{3})^2-4k_\perp^{-2}(\bar{\kappa}_i+\bar{\kappa}_e-\frac{4}{3})}}{2}. \tag{38}$$

The unstable part of the solution for which: $\mathrm{Im}(\lambda)>0$, is plotted in Fig. 2 for various parameters. The waves are unstable when $\bar{\kappa}_i+\bar{\kappa}_e>4/3$. Notice the well-known ion FLR

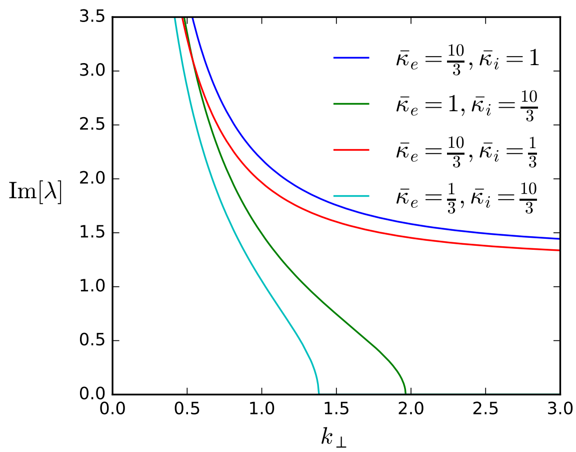

Figure 2. Dispersion diagram for the unstable solution. By comparing the red and green curves, we see the effect of ion FLR stabilization.

stabilization[39] by the first term in the radicand in Eq. (38). The stabilizing effect is clearly illustrated by the blue and green curves in Fig. 2 which have the same interchange drive "$\bar{\kappa}_i+\bar{\kappa}_e$" but when $\bar{\kappa}_e>\bar{\kappa}_i$ (blue) the growth rate is significantly higher than when $\bar{\kappa}_e<\bar{\kappa}_i$ (green). Only for very low $k_\perp$ (not visible in Fig. 2) the growth rate of the green curve exceeds the blue curve.

The linear fluctuations are related by

$$\frac{\phi_{\boldsymbol{k}}}{p_{i\boldsymbol{k}}}=\frac{|\phi_{\boldsymbol{k}}|}{|p_{i\boldsymbol{k}}|}e^{i(\delta_\phi-\delta_{p_i})}=\frac{3\lambda(3\lambda-2\xi^{-1})}{3\lambda(3\bar{\kappa}_i-2)+2(3\bar{\kappa}_e-2)}, \tag{39}$$

$$\frac{p_{e\boldsymbol{k}}}{p_{i\boldsymbol{k}}}=\frac{|p_{e\boldsymbol{k}}|}{|p_{i\boldsymbol{k}}|}e^{i(\delta_{p_e}-\delta_{p_i})}=\frac{(3\bar{\kappa}_e-2)(3\lambda-2)}{3\lambda(3\bar{\kappa}_i-2)+2(3\bar{\kappa}_e-2)}. \tag{40}$$

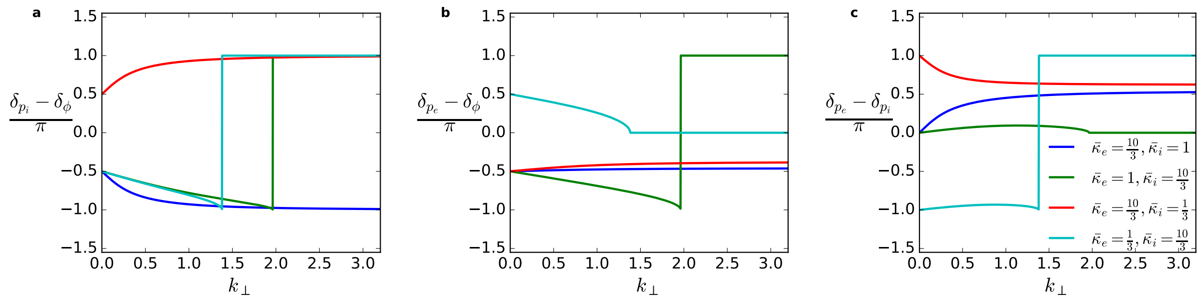

Figure 3. Quasi linear calculation of the phase shift between a) ion pressure and electric potential fluctuations, b) electron pressure and electric potential fluctuations, and c) ion and electron pressure fluctuations as functions of $k_\perp$. Line colors are defined in the caption of Fig. 2.

