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ABSTRACT.

A fast method of calculation of the flux surfaces and magnetic fields external to a tokamak plasma in air-core tokamaks is presented. The plasma is represented by a set of wires with given current and position. Central solenoid, stray field compensation and shaping coils are represented by sets of wires with given current and location. Radial and vertical field coils are represented by wires with fixed location but variable total current. The radial and vertical force balances are satisfied by fitting the currents in the vertical and radial field coils, to obtain correct values for the force integrals over a fixed boundary, external to the plasma. The calculation was implemented in a code which is suitable for general design work on magnetic configurations, divertor geometry, and poloidal field systems. The code is interactive, and has a turnaround time of about 2 minutes.

1 INTRODUCTION

The objective of this work was to develop a fast, interactive, code for the prediction of the magnetic geometry external to the plasma in an air core tokamak, when currents in the central solenoid and in the shaping coils are given. The main application is in the initial design of the poloidal circuits and divertor geometries, where large numbers of configurations may have to be assessed, and where parametric studies are necessary.

There are two general approaches to the problem. The first is to solve the vacuum Grad—Shafranov equation in the region external to the plasma, and the complete Grad—Shrafanov equation in the internal region, and to iteratively match the solutions at the plasma boundary. This approach is implemented in the large equilibrium prediction codes, such as INVERSX [1], ODIN [2] and PROTEUS [3]. In these codes, the value of the poloidal β_p, and a prescription for the profile of the kinetic pressure, are provided by the user. In general, these codes use a large amount of CPU time, and have long turn—around times. In addition, it is often difficult to find convergence, in particular for plasma geometries that are not symmetric around the z=0 plane, eg. single null divertor plasmas.

The second approach is to represent the plasma current and the external currents by a set of wires, and simply to calculate the total field due to the wire distribution. For an air core machine, the Biot—Savart law can be used to calculate the field. For a machine with an iron yoke that is not saturated, the Greens function for a wire in any position can be found by matching the coefficients of a series of special functions, which are general solutions of the Laplace equation over the domain bounded by the iron. This method is applied in the Jakob [4] code. This code, however, contains no calculation of the force balance. The magnetic configuration obtained is therefore not necessarily an equilibrium.

Clearly, when one is interested in the fields external to the plasma it is not necessary to carry out the integration of the Grad—Shrafanov equation inside the plasma. For such problems, one would be better served with a wire code, provided that this could be extended so that the radial and vertical force balances are satisfied. In this note, we describe how these force balances can be satisfied by fitting the currents in the vertical and radial field coils respectively. The fit is a two parameter minimisation (the coil currents) with two residuals, ie. the two fixed boundary integrals (eg. over the vacuum vessel) representing the radial and vertical forces. The calculation was implemented in a code. Details of this code, and information about input requirements and output facilities, are given in a document by Gaze [5].

2 NOTATIONS AND THE FORCE BALANCES

In this section, we follow the notation and definitions given by Braams [6]; exceptions are indicated. T is the volume of the vacuum vessel, δT is the surface bounding T, Ω is a poloidal cross–section of the vacuum vessel and $\delta\Omega$ is the contour bounding Ω. dV is the volume element of T, dS the area element of Ω, dA the area element of δT and ds is the line element of $\delta\Omega$. Hence we have dV = 2πr dS and dA = 2πr ds. r is the radial coordinate of a cylindrical system r, φ, z. The normal and tangential derivatives on $\delta\Omega$ are written as $\delta/\delta n$ and $\delta/\delta s$ resp. Vectors, and tensors, are bold–faced.

The basic force balance equation, in terms of the stress tensor **T**, is $\nabla.\mathbf{T} = 0$, where

$$\mathbf{T} = p\mathbf{I} + \tfrac{1}{2}\mu_0^{-1} (B^2\,\mathbf{I} - 2\mathbf{BB}) \tag{1}$$

where p is the kinetic pressure, μ_0 is the vacuum permeability, **B** is the total magnetic field and **I** is the unit tensor. The pressure is assumed to be isotropic, and there is no plasma rotation. Taking the divergence of (1), multiplying by an arbitrary axi–symmetric vector function **Q**, separating toroidal fields B_t and poloidal fields B_p, and integrating over the volume of the torus, gives (eqn. 43 in [6]):

