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A new tool for modelling ion cyclotron resonance heating wave propagation and damping in non-axisymmetrical magnetic confinement fusion machines

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Introduction & scope

Ion cyclotron resonance heating (ICRH) is a routinely used method to bring plasmas to fusion relevant temperatures in magnetic confinement fusion machines (see e.g. [1]). Properly modelling the dielectric response is a challenge both from the physics and the computational point of view. More often than not simplifications are made, first of all by truncating the dielectric tensor at the leading order terms in finite Larmor radius corrections and secondly by simplifying the geometry. A new solver is under development at LPP-ERM/KMS to help understand the ICRH wave propagation and damping in stellarators such as Wendelstein-7X. It is adopting the basic philosophy pioneered by Jaeger [2] for the AORSA code, and intends to complement the physics described in the SCENIC code (see e.g. [3]) with a rigorous computation of the wave polarisation at higher cyclotron harmonics. Whereas SCENIC is much more advanced in many respects (it is coupled to an equilibrium as well as a Fokker-Planck solver and capable of treating non-Maxwellian minority populations), the key idea here is to assess the importance of higher order finite Larmor radius corrections while keeping the zero order distribution functions simple (Maxwellian) and the geometry prescribed. At this stage 2 roads are being explored in parallel: (i) a "brute force" technique that puts the computational effort on supercomputers as it requires inversion of massive, full matrices, and (ii) a technique that explores the possibility of reducing the computational effort required by semi-analytically solving some of the required integrals by hand and exploiting the fact that the net effect of very fast oscillations potentially offers a justification for reducing the full matrix to a sparse(r) one. The work presented here is part of an ongoing longer term project and only sketches preliminary findings.

Wendelstein 7-X

The purpose of the Wendelstein 7-X (W7-X) reactor - built and operated at the Max Planck Institute for Plasma Physics in Greifswald (Germany) - is to assess the potential of a *stellarator* as a future fusion reactor. On the road to commercialisation of fusion-based electricity production, demonstrating that fusion-born α particles are confined long enough to ensure the plasma stays at fusion-relevant temperatures without auxiliary heating once ignition has been reached is a key issue. One of the objectives of W7-X is to demonstrate that fast particle confinement is sufficient in stellarators to

ensure this "self-heating", the latter relying on the energy transfer via Coulomb collisional slowing-down of fast particles on the bulk ions and electrons. W7-X is not designed to withstand abundant fusion-born α particles but will use ion cyclotron resonance heating to bring ions to high energies allowing subsequently to study this crucial confinement issue. The geometry of stellarators is fully 3-dimensional (see Fig.1) and heating schemes do not necessarily guarantee total absorption in a single transit over the plasma. Therefore, since edge reflections and local wave trapping are ingredients that need to be accounted for, dedicated tools are required to model wave propagation and damping in such a device.

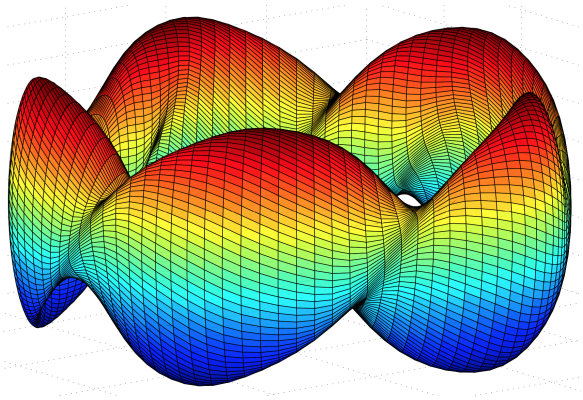

Figure 1: A typical shape of the geometry of the plasma in W7-X [4].