From these expressions the corresponding phase shifts can be calculated (see Fig. 3). As expected the phase shifts between pressure and electric potential fluctuations plotted in Figs. 3a and 3b show that the interchange drive term in Eq. (26) according to Eq. (30) transforms thermal energy into fluctuating energy when the waves are unstable, see Fig. 2. We also observe that in the cases where the inverse profile gradient length scales $\bar\kappa_e = 1/3$ (cyan) and $\bar\kappa_i = 1/3$ (red) are below unity, the direction of the energy flux is reversed even though the waves are unstable.

For the analysis of the diamagnetic Reynolds stress given in Eq. (31), we employ the quasi-linear approximation. By expressing the ion pressure fluctuations in terms of the potential fluctuations, we get

$$\langle u_y \frac{\partial}{\partial y} p_i \rangle = -2 \sum_{k_y>0} k_y \left(|\phi_{k_y}|^2 \delta'_\phi \,\mathrm{Re}\left[\frac{p_{ik_y}}{\phi_{k_y}}\right] + \frac{1}{2}(|\phi_{k_y}|^2)' \,\mathrm{Im}\left[\frac{p_{ik_y}}{\phi_{k_y}}\right]\right). \tag{41}$$

The first term (see Eq. (29)) equals the Reynolds stress times the real part of the ratio of the ion pressure to the potential. The magnitude of the first term in the diamagnetic Reynolds stress relative to the standard Reynolds stress is therefore simply given by the magnitude of $\mathrm{Re}[p_{ik_y}/\phi_{k_y}]$. In the quasi-linear treatment this factor can be calculated using Eq. (39) employing the solution given in Eq. (38). When the absolute value of $\mathrm{Re}[p_{i\boldsymbol{k}}/\phi_{\boldsymbol{k}}]$ exceeds unity, the first term in the diamagnetic Reynolds stress exceeds the standard Reynolds stress and equivalently the diamagnetic Reynolds stress production term dominates. Quasi-linear calculations of $\mathrm{Re}[p_{i\boldsymbol{k}}/\phi_{\boldsymbol{k}}]$ as a function of $k_\perp$ and κ_i are shown for two values of κ_e in Figs. 4a and 4c. In Figure 4a $\bar\kappa_e = 1$. In this case $|\,\mathrm{Re}\left[\frac{p_{i\boldsymbol{k}}}{\phi_{\boldsymbol{k}}}\right]| > 1$ when $\bar\kappa_i < 1.8$. When steepening

