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A theoretical interpretation of the main scrape-off layer heat flux width scaling for tokamak inner wall limited plasmas

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Abstract. The International Tokamak Physics Activity Topical Group on scrape-off layer (SOL) and divertor physics has amassed a database comprising hundreds of reciprocating Langmuir probe measurements of the main scrape-off layer heat-flux width λ_q in inner-wall limited discharges. We have carried out an analysis, based on turbulent transport theory, of the variation of λ_q with respect to the plasma dimensionless parameters. Restricting our analysis to circular plasmas, we find that a model based on non-linearly saturated turbulence can well reproduce the λ_q values found in the database.

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1. Introduction

Since the ramp-up phase of ITER plasmas are expected to be mostly inner-wall limited (IWL) [1], it is important to establish a predictive capability describing the scrape-off layer (SOL) heat flux width, $\lambda_q = -q_\parallel/\nabla q_\parallel$ ($q_\parallel \sim n c_s T$), in this configuration. The original ITER heat load specifications assumed that λ_q in IWL discharges would follow a power-law scaling originally obtained for L-mode diverted plasmas [2]. In recent years, however, this assumption has been clearly shown to be flawed.

An extensive study of limiter discharges in Tore Supra demonstrated that the scaling was not obeyed in this device [3]. Stimulated by these observations, the Divertor and SOL Topical Group of the International Tokamak Physics Activity (ITPA), embarked on a multi-machine database effort to characterize the main SOL λ_q and confirm the choices made by the ITER Organization (IO) in the design of the ITER inner wall toroidal shaping. As part of this effort, dedicated experiments on JET subsequently found clear evidence of a narrow feature in the SOL heat flux close to the last closed flux surface (LCFS), although insufficient data were obtained to obtain a scaling for either the near-SOL feature or the broader profile width in the main SOL [4]. These JET experiments in turn provoked a new multi-machine effort to investigate this narrow feature [5–8], ultimately leading to a re-design of the ITER inner wall toroidal profile shape taking the narrow feature into account [9]. The main SOL heat-flux database obtained, as well as the scalings derived from the ITPA study, are described in a companion paper of this special issue [10]

The database is comprised of hundreds of reciprocating Langmuir probe measurements performed in IWL discharges in a large number of tokamaks. Each reciprocation is fitted using an exponential power law with a single decay length, *i.e.* neglecting the narrow feature, which is rather challenging to measure using fast probes and which is best inferred from infrared thermography of the limiter surface [9].

Within the companion paper, only known ITER parameters, such as SOL power, toroidal field B_ϕ, major radius R, plasma current I_p, safety factor q, etc, were considered as regression parameters in order to minimize the error in the possible scalings. The principal finding of that study is that several scalings can be constructed from these engineering parameters, all with similar coefficients of determination (R^2 parameter). Fortunately, all scalings project approximately the same main-SOL λ_qs for ITER inner wall start-up plasmas. However, a straightforward physical interpretation of the data is not possible.

In an attempt to seek a physics-based understanding of this database, we have reassessed its contents from a completely different perspective. Rather than concentrating on finding suitable scaling parameters directly from known ITER quantities, our approach is to use SOL turbulent transport theory to guide our choices of parameters. In this approach, the scaling parameters are *dimensionless.* A priori, it may seem like this approach has a serious disadvantage. The dimensionless plasma parameters required by the theory involve the last closed flux surface (LCFS) local temperature and density, and are more difficult to determine from experimental reciprocating Langmuir probe measurements. The latter are often noisy and subject to systematic error, particularly as the probes penetrate deeper into the SOL where the profile steepens rapidly. In addition, the location of the LCFS is based on magnetic equilibrium reconstruction, which precise to a few mm's, and the LCFS is often not reached by the probes. As we will show, however, the experimental uncertainty related to these parameters has little effect on the final result.

