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Influence of tangential drifts on neoclassical transport in optimized stellarators

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Introduction

In general, the orbit-averaged radial magnetic drift is non-zero for trapped particles in stellarators. Stellarators in which the orbit-averaged radial magnetic drift vanishes are called omnigeneous. Although exactly omnigeneous configurations are not mathematically forbidden, achieving perfect omnigeneity in practice requires such an accuracy in the design and placement of the coils that, even in future devices, deviations from omnigeneity are unlikely to be negligible. These deviations are more deleterious at small collisionalities. The $1/\nu$ regime has been recently treated in stellarators close to quasisymmetry [1], which are particular cases of stellarators close to omnigeneity. Here, the techniques learnt in [1] are generalized to stellarators close to omnigeneity and applied to collisionality values below the $1/\nu$ regime.

Omnigeneous stellarators and stellarators close to omnigeneity

We use spatial coordinates $\{\psi,\alpha,l\}$, where ψ determines the flux surface, α is a poloidal angle (this is simply to fix ideas; α might have a different helicity and the treatment would be analogous) that labels magnetic field lines once ψ has been fixed, and l is the arc length over the magnetic field line selected by fixing ψ and α.

Passing particles always have vanishing average radial magnetic drift. A stellarator is called omnigeneous if the orbit-averaged radial magnetic drift is zero for all trapped particles [2]. This is equivalent to saying that $\partial_\alpha J = 0$, where

$$J = 2\int_{l_{b1}}^{l_{b2}} |v_{||}| \mathrm{d}l \tag{1}$$

is the second adiabatic invariant, and l_{b1} and l_{b2} are the bounce points. Hence, a stellarator is omnigeneous if and only if J is a flux function.

Below, we write magnetic fields for stellarators close to omnigneity as

$$\mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B}_1, \tag{2}$$

where $\mathbf{B}_0$ is omnigeneous and $\delta\mathbf{B}_1$ is a perturbation with $0 \le \delta \ll 1$ and $\mathbf{B}_1 \sim \mathbf{B}_0$. We assume that $|\nabla \ln B_0|^{-1} \sim |\nabla \ln B_1|^{-1} \sim L_0$.

Low-collisionality drift-kinetic equation: $\rho_{i*} \ll \nu_{i*} \ll 1$

Let us use coordinates $\{v, \lambda, \sigma\}$ in velocity space, where v is the magnitude of the velocity, $\lambda = v_\perp^2/(v^2 B)$ is the pitch angle coordinate and σ is the sign of the parallel velocity. In the standard drift-kinetic expansion, the distribution function $F_i(\psi, \alpha, l, v, \lambda, \sigma)$ is expanded as $F_i = F_{i0} + F_{i1} + O(\rho_i^{*2} F_{i0})$, where $F_{i1} \sim \rho_{i*} F_{i0}$, F_{i0} is a Maxwellian distribution with density n_i and temperature T_i constant on flux surfaces and $\rho_{i*} \ll 1$ is the ion gyroradius over L_0. The electrostatic potential is expanded as $\varphi(\psi, \alpha, l) = \varphi_0(\psi) + \varphi_1(\psi, \alpha, l)$, with $\varphi_0(\psi) \sim T_i/(Z_i e)$ and $\varphi_1/\varphi_0 \sim \rho_{i*}$. Here, $Z_i e$ is the charge of the ions and e is the proton charge.

The drift-kinetic equation for the non-adiabatic component of the distribution function $G_{i1} = F_{i1} + (Z_i e \varphi_1/T_i) F_{i0}$ is[1]

$$v_{||} \hat{\mathbf{b}} \cdot \nabla G_{i1} + \Upsilon_i \mathbf{v}_{M,i} \cdot \nabla \psi F_{i0} = C_{ii}^{\ell}[G_{i1}], \tag{3}$$

where $\mathbf{v}_{M,i}$ is the ion magnetic drift,

$$\Upsilon_i = \frac{n_i'}{n_i} + \left(\frac{m_i v^2}{2T_i} - \frac{3}{2} \right) \frac{T_i'}{T_i} + \frac{Z_i e \varphi_0'}{T_i}, \tag{4}$$

primes stand for differentiation with respect to ψ and C_{ii}^{ℓ} is the linearized ion-ion collision operator. In this paper we focus on ion transport and assume that a mass ratio expansion $\sqrt{m_e/m_i} \ll 1$ has been taken, so that the ion-electron collision term has been dropped. Here, m_e is the electron mass and m_i is the ion mass.