$$\int_T [p\,\nabla.\mathbf{Q} + \tfrac{1}{2}\mu_0^{-1} (B_t^2 - B_{t0}^2)\, r^2\,\nabla.(r^{-2}\,\mathbf{Q}) - \mu_0^{-1}\,\mathbf{B}_p.(\nabla\mathbf{Q} - \tfrac{1}{2}(\nabla.\mathbf{Q})\mathbf{I}).\mathbf{B}_p]dV$$

$$= \mu_0^{-1} \int_{\delta T} [\tfrac{1}{2}B_p^2\,(\mathbf{Q}.\mathbf{n}) - (\mathbf{Q}.\mathbf{B}_p)(\mathbf{B}_p.\mathbf{n})]dA \tag{2}$$

where **n** is the normal to the integration contour, and B_{t0} is the vacuum toroidal field.

Substituting different functions for the vector **Q** results in different expressions relating volume integrals of internal plasma pressures to surface integrals of external fields. In particular, there are the following well–known cases: $\mathbf{Q} = \mathbf{e}_r$ (where $\mathbf{e}_r$ is the radial unit vector) produces the radial force balance equation (also known as the 2nd Shafranov integral S_2); $\mathbf{Q} = \mathbf{e}_z$ produces the vertical force balance equation; $\mathbf{Q} = r\mathbf{e}_r + z\mathbf{e}_z$ produces the first Shafranov integral S_1, and $\mathbf{Q} = z\mathbf{e}_z$ produces the third Shafranov integral (note that for the third integral S_3 we follow the definition by Lazzaro [7] rather than Braams [6], who uses $r\mathbf{e}_r$).

Explicitly writing the radial force balance, and using $dA = 2\pi r ds$ to obtain a contour integral, yields:

$$S_2 \stackrel{\text{def}}{=} \int_T [p - \tfrac{1}{2}\mu_0^{-1} (B_t^2 - B_{t0}^2) + \tfrac{1}{2}\mu_0^{-1} B_p^2]\, r^{-1}\, dV$$

$$= 2\pi\, \mu_0^{-1} \int_{\delta T} [\tfrac{1}{2} B_p^2\, \mathbf{e}_r.\mathbf{n} - (\mathbf{e}_r.\mathbf{B}_p)(\mathbf{n}.\mathbf{B}_p)]\, r\, ds \qquad (3)$$

The l.h.s. represents the internal kinetic and magnetic pressures, the r.h.s. represents the force exerted by the field on the plasma boundary. The plasma is in radial force balance if eqn (3) is satisfied. (Note that if the integral is taken over the plasma boundary instead of the vacuum vessel, the second term on the r.h.s. disappears).

In the equation for the vertical force balance the l.h.s. of eqn (2) vanishes. Hence the vertical force balance equation is:

$$0 = 2\pi\, \mu_0^{-1} \int_{\delta T} [\tfrac{1}{2} B_p^2\, \mathbf{e}_z.\mathbf{n} - (\mathbf{e}_z.\mathbf{B}_p)(\mathbf{n}.\mathbf{B}_p)]\, r\, ds \qquad (4)$$

The plasma is thus in vertical force balance if eqn (4) is satisfied.

Hence, both the radial and the vertical force balances are now formulated as contour integrals over a fixed contour lying outside the plasma. Force balance is achieved if the value of the contour integral (3) equals a pre–determined value of S_2 — representing the internal pressures — and the contour integral (4) equals 0. In the code the values of the current in sets of vertical field coils and radial field coils respectively are iterated in a least squares routine so as to minimise the sum of squares of the deviation between the contour integrals and their desired values.