Despite the uncertainties introduced by requiring absolute values of local plasma parameters, our approach has advantages and some beneficial side effects. First, the theoretical analysis is not constrained by mutual correlations between the regression parameters, which limits the combination of parameters available in [10]. Second, the ITPA database allows recently proposed theoretical models to be tested [11, 12]. Third, it is possible to use apply hybrid theory/data analysis to the database, leading to new, possibly more precise descriptions of the SOL width. Fourth, and most important, our approach allows us (for a subset of the data, at least) to better interpret and understand the physical origin of the variation of λ_q with the plasma parameters.

At the time the database was being compiled, there was no theory-based SOL model with credible predictive capability for λ_q. Since then, 3-D non-linear, flux-driven turbulence simulations of SOL dynamics have revealed, (a) that the turbulent modes saturate through the *gradient removal* mechanism [11], (b) that turbulence is driven by ballooning or drift type modes [13, 14], and (c) that the transport levels are strongly affected by parallel dynamics effects, such as collisionality and electromagnetic flutter, and by the normalized plasma size [15, 16]. The combination of these elements led to the resistive ballooning mode (RBM) scaling [12], which compared favorably against a small experimental database gathered from existing published SOL widths in limiter discharges. The work here presented shows, in fact, that the mechanisms set forward in Refs. [11–14, 16], describing SOL transport dynamics in circular IWL discharges, are consistent with the ITPA database.

2. On the choice of model and parameters

The cold-ion electrostatic drift-reduced Braginskii equations [17, 18], expressed for circular flux-surface geometry (see appendix), are used as the basis of our analysis. Since it is typically observed that $T_i > T_e$ in the SOL [19], we make some remarks on the neglect of T_i in the model. The effects of finite T_i are exhaustively analyzed for the IWL configuration by Mosetto *et al.* [14], finding altogether a weak contribution on the SOL transport dynamics. Purely ion-temperature-driven modes, such as the ion temperature gradient (ITG) instability, are ruled out. The only noteworthy effect, at transport relevant wavenumbers, is a slight enhancement of the RBM instability leading to a factor of $(1 + T_i/T_e)^{1/7}$ in the final expression of the RBM scaling [12].

The weak influence found for T_i effects in turbulence simulations, combined with the lack of T_i profiles in the database, motivate our choice of a cold ion model. The effects of T_i could be easily reintroduced in the present work, if T_i data became available. The main qualitative parameter trends and conclusions of the study should remain unaffected, although details of the regression fits may be altered.

To obtain the final form of the drift-reduced Braginskii equations shown in the appendix, we follow [18], where temperature and density are normalized to their values at the LFCS, T_{e0} and n_0, and we choose a reference perpendicular length $L_\perp = \rho_s = c_s/\omega_{ci}$ ($c_s = \sqrt{T_{e0}/m_i}$, $\omega_{ci} = eB_\phi/m_i$), a reference parallel length $L_\parallel = R$, and a reference time $\tau_{\rm ref} = R/c_s$ (here we use SI units to define these physical quantities, except the temperature which is expressed in eV). Normalization and linearization of the drift-Braginskii equations *naturally* yields the following set of dimensionless parameters:

$$\rho_\star = \rho_s/R \tag{1}$$

$$\nu = \frac{e^2 nR}{m_i c_s \sigma_\parallel} \tag{2}$$

$$q \approx q_{95} \sim \frac{a}{R}\frac{B_\phi}{B_\theta} \tag{3}$$

where $\sigma_\parallel$ is the parallel Spitzer conductivity assuming a pure deuterium plasma.

Here we have replaced q at the LCFS, which appears in the theory, with q_{95}, which is the quantity available in the database. The parameters describe, in dimensionless form, the plasma size ($\rho_\star$), the Spitzer resistivity (ν), and the connection length (q_{95}). If the heat-flux widths were solely determined by turbulent transport, these parameters should fully explain the variation of the λ_q in the database. There is a total of 317 entries for which these dimensionless quantities can be computed, including 120 entries from ToreSupra, 23 from DIII-D, 84 from COMPASS, 27 from JET, 3 from CASTOR, 2 from EAST, 39 from HL-2A, 1 from KSTAR, and 18 from Alcator C-Mod. Entries from other tokamaks are neglected due to the local n_{e0} and T_{e0} data not being compiled into the database. We include both circular ($\kappa < 1.2$) and shaped discharges in the analysis, which allows us to indirectly evaluate the importance of shaping in the transport dynamics.

