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A Parallel 3D Discontinuous Galerkin Framework for Nonlinear Resistive MHD Simulations in Tokamak and Stellarator Geometries

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Introduction

The numerical simulation of nonlinear resistive MHD instabilities arising in the plasma of fusion devices exhibits substantial resolution requirements, both in poloidal and toroidal direction, demanding for parallelization and strong scaling of the MHD solver. In addition, with the upcoming Wendelstein 7-X Stellarator experiments, numerical tools being able to simulate fully three-dimensional, non-axisymmetric configurations are necessary.

To address these issues, we investigate the high order *discontinuous Galerkin* (DG) schemes. They are conservative schemes, with a continuous polynomial representation locally within each element. Elements couple only to direct face neighbors, and the discontinuous solution at the element interface is resolved via numerical flux functions. Due to the locality of the high order operator, the DG code exhibits high parallel efficiency [4] both for weak and strong scaling. The DG method presented is using 3D unstructured hexahedral meshes, allowing to discretize Tokamaks and Stellarators within the same framework. We briefly sketch the method and the equations as well as the code framework, which involves the generation of 3D high order meshes from MHD equilibria. Also a benchmark simulation of an internal kink MHD instability is presented and compared to the the results with the MHD solver JOREK [1, 2, 3], a well-established tool for Tokamak geometries.

DG method and MHD equations

The starting point for the Discontinuous Galerkin Spectral Element Method (DG-SEM) [5] is the conservation form of the MHD equations

$$U_t + \nabla_x \cdot \mathbf{F}(U, \nabla U) = 0, \tag{1}$$

where U_t is the time derivative of the vector of conserved quantities $U = (\rho, v, \tilde{\mathscr{E}}, B)$, with total energy $\tilde{\mathscr{E}} := \rho e + \frac{1}{2\mu_0}|B|^2$, and the flux has an advection and a viscous part: $F = F^a - F^v$

$$\begin{aligned} F^a = &\quad (\rho v \quad , \quad \rho vv + \tilde{p}\underline{\underline{\delta}} - \tfrac{1}{\mu_0}BB \quad , \quad \left(\tilde{\mathscr{E}} + \tilde{p}\right)v - \tfrac{1}{\mu_0}B(B\cdot v) \quad , \quad vB - Bv \quad), \\ F^v = &\quad (0 \quad , \quad \underline{\underline{\sigma}} \quad , \quad \underline{\underline{\sigma}}v - q_h - \tfrac{\eta}{\mu_0}(J\times B) \quad , \quad \tfrac{\eta}{\mu_0}\left((\nabla B)^T - \nabla B\right) \quad) \end{aligned} \tag{2}$$

with total pressure $\tilde{p} := p + \frac{1}{2\mu_0}|B|^2$, viscous stress tensor $\underline{\underline{\sigma}}$, heat flux q_h, current $J := \frac{1}{\mu_0}(\nabla \times B)$ and resistivity η. The divergence constraint $\nabla \cdot B = 0$ is solved with a Generalized Lagrange

Multiplier, see [6]. The solution in each element is represented by a tensor product of 1-D Lagrange polynomials of degree N vector

$$U(\xi) = \sum_{ijk=0}^{N} \hat{U}_{ijk} \psi_{ijk}^{N}(\xi), \qquad \psi_{ijk}^{N}(\xi) = \ell_i^N(\xi^1)\ell_j^N(\xi^2)\ell_k^N(\xi^3). \tag{3}$$

The Galerkin approach and a spatial integration by parts yields the weak formulation of each element E

$$\frac{\partial}{\partial t}\int_E U\phi^N \, d\mathbf{x} = -\oint_{\partial E} F_n^* \phi^N \, ds + \int_E \mathbf{F}(U, \nabla_x U)\cdot\nabla\phi^N \, d\mathbf{x}. \tag{4}$$

where the volume integral acts only on element-local polynomials and the surface integral represents the coupling between elements, where the discontinuous solution at the interface is resolved via the numerical flux function $F_n^*(U^-, U^+, \mathbf{n})$, using a Riemann solver. The integrals are solved with numerical quadrature. The DG-SEM is a 'collocation' method, where we use Gauss-Lobatto points for both integration and interpolation, leading to a diagonal mass matrix and a highly efficient dimension-by-dimension operator, see [4].