The desired value for S_2, called $S_{2,est}$, is obtained as follows. The integral quantities, representing the internal plasma pressures, ie. normalised kinetic pressure β_p, the internal inductance l_i, and the diamagnetic parameter μ_I are defined as follows [6], with a normalisation parameter c:

$$c = \tfrac{1}{4}\,\mu_0\,r_c\,I_p^2$$

$$\beta_p = c^{-1} \int_T p\,dV,$$

$$l_i = c^{-1} \int_T \tfrac{1}{2}\mu_0^{-1}\,B_p^2\,dV,$$

$$\mu_I = -c^{-1} \int_T \tfrac{1}{2}\mu_0^{-1}\,(B_t^2 - B_{t0}^2)\,dV \qquad (5)$$

where r_c is the plasma current centre, determined by the current moment integral over the external contour and I_p is the total plasma current. We note that the volume integral in equation (3) contains a factor r^{-1}, which is not present in the definition equations (5). Hence, following [7] with an adapted notation, we write:

$$S_2 = c\,R_t^{-1}\,(\beta_p + \mu_I + l_i) \qquad (6)$$

where R_t is defined as

$$R_t = \int_T g\,dV \,/ \int_T r^{-1}\,g\,dV$$

$$g = p - \tfrac{1}{2}\mu_0^{-1}\,(B_t^2 - B_{t0}^2) + \tfrac{1}{2}\mu_0^{-1}\,B_p^2 \qquad (7)$$

For the purpose of obtaining an estimate $S_{2,est}$, it is thus necessary to provide values for R_t, β_p, μ_i and l_i. Lazzaro [7] gives a relation between β_p and μ_I for the case of elongated plasmas:

$$\mu_I = \beta_p - 2E \,/\, (1 + E^2) \qquad (8)$$

where E is the elongation. Furthermore, because $R_t \approx r_c$, we approximate $R_t =: \eta_{r,est}\,r_c$, where $\eta_{r,est}$ is normally set to 1. $S_{2,est}$ is then calculated as:

$$S_{2,est} = c\ \eta^{-1}_{r,est}\ r^{-1}_{c}\ (\ 2\beta_{p,est} + l_{i,est} - 2E_{est}\ /\ (1 + E^2_{est}) \qquad (9)$$

where $\eta_{r,est}$, $\beta_{p,est}$, $l_{i,est}$ and E_{est} are input parameters for the code, E_{est} represents the expected elongation. We note that in principle it is superfluous to provide input values for β_p and l_i separately, because they appear only in combination in the radial force balance. Ie. the fit will ensure that the sum $\beta_p + \frac{1}{2}l_i$ is approximately correct, but the values of β_p and l_i individually may still differ from the desired values. The individual values depend on the distribution of the current carrying wires that represent the plasma.

3 PRACTICAL IMPLEMENTATION

The above force balance calculation is implemented in a code. The plasma is represented by a set of wires at fixed positions and with fixed currents. The central solenoid of the tokamak, the stray field compensation coils, divertor coils and shaping coils, are also represented by wires or sets of wires at fixed positions with fixed currents. The set of coils that provides the vertical field is set at fixed positions, and has a fixed ratio of currents between the coils, while the total current in the set is a free parameter in the least squares routine that calculates the radial force balance. The radial field coils are treated in the same manner.

The fields and flux due to each of the wires, on the grid points of the 'machine grid',is obtained via geometrical scaling and interpolation in look–up tables. There are three look–up tables, containing respectively the radial and vertical components of the magnetic field and the flux for a single wire loop of unit radius. The look–up tables are made by numerically integrating the Biot–Savart equation, and the equation for the vector potential (note flux $\psi = 2\pi\ r\ A_{\varphi}$) around the unit wire loop for every point of a fine grid. This is a lengthy calculation, but it has to be performed only once.

4 DETERMINATION OF X–POINTS AND LAST CLOSED FLUX SURFACE

After the force balance calculation has been performed (the relevant contour integrals are taken over the vacuum vessel), the currents in all the coils of the machine, for the particular equilibrium, are known. The code then proceeds by calculating the components of the magnetic field and the flux on the machine grid, by summing the contributions from all the wires.

The code searches for X–points by executing a search for a minimum in the absolute value of the field in the top and bottom divertor areas. Usually two X–points are found, and their flux values are determined.

An (optional) input to the code is a file containing coordinates of a toroidal field coil and a vacuum vessel. The vacuum vessel coordinates should describe the inner surface of the vessel, including internal elements. The code calculates the flux and poloidal field on the vessel and determines whether the plasma is bounded by the vessel or by one or both of the X–points. The last closed flux surface is determined.