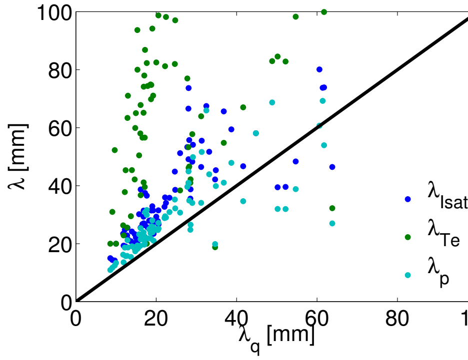

Figure 1. The profile lengths $\lambda_{I_{sat}}$, λ_T, and λ_p are shown for the CASTOR and COMPASS tokamaks. The pressure profile widths, λ_p, are about 20% longer than the heat flux widths λ_q.

We have also introduced $L_q = \lambda_q/\rho_s$, a dimensionless heat-flux width, under the assumption that

$$\lambda_q \propto L_p = -p/\nabla p. \tag{4}$$

This assumption is necessary because the database does not contain L_p, which is the quantity predicted by the theory. For the CASTOR and COMPASS tokamaks we show $\lambda_{I_{sat}}$ ($I_{sat} = nc_s$), λ_T, and λ_p in figure 1. We find that λ_p and λ_q are roughly proportional, with $\lambda_p \approx 1.2\lambda_q$. Supported by this finding, we introduce a proportionality constant between L_q and L_p through a best fit between the theoretical calculations and the experimental data, which always remains within order unity.

Some comments should be made regarding the use of a purely turbulent transport model. A possible caveat in the transport model is the neglect of neutral particles, which could drive parallel temperature gradients. However, we have evaluated the neutral collision length to be at least $8\lambda_q$ at the LCFS of the discharges in the database. Therefore, neutral particles should ionize well inside the plasma and fuel the SOL as they are expelled through the LCFS.

Another possible concern would be the lack of effects of impurities in the theory, in particular, to describe λ_q in machines with carbon walls. This effect could also lead to parallel temperature gradients, which would translate to a poloidal dependence of λ_q in the measurements. In recent experiments carried out in Alcator C-Mod, which is a high density, high-Z wall device, λ_q was found to be poloidally uniform [20]‡. In fact, the poloidal angle at which the Langmuir probe measurements are carried out does not appear to be an important factor in the scalings reported by Horacek *et al* [10]. We proceed, thus, under the assumption that parallel temperature gradients are small. In the end, this choice is vindicated by the good theory-experiment agreement found.

‡ The lack of carbon in C-Mod means potentially lower impurity radiation in the contact area, since molybdenum sputters at a higher temperature than carbon. On the other hand, high density should increase the importance of heat conduction compared to convection.