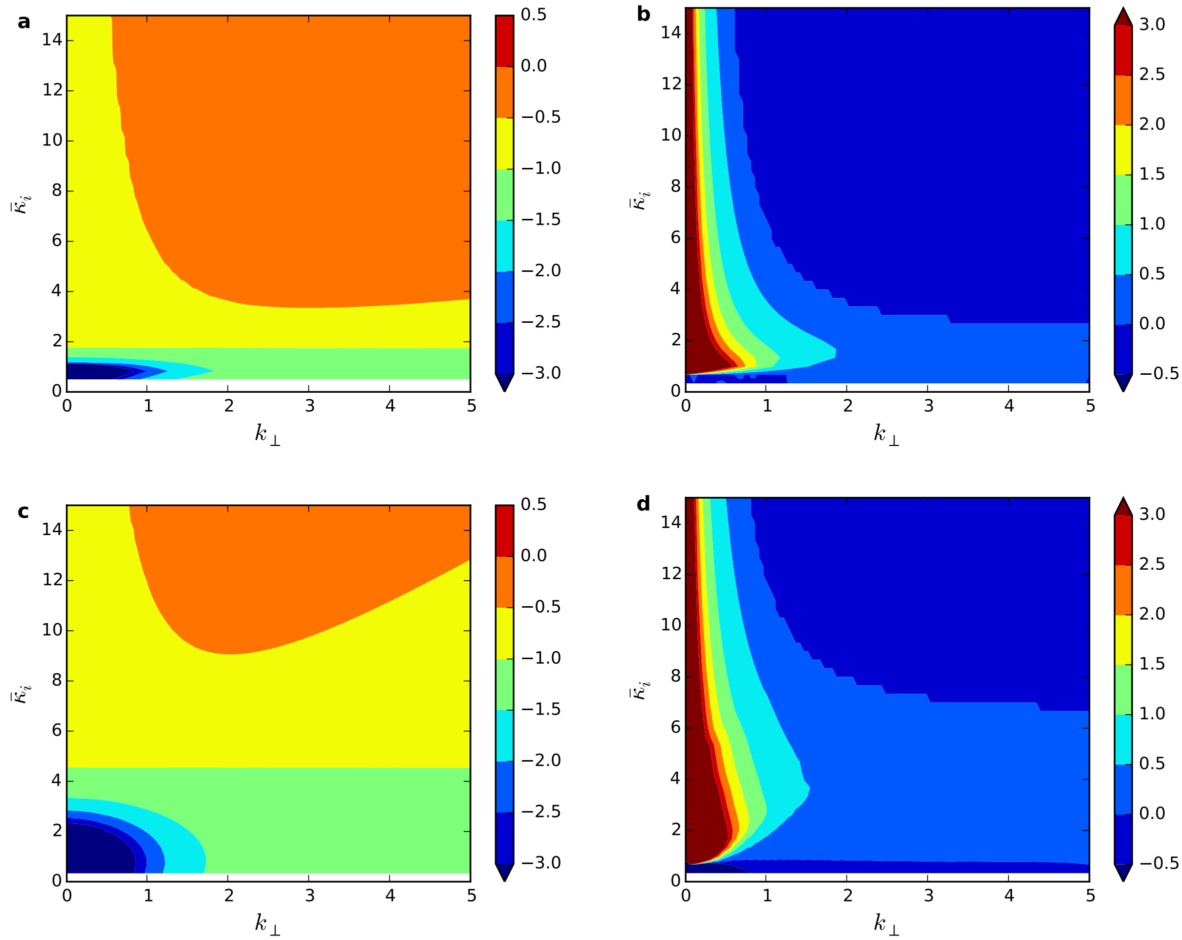

Figure 4. Comparison of diamagnetic and standard Reynolds stress. Quasi-linear calculations of $\text{Re}\left[\frac{p_{i\boldsymbol{k}}}{\phi_{\boldsymbol{k}}}\right]$ for (**a**) $\bar{\kappa}_e = 1$ and (**c**) $\bar{\kappa}_e = 10$, and $\text{Im}\left[\frac{p_{i\boldsymbol{k}}}{\phi_{\boldsymbol{k}}}\right]$ for (**b**) $\bar{\kappa}_e = 1$ and (**d**) $\bar{\kappa}_e = 10$. When the absolute value of the real part in (**a**) and (**c**) is above unity, the first term in the diamagnetic Reynolds stress given in Eq. (41) exceeds the standard Reynolds stress.

the electron pressure gradient $\bar{\kappa}_e = 10$, the region were $|\,\text{Re}\left[\frac{p_{i\boldsymbol{k}}}{\phi_{\boldsymbol{k}}}\right]| > 1$ is extended and holds for all $\bar{\kappa}_i < 4.5$ as shown in Fig. 4c.

The magnitude of the second term of the diamagnetic Reynolds stress given in Eq. (41) depends on the radial gradient of the fluctuating kinetic energy and is therefore only able to drive or damp the mean flow if the fluctuating kinetic energy is radially inhomogenous $\frac{1}{2}(|\phi_{\boldsymbol{k}}|^2)' \neq 0$. The magnitude of the fluctuating kinetic energy is not readily accessible through quasi-linear calculations and must be obtained via non-linear numerical calculations. However, the fluctuating energy is multiplied by $\text{Im}\left[\frac{p_{i\boldsymbol{k}}}{\phi_{\boldsymbol{k}}}\right]$, and hence regardless of the radial structure of the fluctuating kinetic energy this must be finite for this part of the diamagnetic