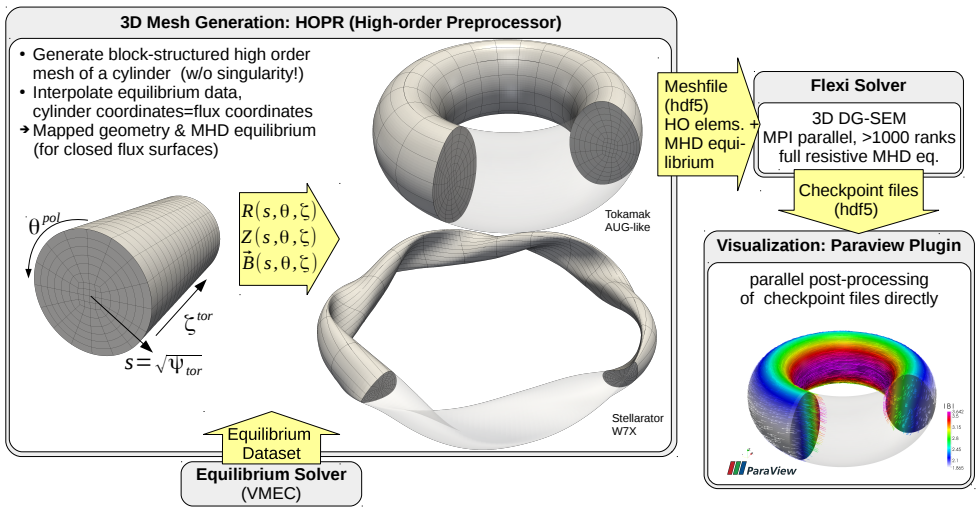

Figure 1: *Mesh generation using VMEC equilibria and parallel code framework.*

Mesh Generation and Code Framework

To start an MHD simulation, an MHD equilibrium, fulfilling $\nabla p = J \times B$, is needed on a given mesh. Currently, the computational domain represents the plasma inside closed flux surfaces, with the domain boundary following a flux surface. A 3D mesh is generated following the strategy depicted in Fig. 1, where we start with a block-structured high order mesh of a cylindrical domain (s, θ, ζ) with the radius as the flux coordinate s. The domain is mapped using VMEC [7] equilibrium data, allowing for both Tokamak or Stellarator configurations. After the mesh generation, the parallel work flow using the DG-SEM solver *Flexi* [4], together with parallel post-processing tools, can be started.

Comparison with Jorek for an Internal Kink Instability

The computational domain is a torus with circular cross-section $R_0 = 10, a = 1$. The input profiles for the Grad-Shavranov solver in JOREK to compute the ideal MHD equilibrium are given as a function of the normalized poloidal flux $\psi_n \in [0;1]$

$$\rho/\rho_0 = 1 - 0.9\psi_n\,, \quad T/T_0 = 1 - 0.8\psi_n\,, \quad F^2 = F_0^2 - 2\left(\frac{\partial \psi_n}{\partial \psi}\right)(2\psi_n - \psi_n^2) \tag{5}$$

with $\rho_0 = 1, p_0 = T_0 = 2\times 10^{-3}, F_0 = 10$, leading to a $B_0 = 1$ and $\beta = 4\times 10^{-4}$. From the G-S solve, the q-profile ranging between $[0.74;1.6]$, and the poloidal flux at the edge $\psi_{\text{edge}} - \psi_{\text{axis}} = 0.4972$ are obtained. Both are needed for the VMEC input together with the pressure profile. Both the JOREK and the VMEC equilibrium agree up to numerical precision. We generate the mesh using the VMEC data and interpolate density, pressure and magnetic field to the high order mesh, to start the *Flexi* simulation. The *Flexi* mesh has a poloidal $\times$ toroidal resolution of 832×16 elements of polynomial degree $N = 4$, resulting in ~ 1.6 Mio. DOF. The mesh and the input profiles are shown in Fig. 2.

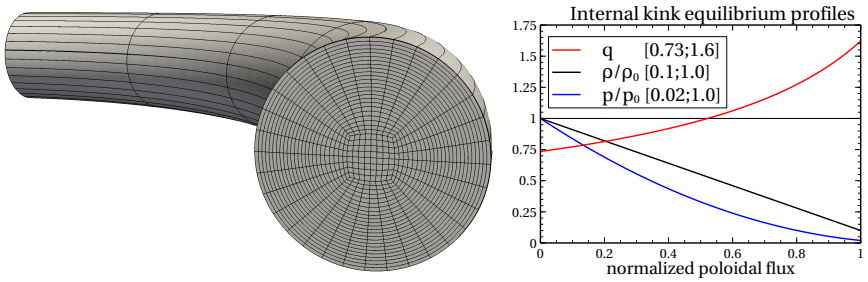

Figure 2: $1/4$ *Flexi mesh with* $R_0 = 10, a = 1$ *and profiles for the equilibrium.*