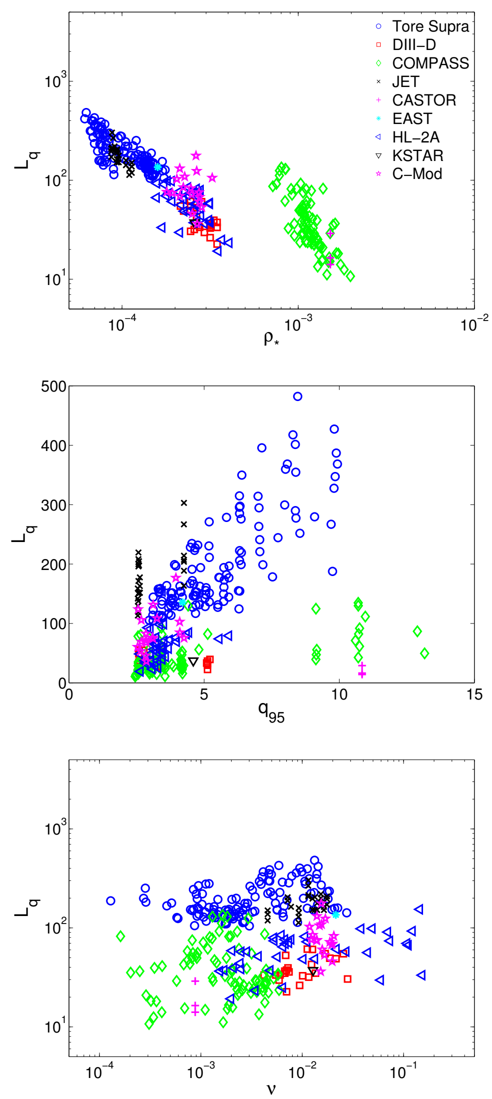

Figure 2. Variation of L_q with respect to $\rho_\star$ (top), q_{95} (center), and ν (bottom)

3. Variation of L_q with the dimensionless parameters

At first sight, it appears that $L_q = \lambda_q/\rho_s$ is correlated with two of the dimensionless parameters. Figure 2 illustrates the variation of the L_q with respect to $\rho_\star$, q_{95}, and ν. In each panel, we show a scatter plot of L_q as a function of one dimensionless parameter. The top panel suggests a power law dependence of L_q with respect to the plasma size. The centre panel indicates a predominantly linear dependence of L_q on q_{95}, which resembles the $\sim I_p^{-1}$ scaling found both in the RBM scaling for IWL discharges and also for the inter-ELM scaling for H-mode diverted plasmas [21]. As an aside, we observe that the COMPASS data (green diamonds) have a much shallower slope as a function of q_{95} than the rest of the data points. Additionally, for the COMPASS data, a very large range of L_q is possible for essentially the same $\rho_\star$. The bottom panel reveals a weak or zero dependence of L_q on ν, as anticipated by the turbulent transport theory [12, 16]. However, many entries in the database have $\nu \sim 10^{-3}$ and below, which is an order of magnitude smaller than the value assumed in Ref. [12]. This implies that inertial effects could become important in the transport theory. The values of $\beta_e = 2\mu_0 p_{e0}/B_\phi^2$ and $\alpha = q_{95}^2\beta_e/(\rho_\star L_q)$ are small for all entries in the database, and therefore it is appropriate to neglect electromagnetic effects.

4. Transport theory and quasi-linear modeling

In order to compute L_q, we consider a simple transport equation for p involving only the leading order terms in the pressure balance equation in steady state, $\nabla_\perp \cdot (p v_{\mathbf{E}\times\mathbf{B}}) \sim \nabla_\parallel \cdot (p c_s)$. The dominant fluxes involve $\mathbf{E}\times\mathbf{B}$ cross field transport driven by non-linearly saturated mesoscale turbulence, $\sim \gamma p/(k_\theta L_p)$, where γ is the linear growth rate of the turbulent mode and k_θ is the poloidal wavenumber, and sheath losses $\sim p c_s/q_{95}$. The balance of these fluxes gives the *gradient removal* SOL width

$$L_{q,\mathrm{gr}} \propto L_{p,\mathrm{gr}} = \frac{q_{95}}{c_s}\left(\frac{\gamma}{k_\theta}\right)_{max}, \tag{5}$$

which is valid under the assumption of small parallel temperature gradients.

We consider first the particular case of non-linearly saturated RBM turbulence. RBMs are chosen as a hypothesis for IWL SOL transport because quasi-linear and non-linear calculations demonstrated clear evidence of RBMs dominating transport at IWL relevant parameters ($q = 3$–8, $\nu = 0.01$, weak magnetic shear) [13]. A solution for equation 5 can be obtained analytically for RBMs as follows [12, 16]. First, a dispersion relation for RBMs is obtained from the reduced resistive MHD equations. Then, the maximum flux that can be driven by RBMs is found by maximizing γ/k_θ that appears in equation 5, *i.e.* we solve $\partial_{k_\theta}(\gamma/k_\theta) = 0$ starting from the dispersion relation. After straightforward algebra, we find that the flux is maximized for $\gamma = \gamma_{\mathrm{RBM}} \approx \sqrt{2/(\rho_\star L_p)}$ and $k_\theta = k_{\theta,\mathrm{RBM}} = \nu^{-1/2} q^{-1} \gamma_{\mathrm{RBM}}^{-1/2}$. Using γ_{RBM} and $k_{\theta,\mathrm{RBM}}$ in equation 5 leads to the following expression:

$$L_{p,\mathrm{RBM}} = 2^{3/7}\nu^{2/7}\rho_\star^{-3/7}q_{95}^{8/7}. \tag{6}$$

We then carry out a least-squares fit to the database to find a proportionality constant between L_p and L_q. The heat-flux widths are given by the expression:

$$L_{q,\mathrm{RBM}} \approx 1.73 L_{p,\mathrm{RBM}}. \tag{7}$$

The top panels of figure 3 show a comparison between equation 7 and the experimental database. On the left figure, we compare the *normalized* heat-flux widths L_q with the theoretical predictions, while on the right figure the same comparison is repeated, however, showing $\lambda_q = L_q \rho_s$ $[m]$. The quality of the comparison is given by R^2, which is defined here as the square of the Pearson correlation coefficient. The normalized flux-widths from equation 7 match the database quite well, with $R^2 = 0.73$, which in fact increases to $R^2 = 0.81$ for nearly circular discharges ($\kappa < 1.2$, shaping effects are not considered in [12]). This level of agreement is as good as the agreement found between equation 6 and non-linear turbulence simulations in our previous work. One possible issue, however, is that the range of L_q is dominated by Tore Supra data, which

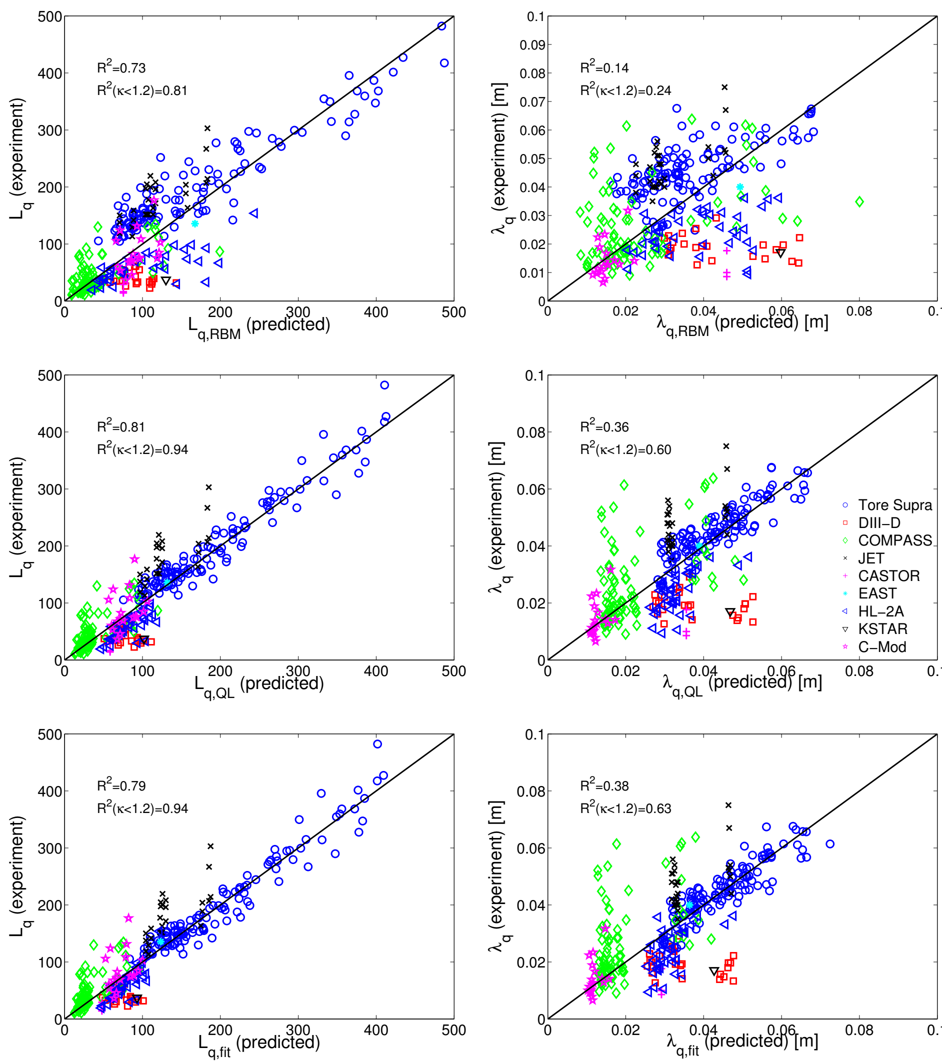

Figure 3. Resistive ballooning mode theory (top, equation 7), quasi-linear transport theory (center, equation 8), and direct fit of L_q using dimensionless parameters (bottom, equation 9) are compared against the heat-flux widths expressed as $L_q = \lambda_q/\rho_s$ (left) and λ_q $[m]$ (right).

have the greatest variation of SOL widths, and whose configuration is best captured by our model.

The absolute values of λ_q, on the other hand, are not well described by the resistive ballooning mode theory. Previous non-linear simulation results already hinted at this result: when $\nu < 10^{-2}$, inertial effects can become important and therefore the RBM hypothesis must be relaxed.

