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Fitting of the Thomson scattering density and temperature profiles on the COMPASS tokamak[a)]

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The high-resolution Thomson scattering system on the COMPASS tokamak provides radial profiles of electron temperature and density in the central and edge plasma region. The spatial resolution in the edge plasma region is optimized for edge transport barrier studies. Formation of the so-called pedestal (steep gradient region) in the H-mode edge profiles is observed. Both the temperature and density pedestals are fitted with modified hyperbolic tangent function, which is a convenient method to obtain the pedestal parameters. A new technique for fitting the full radial profiles in H-mode discharges is described. Deconvolution with the diagnostic instrument function is applied on the profile fit, taking into account the dependence on the actual magnetic configuration.

I. Introduction

Study of a transport barrier formed in the tokamak plasma edge region in the high confinement mode (H-mode) is important for future fusion machines like ITER [Beurskens], [Snyder]. A steep pressure gradient region, also called the pedestal, is characteristic for the H-mode plasma edge. High pressure gradients in the pedestal lead to edge-localized modes (ELMs), which can be potentially dangerous in large tokamaks. Systematic pedestal measurement and description for scaling of the observations from existing to future tokamaks is necessary [Kallenbach], [Lackner]. Methods of determining the pedestal parameters of the H-mode radial profiles of electron temperature (T_e) and density (n_e) obtained by Thomson scattering (TS) diagnostic on the COMPASS tokamak are described.

The COMPASS tokamak ($R = 0.56$ m, $a = 0.2$ m, $B_T = 0.8$-2.1 T, I_{plasma} up to 400 kA) at the Institute of Plasma Physics in Prague operates in divertor plasma configuration with ITER-like plasma cross-section [Panek ?]. Ohmic and neutral beam injection (NBI) assisted H-modes are achieved reliably. NBI assisted H-mode is provided by two 0.3 MW, 40 keV beams. Different types of ELMs have been observed, including the type I ELMs [Panek 2015].

The high resolution TS diagnostics (HRTS) on the COMPASS tokamak was designed to measure radial profiles of electron temperature and electron density in the central and the edge region of the plasma column with spatial resolution optimized for edge transport barrier studies (~1/100 of the tokamak minor radius in the edge TS system) [Bilkova NIMA], [Bohm 2014], [Aftanas jinst]. It consists of two semi-independent core and edge subsystems sharing the laser beams of two Nd:YAG lasers, with 30 Hz repetition rate and 1.5 J pulse energy each. An overlapping region of core and edge TS validates the set-up of the two systems, together collecting the scattered radiation from 54 spatial points. The collected light is transferred through fibre bundles into polychromators where it is processed using a set of spectral filters, avalanche photodiodes, and fast analogue-to-digital converters.

In this paper, **section II** presents a conventional technique of fitting the modified hyperbolic tangent function (mtanh) to the edge region of H-mode radial HRTS profiles of electron temperature (T_e) and density (n_e). In **section III**, a new technique for fitting of the full radial profile by a single analytical function is described. Advantages and applications are discussed. **Section IV** shows the deconvolution of TS profiles with the instrument function with respect to the actual magnetic configuration, and conclusions are presented in **section V**.

II. Fitting of the edge pedestal

During H-mode, a formation of characteristic edge profiles with pedestals is observed. It is generally accepted that the electron temeperature T_e and density n_e pedestals can be fitted by a so-called modified hyperbolic tangent function (mtanh), which has five parameters with straightforward meaning and was introduced by [Groebner,Carlstrom]. The function that describes the shape of the edge transport barrier:

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$$F_{\mathrm{ped}}(r,b)=\frac{b_{\mathrm{height}}-b_{\mathrm{SOL}}}{2}\left[\mathrm{mtanh}\left(\frac{b_{\mathrm{pos}}-r}{2b_{\mathrm{width}}},b_{\mathrm{slope}}\right)+1\right]+b_{\mathrm{SOL}}$$

$$\mathrm{mtanh}\left(x,b_{\mathrm{slope}}\right)=\frac{\left(1+b_{\mathrm{slope}}x\right)e^{x}-e^{-x}}{e^{x}+e^{-x}},$$

where b_{height} is the pedestal height, b_{SOL} is the pedestal offset in the scrape-off-layer, b_{pos} is the pedestal position, b_{width} is a quarter of the pedestal width and b_{slope} is the slope of the inner side of the profile. The pedestal height and width are important and widely used pedestal scaling parameters (e.g. [Beuerskens]). Note that by r we denote the minor radius, which can be replaced by any other quantity with similar meaning, such as normalized poloidal magnetic flux ψ_{norm} or its square root ρ; the meaning of the fit parameters depends on the chosen radial coordinate.