Define the ion collisionality as $\nu_{i*} = \nu_{ii} L_0 / v_{ti}$, where ν_{ii} is the ion-ion collision frequency, $v_{ti} = \sqrt{T_i/m_i}$ is the ion thermal speed. If $\nu_{i*} \ll 1$, one can perform an expansion in powers of the collisionality. To lowest order one finds that G_{i1} is homogeneous along the coordinate l. The function G_{i1} is found from averages of the drift-kinetic equation to next order in ν_{i*}. For trapped particles we average over the orbit,

$$\int_{l_{b1}}^{l_{b2}} |v_{||}|^{-1} C_{ii}^{\ell}[G_{i1}] \, \mathrm{d}l = \left(\int_{l_{b1}}^{l_{b2}} |v_{||}|^{-1} \mathbf{v}_{M,i} \cdot \nabla \psi \, \mathrm{d}l \right) \Upsilon_i F_{i0}. \tag{5}$$

For passing particles we take the flux surface average, that we denote by $\langle \cdot \rangle_\psi$. That is,

$$\left\langle B v_{||}^{-1} C_{ii}^{\ell}[G_{i1}] \right\rangle_\psi = 0. \tag{6}$$

From the last two equations, it is clear that $G_{i1} \sim \nu_{i*}^{-1} \rho_{i*} F_{i0}$. This defines the $1/\nu$ regime.

[1]The ambipolarity condition and the quasineutrality equation that allow to solve for φ_0 and φ_1 are discussed in [3].

Drift-kinetic equation for collisionalities below the $1/\nu$ regime: $\nu_{i*} \ll \rho_{i*}$

In general, if $\nu_{i*} \lesssim \rho_{i*}$ the drift-kinetic expansion breaks down because G_{i1} becomes as large as F_{i0}. In addition, the drift-kinetic equation becomes radially non-local because there is no reason, in principle, to neglect terms like $\mathbf{v}_{M,i} \cdot \nabla\psi \partial_\psi G_{i1}$ (in this paper we do not discuss large aspect ratio effects). However, if $\delta \ll 1$ there is a rigorous way to carry out the expansion and to derive a radially local drift-kinetic equation [3]. If $\delta \ll 1$, it is possible to show that $G_{i1} = g_i(\psi, \alpha, v, \lambda)$ in the trapped region and that it vanishes in the passing region. The correct ansatz to deal with the regime $\nu_{i*} \ll \rho_{i*}$ is $g_i = \delta g_i^{(1)} + \ldots$, where $g_i^{(1)} \sim F_{i0}$. Analogously, we take $\varphi_1 = \delta \varphi_1^{(1)} + \ldots$, where $\varphi_1^{(1)} \sim \varphi_0$.

Expanding in δ, we find a radially-local drift-kinetic equation valid[2] for $\nu_{i*} \ll \rho_{i*}$. Namely,

$$-\partial_\psi J^{(0)} \partial_\alpha g_i^{(1)} + \partial_\alpha J^{(1)} \Upsilon_i F_{i0} = \sum_\sigma \frac{Z_i e \Psi_t'}{m_i c} \int_{l_{b10}}^{l_{b20}} \frac{\mathrm{d}l}{|v_{||}^{(0)}|} C_{ii}^{\ell(0)}[g_i^{(1)}],$$

where Ψ_t is the toroidal magnetic flux over 2π, l_{b10} and l_{b20} are the orbit bounce points calculated using B_0, c is the speed of light,

$$\partial_\psi J^{(0)} = -\int_{l_{b_{10}}}^{l_{b_{20}}} \frac{\lambda v \partial_\psi B_0 + 2Z_i e/(m_i v) \partial_\psi \varphi_0}{\sqrt{1-\lambda B_0}} \mathrm{d}l$$

and

$$\partial_\alpha J^{(1)} = -\int_{l_{b_{10}}}^{l_{b_{20}}} \frac{\lambda v \partial_\alpha B_1 + 2Z_i e/(m_i v) \partial_\alpha \varphi_1^{(1)}}{\sqrt{1-\lambda B_0}} \mathrm{d}l.$$

We have employed a superindex (0) for quantities corresponding to $\mathbf{B}_0$ and a superindex (1) for perturbed quantities.