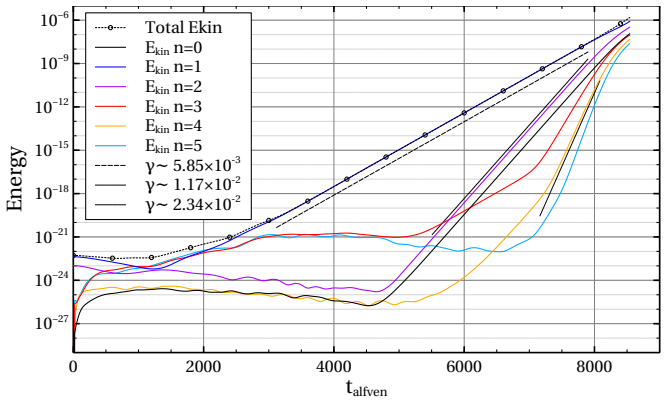

Figure 3: *Growth of toroidal kinetic energy modes over time,* $\eta = 10^{-5}, \mu = 10^{-6}$.

The simulation was running 13h on 1664 MPI ranks (104 nodes) on Helios, simulating 8500 Alfven time units. The resistivity, using the same normalization as JOREK, is set to $\eta = 10^{-5}$ and the viscosity to $\mu = 10^{-6}$. The corresponding growth of the toroidal kinetic energy modes are shown in Fig. 3. In comparison, the linear growth rate of the $(0,1)$ mode in *Flexi* of $\gamma = 0.00585$ does not exactly match the one in JOREK with $\gamma = 0.0071$, and the difference must be

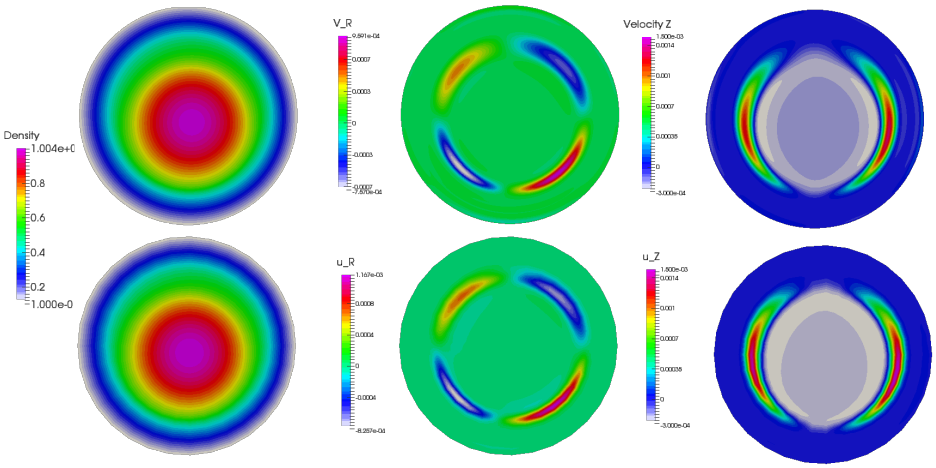

Figure 4: *Solution comparison of Flexi (upper row) and JOREK (lower row). Left to right: density, radial and vertical velocity.*

further investigated. However, the eigenfunctions of the instability agree very well, as shown in Fig. 4.

Conclusion

A new high order 3D MHD solver *Flexi* on 3D hexahedral meshes has been successfully applied to a closed flux surface Tokamak scenario and compared to results from the JOREK code. It was also already tested to strong anisotropic diffusion. The Flexi code exhibits high scalability on HPC systems and supports general 3D geometries, thus we want to provide highly resolved nonlinear MHD simulations for both Tokamak and Stellarator geometries. Future numerical developments include implicit time integration, imposition of a strong divergence constraint and mesh alignment techniques to reduce resolution requirements.

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References

[1] O. Czarny and G. Huysmans, Journal of Computational Physics **227**, 7423 (2008)
[2] G. Huysmans and O. Czarny, Nuclear Fusion **47**, 659 (2007)
[3] Haverkort J.W., de Blank H.J., Huysmans G., Pratt J., Koren B. , Journal of Comp. Physics **316**(281), (2016)
[4] F. Hindenlang, G. Gassner, C. Altmann, A. Beck, M. Staudenmaier, C.-D. Munz, Computers&Fluids **61** (2012)
[5] K. Black, Kybernetika **35**, 1 (1999)
[6] A. Dedner, F. Kemm, D. Kröner, C.-D. Munz, T. Schnitzer, M. Wesenberg, Journal of Comp. Physics **175**, 645 (2002)
[7] S.P. Hirshman, W.I. van Rij and P. Merkel, Comp. Phys. Comm. **43**, 143 (1986)