III. Fitting of the full H-mode profile

Besides the conventional way of fitting the edge profiles by the modified hyperbolic tangent function, there is a motivation to find an empirical function with similarly clear meaning of parameters that could be used to fit the core profiles. Formation of profiles with consistent, characteristic shape is expected, especially in the case of T_{e}, where a peaked core profile is related to the way of plasma heating, and a convex shape above the pedestal (so-called stiffness of the profile) is judged by the nature of turbulence-driven transport. There is also a flat, non-stiff central region of the T_{e} profile, observed on many devices and described in transport models [Garbet 2004], which is further flattened in presence of sawtooth instability. It is desirable to utilize the observed consistency in a fitting function.

A common approach is to separate the edge and core profiles at a selected radial position r_{split}, and fit the core part by a general polynomial of a given order constrained by boundary conditions such as zero derivative at r=0 and smooth connection to the F_{ped} function at $r = r_{\mathrm{split}}$. Introduction of the boundary conditions leads to a complicated mathematical formula, where the role of individual parameters is practically unfathomable. In this paper we present a relatively simple empirical formula satisfying both the boundary conditions mentioned above:

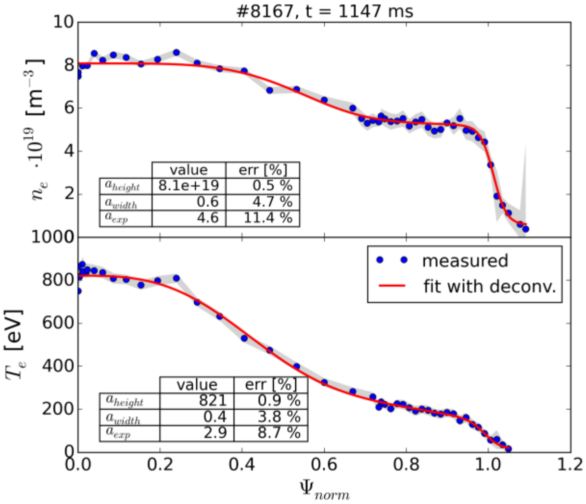
$$F_{\mathrm{full}}(r,a,b)=F_{\mathrm{ped}}(r,b)+\left[a_{\mathrm{height}}-F_{\mathrm{ped}}(r,b)\right]\cdot e^{-\left(\frac{r}{a_{\mathrm{width}}}\right)^{a_{\mathrm{exp}}}}$$

The smooth transition between the core and edge part is achieved by the combination of the pedestal function F_{ped} with an exponential term similar to a Gaussian but with a general power a_{exp}: in the edge part ($r > a_{\mathrm{width}}$), the exponential term vanishes and the function equals to F_{ped}, whereas in the plasma centre ($r \rightarrow 0$), the exponential term approaches one, F_{ped} is cancelled out and the Gaussian shape dominates:

$$F_{\mathrm{full}}(r,a,b)\approx a_{\mathrm{height}}\cdot e^{-\left(\frac{r}{a_{\mathrm{width}}}\right)^{a_{\mathrm{exp}}}} \quad ; \quad r\rightarrow 0$$

This "modified Gaussian function" has only 3 free fitting parameters in addition to the 5 fixed parameters of F_{ped}, which are fully determined from the pedestal part. The role of the parameter a_{height} is obviously the ordinate in the plasma centre, but the remaining two define the width and shape of the peaked core profile in a less straightforward way; a future improvement of this fitting function is foreseen to provide parameters with a clearer effect on the gradient in the core profile. An important feature of the function F_{full} is the fact that it doesn't depend on the assessment of r_{split}, unlike a constrained polynomial which has a rather complex dependency on this uncertain parameter.

Fig. C shows the function F_{full} fitted to radial profiles of n_{e} and T_{e} measured by HRTS during an ELM-free H-mode on COMPASS; the inserted table contains the function's parameters and their errors, which were obtained by least squares method with all parameters of F_{ped} being fixed from a previous step. Testing on a range of different scenarios has shown that the three free parameters are sufficient for the function to conform to the typical shape of the core T_{e} profiles, including profiles obtained just after a sawtooth crash—these have a notably broad and flat central part, which is very difficult to fit by any low-order polynomial. However, the function's compatibility with the core n_{e} profiles is much less robust; it fits well only in cases of peaked profiles with the core n_{e} value above the one at the top of the pedestal, such as in Fig. C. Apart from T_{e}, the function can be systematically fitted also to the core profiles of electron pressure p_{e}, which have very similar shape.



	value	err [%]
a_{height}	8.1e+19	0.5 %
a_{width}	0.6	4.7 %
a_{exp}	4.6	11.4 %

	value	err [%]
a_{height}	821	0.9 %
a_{width}	0.4	3.8 %
a_{exp}	2.9	8.7 %

FIG. C. Example of the full profile fit.