Let us define the poloidal frequency $\omega_\alpha = m_i c \partial_\psi J^{(0)}/(Z_i e \Psi_t' \tau_b^{(0)})$, where $\tau_b^{(0)}$ is the orbit time in the magnetic field $\mathbf{B}_0$. Typically, $\omega_\alpha \sim \rho_{i*} v_{ti}/L_0$ and the drift-kinetic equation is solved by expanding in $\nu_{ii}/\omega_\alpha \sim \nu_{i*}/\rho_{i*} \ll 1$. To lowest order in the ν_{ii}/ω_α expansion one obtains $g_i^{(1)} = g_0 + \ldots$, with

$$g_0 = \frac{1}{\partial_\psi J^{(0)}} \left(J^{(1)} - \frac{1}{2\pi} \int_0^{2\pi} J^{(1)} \mathrm{d}\alpha \right) \Upsilon_i F_{i0}.$$

Energy flux when $\nu_{i*} \ll \rho_{i*}$

It is easy to realize that g_0 does not contribute to the energy flux, Q_i. The physical effect that explains neoclassical transport when $\nu_{i*} \ll \rho_{i*}$ depends on certain properties of ω_α.

Customarily, for non-zero φ_0' there exists a minimum value of v for which $\omega_\alpha = 0$ for some value of λ (the value of λ for each v is usually unique). We denote this value of v by $v_{\min}$. When

[2] As explained in [3], it is expected that this drift-kinetic equation also ceases to be valid for sufficiently low collisionality.

$v \geq v_{\min}$, we define $\lambda_r(\psi, v)$ as the value of λ such that $\omega_\alpha(\psi, v, \lambda_r) = 0$. Of course, λ_r is a function of ψ and v, $\lambda_r \equiv \lambda_r(\psi, v)$.

(i) $\sqrt{\nu}$ *regime*

If $v_{\min} \gg v_{ti}$, transport is dominated by the discontinuity of g_0 at the trapped/passing boundary. This discontinuity originates a collisional layer of size $\Delta\lambda \sim B_0^{-1}(\nu_i/\omega_\alpha)^{1/2}$, and the energy flux can be shown to scale as

$$Q_{i,\sqrt{\nu}} \sim \delta^2 \frac{\nu_{ii}^{1/2}}{\omega_{\alpha,c}^{3/2}} \rho_{i*}^2 n_i m_i v_{ti}^4 L_0^{-1} S_\psi,$$

where S_ψ is the area of the flux surface. This is the $\sqrt{\nu}$ regime.

(ii) *Superbanana-plateau regime*

If $v_{\min} \lesssim v_{ti}$, transport is dominated by the divergence of g_0 at the resonant values of the pitch-angle coordinate, $\lambda_r(\psi, v)$. In this case, the energy flux turns out to be independent of the collisionality,

$$Q_{i,\mathrm{sb-p}} \sim \delta^2 \rho_{i*} n_i m_i v_{ti}^3 S_\psi.$$

Additive formula for the energy flux

Since the layers corresponding to the $\sqrt{\nu}$ and to the superbanana-plateau regimes are small and located at different regions of phase space, their contributions to transport are additive. This means that we can write, for $\nu_{i*} \ll \rho_{i*}$,

$$Q_i = Q_{i,\sqrt{\nu}} + Q_{i,\mathrm{sb-p}}.$$

An explicit expression for this formula is provided in [3].

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References

[1] I. Calvo et al., *Plasma Phys. Control. Fusion* **56**, 094003 (2014).

[2] J. R. Cary *et al.*, *Phys. Plasmas* **4**, 3323 (1997).

[3] I. Calvo F. I. Parra, J. L. Velasco and J. A. Alonso, “The effect of tangential drifts in stellarators close to omnigeneity", in preparation.