5 FURTHER CALCULATIONS ON THE LAST CLOSED FLUX SURFACE

After the last closed flux surface has been found, a number of further calculations is performed in order to extract the following information: elongation E, β_p, l_i, safety factor q_{cyl} and q_{95}, and poloidal magnetic field perpendicular to the toroidal field coil.

The elongation is found by finding the inner (R_{min}) and outer (R_{max}), and top (z_{max}) and bottom (z_{min}) points of the surface. The major radius is calculated as $R_{maj} = (R_{max} + R_{min}) / 2$, and the horizontal and vertical minor radii as $a = (R_{max} - R_{min}) / 2$ and $b = (z_{max} - z_{min}) / 2$. The elongation is $E = b / a$, and the aspect ratio $\epsilon = a / R_{maj}$. β_p, l_i and R_t are found by evaluating the S_1, S_2 and S_3 integrals over the last closed flux surface. We have – in addition to equation (3) for S_2 – the following relations (the equations are given here, for generality, for integration over a surface outside the plasma that does not necessarily coincide with a flux surface. In the actual application here, where the surface of integration coincides with a flux surface, the terms containing $\mathbf{B}_p.\mathbf{n}$ vanish):

$$S_1 \overset{\text{def}}{=} \int_T \left[3p + \tfrac{1}{2}\mu_0^{-1}\left(B_t^2 - B_{t0}^2\right) + \tfrac{1}{2}\mu_0^{-1} B_p^2\right] dV$$

$$= 2\pi\, \mu_0^{-1} \int_{\delta T} \left[\tfrac{1}{2}B_p^2\, \mathbf{r}.\mathbf{n} - (\mathbf{r}.\mathbf{B}_p)(\mathbf{n}.\mathbf{B}_p)\right] r\, ds$$

$$= c\,(3\beta_p + l_i - \mu_I) \tag{10}$$

$$S_3 \overset{\text{def}}{=} \int_T \left[p + \tfrac{1}{2}\mu_0^{-1}\left(B_t^2 - B_{t0}^2\right) + \tfrac{1}{2}\mu_0^{-1} B_{p,z}^2\right] dV$$

$$= 2\pi\, \mu_0^{-1} \int_{\delta T} \left[\tfrac{1}{2}B_p^2\, z\, \mathbf{e}_z.\mathbf{n} - z\,(\mathbf{e}_z.\mathbf{B}_p)(\mathbf{n}.\mathbf{B}_p)\right] r\, ds$$

$$= \beta_p - (\alpha - 1)\, l_i - \mu_I \tag{11}$$

where $\alpha = 2 \int_T B_{p,z}^2\, dV \,/ \int_T B_p^2\, dV$

To solve for β_p, l_i and R_t from S_1, S_2 and S_3, we need an approximation for the volume dependent parameter α. In references [7] and [8], the following expression is used:

$$\alpha = 2\,E \,/\, (1 + E^2) \tag{12}$$

Using this, the quantities follow as:

$$l_i = (2E - c^{-1}\, S_3\, (1 + E^2)) \,/\, (E^2 - 1) \tag{13}$$

$$\beta_p = (c^{-1}\, S_1 - l_i - 2E \,/\, (1 + E^2)) \,/\, 2 \tag{14}$$

$$R_t = (S_1 - 4\, c\, E \,/\, (1 + E^2)) \,/\, S_2 \tag{15}$$

Values for the engineering safety factor q_{cyl} and an approximation for the true MHD q value at the 95% flux surface are calculated as follows:

$$q_{cyl} = (2 \pi \epsilon a E B_{tor}) / (\mu_0 I_p) \qquad (16)$$

$$q_{95} = q_{cyl} (1 + E^2) (1 + 1.5\epsilon^2) / (2E) \qquad (17)$$

As mentioned, an input to the code is a dataset of coordinates of a toroidal field coil (neutral fibre, inner and outer contour) and a vacuum vessel. These elements will be shown in the plots of flux surfaces and magnetic field magnitude. The code will produce an output file containing the values of the poloidal magnetic field component perpendicular to the toroidal field coil. This file can be interfaced to a calculation of the out–of–plane bending stresses and the overturning moment on the coil.