We have obtained a more precise model where we consider transport driven by all possible unstable modes. Since an analytical solution is not possible, we developed a quasi-linear transport code to solve equation 5. The procedure is presented and verified against non-linear simulations in [16]. Only the dimensionless parameters (e.g. $\rho_\star$, q_{95}, and ν), are needed as inputs. The method of solution involves computing γ/k_θ using the linear version of GBS [22] to find the maximum flux. Then, $(\gamma/k_\theta)_{\max}$ is compared to $c_s L_p/q_{95}$, and L_p is adjusted iteratively using a secant method. The iteration stops when left and right hand sides of equation 5 match to the desired precision.

In what follows, the database entries are used as samples to calculate L_p in $\{\rho_\star, q_{95}, \nu\}$ parameter space with the quasi-linear code. Then, we employed a robust regression procedure [23]§ to obtain a power-law scaling describing the variation of the quasi-linear L_p results with the plasma parameters. As before, L_p is adjusted by a single constant to obtain L_q, which is given by:

$$L_{q,\mathrm{QL}} = 0.22\nu^{0.06\pm0.01}\rho_\star^{-0.62\pm0.03}q_{95}^{0.84\pm0.03}. \quad (8)$$

The comparisons between equation 8 and the experimental data are shown in the center panels of figure 3. The center-left panel shows good agreement between the quasi-linear computations and the database, in particular for the circular discharges. Once again, the range is dominated by Tore Supra entries, which are very well matched by our computation. As before, the scatter increases when comparing λ_q rather and L_q, but for circular discharges we obtain $R^2 = 0.60$, which is comparable to the fit qualities obtained in [10]. For shaped discharges, on the other hand, we find poor agreement between the quasi-linear theory and the experiment.

A careful analysis of the quasi-linear scaling was carried out in order to identify the minimal model equations that yield equation 8. The quasi-linear computation was repeated several times, carefully choosing terms that are known to influence the dynamics of resistive and inertial drift and ballooning modes. It was found necessary to retain most terms of the drift-Braginskii equations, with resistive/inertial drift and ballooning modes all being important in determining the SOL width. This is a result of the large range in the parameters $\rho_\star$, ν, and q_{95}, all of which play an important role in determining the dominant linear instability.