Basic equations

Maxwell's equations can be combined to yield the wave equation

$$\nabla \times \nabla \times \vec{E} = i\omega\mu_o \vec{J}_{ant} + k_0^2 \vec{E} + i\omega\mu_o \int d\vec{k}\overline{\overline{\sigma}}(\vec{x},\vec{k}).\vec{E}_{\vec{k}} exp[i\vec{k}.\vec{x}],$$

in which $\vec{E}_{\vec{k}}$ is the $\vec{k}$-mode contribution to the plasma current and $\overline{\overline{\sigma}}$ the corresponding conductivity tensor, related to the dielectric tensor $\overline{\overline{K}}$ by the relation $\overline{\overline{\sigma}} = -i\omega\varepsilon_o\overline{\overline{K}} + i\omega\varepsilon_o\overline{\overline{1}}$. Adopting expressions pioneered by Swanson [5], Jaeger solves this set of 3 equations "as is" using a particularly straightforward version of the collocation method. It imposes the equation to be satisfied at a given number of grid points, the number of which is equal to the number of unknown complex $\vec{k}$-mode amplitudes [2]. The integrals over the continuous variable $\vec{k}$ are approximated by a sum over a suitable, discrete set of $\vec{k}_i$ values. As this yields a matrix system that is full it comes as no surprise that brute force mimicking of Jaeger's adopted strategy requires the use of supercomputers, even for relatively modest densities of the numerical grid. Work is ongoing to allow maximally making use of the CPU time and memory of fast parallel computers but so far our efforts have not allowed to reach a level of accuracy to capture long (fast wave) as well as short (ion Bernstein or ion cyclotron wave) wavelength behaviour in stellarator geometry in a sufficiently trustworthy way. A straightforward technique to potentially

avoid such prohibitively massive number crunching is to use the definition of the $\vec{k}$-space component and re-introduce the local field. Swapping the order of the integrals this yields

$$\nabla \times \nabla \times \vec{E} = + \frac{k_0^2}{(2\pi)^3} \int d\vec{x}' \int d\vec{k} \overline{\overline{K}}(\vec{x},\vec{k}). \left[exp[-i\vec{k}.(\vec{x}-\vec{x}')] \vec{E}(\vec{x}') \right].$$

In the next 2 sections, a method is discussed to solve the associated non-local linear equation system, and a way to tackle the $\vec{k}$-integrals semi-analytically.

The influence matrix method for linear partial differential equations

The "influence matrix" method [6] locally adopts a polynomial approximation of the solution of a system of equations obtaining constraints on the polynomials' coefficients by substituting the solution for its approximation into the equations. Suppose a sufficiently accurate polynomial representation requires m coefficients $c_{1,...,m}$ in total and that there are n system equations. These n conditions only partly determine the choice of constants in the chosen approximation, leaving $\tilde{m} = m - n$ coefficients to be chosen freely. The remaining degrees of freedom allow to determine a set of independent solutions by imposing $c_{1,...,\tilde{m}} = (1,0,...,0)$, $c_{1,...,\tilde{m}} = (0,1,...,0)$, ..., $c_{1,...,\tilde{m}} = (0,0,...,1)$ and finding the $\tilde{m}$ independent local solutions. Consistent with the approximations made, these local "base" solutions allow to reconstitute the general solution to the set of system equations. The solutions in neighbouring elements are connected to the solution in a given element based on physics continuity principles for field components and fluxes; the order of the adopted polynomials needs to be high enough to ensure such continuity can be imposed. All local constraining equations are grouped in a large system matrix. For 1D application, the wave equation at grid point x_i can be written

$$\overline{\overline{N}}_0.\vec{E}_{0,i} + \overline{\overline{N}}_1.\vec{E}_{1,i} + \overline{\overline{N}}_2.\vec{E}_{2,i} = \sum_j \left[\overline{\overline{H}}_0.\vec{E}_{0,j} + \overline{\overline{H}}_1.\vec{E}_{1,j} + \overline{\overline{H}}_2.\vec{E}_{2,j} \right]$$

when the electric field is locally approximated by $\vec{E} = \vec{E}_{0,i} + \vec{E}_{1,i}(x-x_i) + \vec{E}_{2,i}(x-x_i)^2$. Note that the right hand side - in which the dielectric response has been modeled - is a nonlocal term: it contains contributions from all grid points x_j; the left hand side term arises from the $\nabla \times \nabla \times$ operator and is only involving the collocation method grid point x_i. Depending on whether the non-locality only requires "close" neighbours or also involves "wide" neighbours, the corresponding system matrix is more or less sparse.