Optionally, the fitting of the radial and vertical field currents can be omitted, and fixed currents can be supplied for these coils. The code can then be used for calculations of the fluxes provided by the central solenoid and other coils, as well as the external flux required for the plasma. Together with an estimate for the plasma internal flux (eg. $h_i = l_i + 0.5$), the available resistive flux can be estimated.

6 RESULTS

The code is easy to use and is run interactively on an IBM mainframe. A typical calculation, including graphics display, for a configuration with a plasma represented by 10 wires and about 40 wires external to the plasma (mainly in the central solenoid) can be done in about 2 minutes.

The results are illustrated in figures 1 to 4. Figure 1 shows flux surfaces and the separatrix for a hypothetical tokamak with a plasma current of 12MA. In figure 2 the positions of wires representing the plasma and the external coils are listed. In figure 3 the flux in the midplane z=0 is shown as a function of radius. In figure 4 the poloidal field component perpendicular to the TF coil is shown versus the coil arc length, for the neutral fibre and the inner and outer contours of the TF coil.

7 CONCLUSIONS

A code has been developed to predictively calculate plasma boundary and external fields for a given plasma current and position and for given currents in shaping coils, in air core tokamaks. The currents in sets of vertical and radial field coils are adjusted iteratively so as to obtain radial and vertical force balance respectively. The plasma and the external coils are represented by wires or sets of wires.

The code can be run interactively, and has a turn—around time of about 2 minutes. The fast turn—around makes it ideal for initial design work in several areas of tokamak design. Examples are the design of plasma configuration and divertor geometries (including divertor sweeping), design of poloidal coil sets and stray field compensation coils, calculations of flux consumption, and calculations of out—of—plane bending forces on TF coils.

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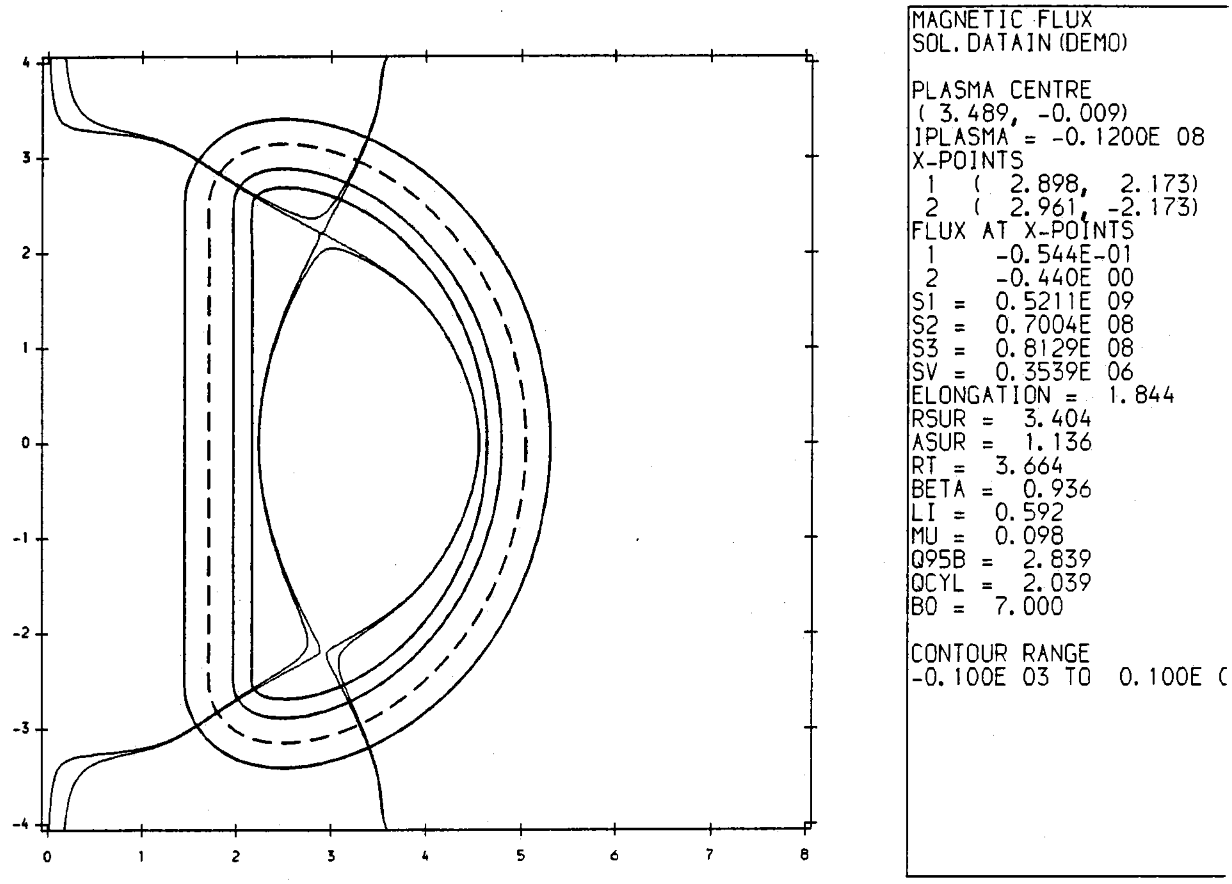