Finally, a scaling for L_q, as a function of ν, $\rho_\star$, and q_{95}, is obtained directly from the data using robust non-linear regression of the circular discharges in the database, which gives

$$L_{q,\mathrm{fit}} = 0.094\nu^{-0.02\pm0.02}\rho_\star^{-0.71\pm0.05}q_{95}^{0.76\pm0.06}. \quad (9)$$

The comparison against experimental data is shown in the bottom panels of figure 3. The centre and bottom panels of figure 3 are quite similar, which is to be expected since expressions 8 and 9 have essentially the same exponents.

5. Conclusion

In conclusion, straightforward analysis of the ITPA database for the main IWL SOL heat-flux widths reveals that a quasi-linear transport theory (equation 8) can produce the same regression fit quality as a brute force non-linear fitting procedure based on the same parameters. It can even achieve a similar degree of accuracy as that obtained by engineering parameter scalings using many more regression parameters. We find, however, that the fitting range is dominated by Tore Supra data. This is also the device whose configuration is best described by our model.

§ Robust non-linear regression algorithms are more reliable than least-squares regression when treating noisy sets of data. This is achieved in part by iteratively adjusting the weights of outlier points to increase the fit quality.

The agreement between theory and measurements is very good for circular discharges, which was the scenario considered by the theory, but poor for shaped discharges. Previous theoretical studies for edge turbulence in closed field line configurations (e.g. [24]) have shown that turbulent transport decreases when elongation or triangularity increase. Extrapolating these results to SOL turbulence, it appears that shaping should induce order unity modifications to λ_q. In fact, we expect that λ_q should decrease with increasing elongation or triangularity. However, the precise combination of effects affecting transport, namely, linear stability, field line length, and flux surface area, is rather intrincate. Consequently, the precise trends forcibly need to be extracted and interpreted with the aid of non-linear SOL turbulence simulations, and will be subject of future work.

The main result of this work can be summarized by expressing equations 8 and 9 in physical units $[\mathrm{m}^{-3}, \mathrm{eV}, \mathrm{m}, \mathrm{T}]$:

$$\lambda_{q,\mathrm{QL}} = 1.93 \times 10^{-4} n_0^{0.07} T_{e0}^{0.06} R^{0.68} q^{0.84} B_\phi^{-0.38} [m] \tag{10}$$

$$\lambda_{q,\mathrm{fit}} = 2.83 \times 10^{-3} n_0^{0.02} T_{e0}^{0.10} R^{0.73} q^{0.76} B_\phi^{-0.29} [m]. \tag{11}$$

Some further relation with engineering parameters can be extracted from these expressions. For simplicity, assume that $\lambda_q \sim R^{0.75} q^{0.75} B_\phi^{0.25}$. Then, introducing $I_p \sim aB_\theta$ we obtain $\lambda_q \sim B_\phi^{0.5} (I_p/a^2)^{-0.75}$.

The SOL transport dynamics assumed in the transport model involves resistive turbulent modes and sonic flows towards the limiter, both of which are strongly affected by the local temperature and density. In the end, it is fortuitous that the dependence on density and temperature almost vanishes from the final result. Therefore λ_q can be estimated using these formulas even if there is large uncertainty on those values.

Finally, we stress that this exercise has demonstrated that a turbulent transport theory involving non-linearly saturated turbulence and sheath losses can describe the dependence of λ_q with respect to the plasma parameters found in a large experimental database. Future avenues of research will concentrate on understanding plasma shaping effects (including the addition of an X-point), and evaluating the effects of turbulent transport on the formation of the near-SOL narrow heat-flux feature.