Semi-analytical treatment

The $\vec{k}$-integral that needs to be evaluated is only significantly differing from zero in $\vec{k}$-space up to a certain value of the wave vector amplitude: the various terms involve the exponentially scaled modified Bessel function $exp[-\lambda]I_N(\lambda)$ which behaves as $\lambda^{-1/2}$ for large λ; $\lambda = \rho_{Larmor}^2 k_\perp^2/2$, ρ_{Larmor} is the Larmor radius and $k_\perp$ the perpendicular wave number. Moreover, the factor $exp[i\vec{k}.(\vec{x}-\vec{x}')]$ is oscillating rapidly as a function of the distance between $\vec{x}$ and $\vec{x}'$ when $|k|$ is large, and hence the net

integrated effect of these oscillations dies away even when the modified Bessel function term does not. The oscillatory integrals that need to be integrated can be subdivided into chunks in $\vec{x}'$ and $\vec{k}$-space. The resulting "building block" integrals can be computed once-and-for-all and stored in an interpolation table using the variables $a = \Delta_k \Delta_x$, $\xi_o = \xi_i + (x_i - x_j)/\Delta_x$ and $\zeta_o = -k_N/\Delta_k$. These integrals can be written in terms of the normalised variables ξ and ζ:

$$I_{i,j,N,m,n} = \Delta_x^{m+1} \Delta_k^{n+1} \int_0^1 d\xi \int_0^1 d\zeta exp[-ia(\zeta - \zeta_o)(\xi - \xi_o)] \zeta^n \xi^m.$$

It can be seen in Fig. 2 that such integrals are well behaved and fade away fairly quickly for ζ_o and a but that the ξ_o behaviour is more rapid and thus requires higher accuracy and a wider ξ_o domain to be retained. The success of the applicability of the present method hinges on how fast these *net* summed contributions fade away for more distant points. Depending on the number of neighbouring x''s that need to be considered for a reference x, the system matrix reduces from a full to a sparser matrix. In 1 dimension, inverting a full matrix does not require excessively much CPU time or memory but the gain that can be achieved when addressing the problem in the actual stellarator geometry - in which not only radial and poloidal but also toroidal modes are coupled - is significant.

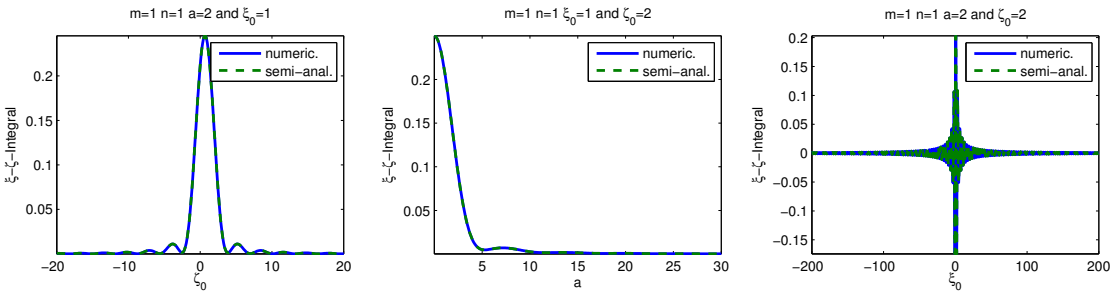

Figure 2: Example of the elementary integrals in terms of ζ_o, a and ξ_o for $m = n = 1$.

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