The fitting function F_{full} has several advantages. The full radial profiles are well represented by a simple formula with meaningful parameters, which facilitates interpretation of the experimental data. The restricted shape of the function emphasizes the significant features of the profile, while ignoring the local extremes created by the scatter of measured points. A monotonic fit, which can be easily secured by constraining its parameters, is in many cases the only physically reasonable result; it is also very useful for analysis of the profile's gradient, which may serve e.g. to transport studies or calculations of magnetic equilibrium and MHD stability. Fitting of this function to scattered data is robust and stable thanks to the low number of parameters and the possibility to provide accurate initial estimates. Implementation of this new technique of profile fitting into a numerical code for analysis of MHD stability of experimental profiles from COMPASS as well as other European tokamaks is expected to reduce the need for human intervention, compared to the present polynomial fitting.

IV. Instrument function and deconvolution of the profiles

The finite resolution and finite size of spatial points of a diagnostic measuring any profile influence the obtained shape: steep gradients (like H-mode n_e pedestal) are broadened. This "convolution" effect is described in [Scannell 2011] in detail. Measurement of T_e by TS diagnostic is influenced also by the underlying density profile, because the scattering spectra broadening analysed in one spatial point is weighted by the signal intensity, corresponding to the density. This leads to an overestimation of the pedestal position, when fitting the profile without deconvolution. The measured profiles are given by

$$n_{e,observed} = \frac{\int n_e(\Psi_{\text{norm}}) I(\Psi_{\text{norm}}) d\Psi_{\text{norm}}}{\int I(\Psi_{\text{norm}}) d\Psi_{\text{norm}}}$$

$$T_{e,observed} = \frac{\int T_e(\Psi_{\text{norm}}) n_e(\Psi_{\text{norm}}) I(\Psi_{\text{norm}}) d\Psi_{\text{norm}}}{\int n_e(\Psi_{\text{norm}}) I(\Psi_{\text{norm}}) d\Psi_{\text{norm}}},$$

where $I(\Psi_{\text{norm}})$ is the instrument function. More rigorous method of convolution is applying the instrument function convolution on raw TS signals, i.e. signals in individual spectral channels of TS diagnostic, instead of resulting n_e and T_e profiles. Nevertheless, the easier method of n_e and T_e profiles convolution is a good approximation for profiles with pedestal width greater than half of the diagnostic spatial point size Δ_{TS} [Scannell 2011].

A method of calculating the instrument function is described in [Frassinetti 2012]. The COMPASS HRTS measurement chord is oriented vertically, which necessitates to map the measured profiles to commonly used coordinates, like r or Ψ_{norm}. A code, which calculates the instrument function in the equilibrium geometry as obtained from the EFIT reconstruction for the corresponding tokamak shot and time, was implemented. The geometry is illustrated in Fig. X. The collection optics view of individual spatial points is approximated by a rectangle in poloidal cross-section, whose position and angle was obtained from HRTS spatial calibration. Laser beams intensity projection into 2D is approximated by a Gaussian curve, with 95% of energy in 1.5 mm diameter in the beam focus (middle of edge TS region) and around 3 mm at the lowest point of the core TS; the beams cross each other on the focusing lens before entering the tokamak and therefore are separated by around 3 mm in the radial direction in the observed region. An example of resulting instrument function of several TS points is shown in Fig. X2. An average full-width half-maximum (FWHM) of the edge TS is $\Delta_{TS} \approx 0.02$ in Ψ_{norm}

The instrument function is calculated for each individual spatial point of the diagnostic. The convolution equations are applied on the expected profile function, returning values of n_e and T_e corresponding to values which would be measured by the diagnostic. The fitting of deconvolved profiles means comparing these simulated values with measured values and optimising the underlying function parameters to get the best fit.