Reynolds stress to play a role. Quasi-linear calculations of Im $\left[\frac{p_{i\boldsymbol{k}}}{\phi_{\boldsymbol{k}}}\right]$ for $\bar{\kappa}_e = 1$ and $\bar{\kappa}_e = 10$ are shown in Fig. 4b and d, respectively. In both cases the magnitude is small for high $k_\perp$ and $\bar{\kappa}_i$ but exceeds unity for low $k_\perp$ for all values of $\bar{\kappa}_i$. Increasing $\bar{\kappa}_e$ by a factor of 10 implies a broadening of the region where Im $\left[\frac{p_{i\boldsymbol{k}}}{\phi_{\boldsymbol{k}}}\right]$ exceeds unity towards higher $\bar{\kappa}_i$. In conclusion the quasi-linear treatment predicts that both parts of the diamagnetic Reynolds stress are mainly active at low $k_\perp$ where the interchange modes are most unstable (see Fig. 2). In these active regions the ability of the diamagnetic Reynolds stress to modify mean flows is as strong or even stronger than the standard Reynolds stress.

Finally, we consider the terms marked "**c**" and "**d**" in the mean flow equation (28) and the corresponding terms marked "**C**","**D**", and "**H**" in the energy integrals (25)-(27). The spectral representations given in Eqs. (32)-(33) show that finite contributions by these terms require that the sines of the phase shifts between ion pressure and electric potential as well as between ion and electron pressure fluctuations are finite. Figures 3a and 3c show that, according to linear theory, these terms yield finite contributions for a wide range of parameters. This observation entails that these mechanisms must be taken into account in the description of mean flows. Specifically, the linear results shown in Fig. 3c reveal that the energy transfer term "**H**", between the ion thermal energy and the residual energy, for most parameters yields an energy transfer from the residual to the ion thermal energy except when electron pressure profiles are nearly flat. The quasi-linear analysis does therefore not indicate the existence of an energy flux from the ion thermal energy via the residual energy to the mean flow energy which bypasses the fluctuating kinetic energy.

V. DISCUSSION AND CONCLUSIONS

In this paper we have investigated how ion temperature dynamics influences azimuthal mean $\boldsymbol{E} \times \boldsymbol{B}$ flows in 2D, electrostatic, interchange driven convection. We consider the dynamics of the *pure* $\boldsymbol{E} \times \boldsymbol{B}$ mean flow disentangled from the diamagnetic fluid drift, which is not responsible for transport over macroscopic distances. Our investigations show that $\boldsymbol{E} \times \boldsymbol{B}$ mean flows may be modified by additional mechanisms beyond the standard Reynolds stress. The standard Reynolds stress is accompanied by a diamagnetic Reynolds stress, which modifies the mean flow in the presence of $\boldsymbol{E} \times \boldsymbol{B}$ and ion pressure fluctuations and a sheared azimuthal $\boldsymbol{E} \times \boldsymbol{B}$ mean flow. Quasi-linear analysis indicates that the standard and diamagnetic Reynolds stresses are equally important. We also demonstrate that taking ion temperature dynamics into account implies two additional mechanisms capable of modifying mean flows. Both mechanisms rely on the magnetic field inhomogeneity. The first mechanism takes the same form as the interchange energy exchange term, which is responsible for feeding free energy from the free thermal energy into $\boldsymbol{E} \times \boldsymbol{B}$ fluctuations in interchange driven instabilities. This mechanism and the interchange energy exchange term are therefore simultaneously active. The second mechanism relies on phase shifted ion and electron pressure perturbations and is in that respect unique because electric potential fluctuations are not needed. This mechanism provides energy transfer between the ion thermal energy and the mean flow energy completely bypassing electric potential fluctuations. However, quasi-linear analysis shows that the direction of the energy flux inhibits mean flows for most parameters.

Our analysis was carried out in a simplified two-dimensional drift fluid model describing interchange driven turbulence in the absence of dynamics parallel to the magnetic field. Naturally our results cannot readily be generalized to a toroidal configuration where parallel dynamics play an important role especially for the turbulence which in that case is more drift-wave like. The introduction of drift wave turbulence most certainly changes the phase shift between electron pressure and electric potential fluctuations which inevitably alter the quasi-linear results presented here. However, our analysis points out that the paradigm of Reynolds stress driven mean flows is incomplete and must be supplemented by other mechanisms apparently equivalently capable of modifying $\boldsymbol{E} \times \boldsymbol{B}$ mean flows.

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