Acknowledgments

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Appendix A. Drift-reduced Braginskii equations

For completeness, we include here the drift-reduced Braginskii equations used as a basis for the transport analysis. The derivation of these model equations was first presented in [17]. Starting from the Braginskii two fluid equations [25], we impose the orderings $d/dt \ll \omega_{ci}$, $k_\perp \gg k_\parallel$, $\beta \ll 1$. The neglect of magnetic flutter is justified by the low β at the LCFS in the discharges studied. We consider a cold ion model, as discussed in section 2. Finite ion temperature effects are quantified in [14], the principal result being that curvature driven modes (*i.e.* resistive ballooning modes) are slightly enhanced with respect to the cold ion model. Furthermore, since the equations are used in a quasi-linear analysis, we include here only those collisional terms that have a noticeable effect on the linear stability – namely, the thermal force and the parallel Spitzer resistivity. The drift-reduced equations, *in normalized units*, read as follows:

$$\partial_t n = -\frac{\rho_\star^{-1}}{B}[\phi, n] - \nabla_\parallel\left(n v_{\parallel e}\right) + \frac{2}{B}\left[\hat{C}\left(p_e\right) - n\hat{C}\left(\phi\right)\right] \tag{A.1}$$

$$\partial_t \omega = -\frac{\rho_\star^{-1}}{B}[\phi, \omega] - v_{\parallel i}\nabla_\parallel \omega + \frac{2B}{n}\hat{C}\left(p_e\right) + \frac{B^2}{n}\nabla_\parallel j_\parallel \tag{A.2}$$

$$\partial_t v_{\parallel e} = -\frac{\rho_\star^{-1}}{B}\left[\phi, v_{\parallel e}\right] - v_{\parallel e}\nabla_\parallel v_{\parallel e} + \frac{m_i}{m_e}\left(\nu\frac{j_\parallel}{n} + \nabla_\parallel \phi - \frac{1}{n}\nabla_\parallel p_e - 0.71\nabla_\parallel T_e\right) \tag{A.3}$$

$$\partial_t v_{\parallel i} = -\frac{\rho_\star^{-1}}{B}\left[\phi, v_{\parallel i}\right] - v_{\parallel i}\nabla_\parallel v_{\parallel i} - \frac{1}{n}\nabla_\parallel p_e \tag{A.4}$$

$$\partial_t T_e = -\frac{\rho_\star^{-1}}{B}[\phi, T_e] - v_{\parallel e}\nabla_\parallel T_e + \frac{4}{3}\frac{T_e}{B}\left[\frac{7}{2}\hat{C}\left(T_e\right) + \frac{T_e}{n}\hat{C}\left(n\right) - \hat{C}\left(\phi\right)\right] + \frac{2}{3}T_e\left(\frac{0.71}{n}\nabla_\parallel j_\parallel - \nabla_\parallel v_{\parallel e}\right), \tag{A.5}$$

where $\omega = \nabla_\perp^2 \phi$ is the vorticity and equation (A.2) has been simplified using the Boussinesq approximation $\nabla \cdot (n d_t \nabla_\perp \phi) \approx n d_t \nabla_\perp^2 \phi$. The parallel current is given by $j_\parallel = n\left(v_{\parallel i} - v_{\parallel e}\right)$. In addition, $[f, g] = \mathbf{B} \cdot (\nabla f \times \nabla g)/B$ is the Poisson bracket, while $\hat{C}(f) = (B/2)\left[\nabla \times \left(\mathbf{B}/B^2\right)\right] \cdot \nabla f$ is the curvature operator. The (scalar) parallel derivative can be expressed as $\nabla_\parallel f = (\partial/\partial\varphi + q^{-1}\partial/\partial\theta)f$.

The reference units used to normalize the equations are R/c_s (time), ρ_s (perpendicular length), R (parallel length), B_ϕ (magnetic field), T_{e0} (temperature), n_0 (density), and e/T_{e0} (potential). The major radius R and the magnetic field B_ϕ are defined at the magnetic axis, while n_0 and T_{e0} are local quantities defined at the LCFS. The ion sound Larmor gyroradius ρ_s is defined using T_{e0} and B_ϕ. The dimensionless parameters $\rho_\star$ and ν are defined in equations 1 and 2.

We consider a circular plasma geometry with a toroidal limiter set at the high-field side equatorial mid plane, with the curvature terms described using the typical $\hat{s} - \alpha$ metric [26]. The quasi-linear transport solver considers a linearized version of these equations [22] to evaluate the required flux $\sim \gamma/k_\theta$.

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