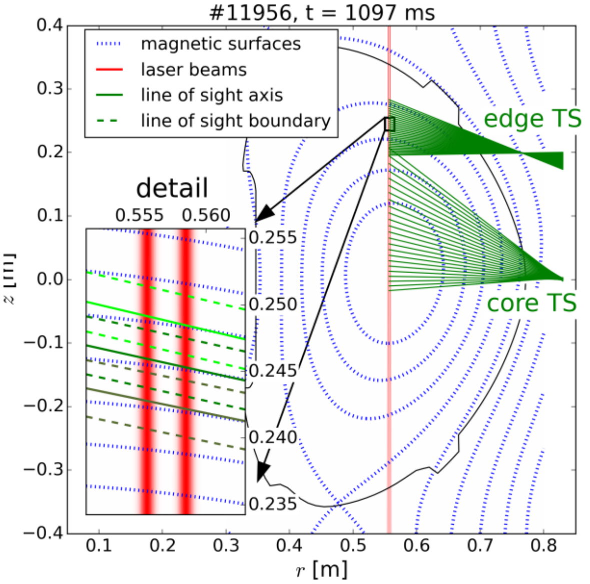


FIG. X. The geometry of the TS instrument function calculation. The detail shows the viewchords of 3 selected spatial points of HRTS.

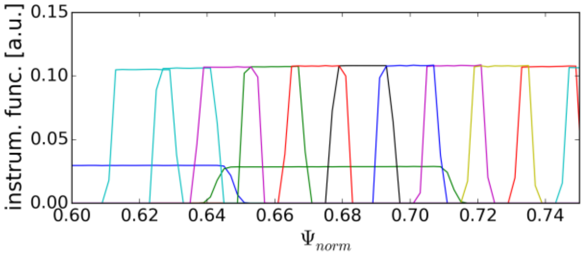
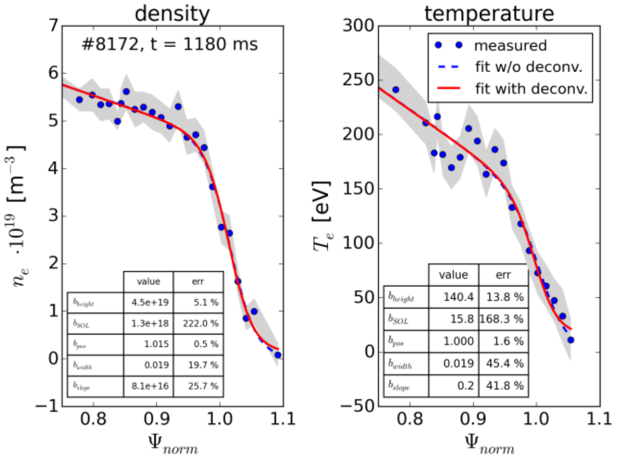


FIG. X2. Example of HRTS instrument function. Detail of region of overlap of core (lower and wider curves) and edge TS is shown.

The experimental n_e and T_e profiles were fitted with least squares method with the mtanh function, including the deconvolution effect. An example of profiles is in Fig. E, showing the experimental points (black points) and the fitted mtanh function without deconvolution (black dashed line) and with deconvolution (red solid line) and their parameters.



	value	err
b_{height}	4.5e+19	5.1 %
b_{SOL}	1.3e+18	222.0 %
b_{pos}	1.015	0.5 %
b_{width}	0.019	19.7 %
b_{slope}	8.1e+16	25.7 %

	value	err
b_{height}	140.4	13.8 %
b_{SOL}	15.8	168.3 %
b_{pos}	1.000	1.6 %
b_{width}	0.019	45.4 %
b_{slope}	0.2	41.8 %

FIG. E. Example of measured pedestal profiles (black dots) fitted with mtanh function, with deconvolution (red line) and without deconvolution (black dashed line).

The profiles obtained from experiments on the COMPASS tokamak feature pedestal widths greater than the edge TS instrument function FWHM. Hence the deconvolution effect is not dominant and the pedestal width is corrected by 5% – 20%. The effect of T_e pedestal displacement is lower than predicted by simulations in [Scannell 2011]; in these simulations, the positions of n_e and T_e pedestals were assumed to be same, while on the COMPASS tokamak T_e pedestal slightly wider than n_e pedestal is observed, both of them having the leg position same, which means the T_e pedestal is more inside the plasma than the n_e pedestal. The T_e pedstal weighting with n_e effect is not so significant then.

V. Conclusions

The newly implemented global fitting method enables robust fitting of TS core and edge profiles on the COMPASS tokamak. The fitted parameters can be easily interpreted and used directly, e.g., for stability analyses. The fitting accuracy is further increased by deconvolution of measured profiles with instrument function, which is constructed from appropriate equilibrium reconstruction. Deconvolution and profile fitting are tested on high resolution Thomson scattering diagnostic. Evaluated convolution effect of this diagnostic is not crutial, however the corresponding systematic error can be now suppressed.

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