Figure 1. Graphics output of the flux surfaces and separatrix. The text output has the following meaning:

- the plasma position given is the position of current centre r_c, z_c as determined from the usual current moment integrals, eg. [6]. Units are [m].
- the plasma current given here is the sum of the currents in the plasma wires.
- the radial and vertical position of the X—points is given. Units are [m].
- Flux at X—points: the value of the flux function at the two X—points (top/bottom). Unit is [Wb].
- S1, S2, S3 are the Shafranov integrals as per eqns. 10,3,11. SV is the vertical force integral as per eqn. 4 (hence it is a small residual value). Units are [N] for S_2 and S_v, [Nm] for S_1 and S_3.
- Elongation from b / a.
- Rsur is the major radius of the flux surface, as defined in section 5. Unit is [m].
- Asur is the horizontal minor radius, as defined in section 5. Unit is [m].
- RT is the radius R_t, as per eqn. 15. Unit is [m].
- BETA, LI and MU as per eqns. 14,13 and 8.
- Q95B and QCYl as per eqns. 17 and 16.
- B0 is the toroidal field. This is an input parameter and is the vacuum field at the radius RSUR. Unit is [T].

TYPE	RADIUS	HEIGHT	CURRENT
VERTICAL	0.5850E 01	0.1200E 01	0.3616E 07
VERTICAL	0.5850E 01	-0.1200E 01	0.3616E 07
RADIAL	0.1700E 01	0.4325E 01	0.1284E 06
RADIAL	0.1700E 01	-0.4325E 01	-0.1284E 06
RADIAL	0.4950E 01	0.3050E 01	0.3467E 05
RADIAL	0.4950E 01	-0.3050E 01	-0.3467E 05
SHAPING	0.2200E 01	0.4325E 01	-0.1440E 08
SHAPING	0.2200E 01	-0.4325E 01	-0.1440E 08
SHAPING	0.2700E 01	0.4325E 01	0.0000E 00
SHAPING	0.2700E 01	-0.4325E 01	0.0000E 00
SHAPING	0.3750E 01	0.3900E 01	0.1200E 07
SHAPING	0.3750E 01	-0.3900E 01	0.1200E 07
SHAPING	0.4950E 01	0.3050E 01	0.3000E 07
SHAPING	0.4950E 01	-0.3050E 01	0.3000E 07
PLASMA	0.3000E 01	0.1000E 00	-0.7500E 06
PLASMA	0.3000E 01	-0.1000E 00	-0.7500E 06
PLASMA	0.3250E 01	0.5000E 00	-0.1500E 07
PLASMA	0.3500E 01	0.1000E 01	-0.1500E 07
PLASMA	0.3750E 01	0.5000E 00	-0.1500E 07
PLASMA	0.4000E 01	0.1000E 00	-0.7500E 06
PLASMA	0.4000E 01	-0.1000E 00	-0.7500E 06
PLASMA	0.3750E 01	-0.5000E 00	-0.1500E 07
PLASMA	0.3500E 01	-0.1000E 01	-0.1500E 07
PLASMA	0.3250E 01	-0.5000E 00	-0.1500E 07

Figure 2. List of the wire currents for the calculation shown in figure 1. Major radius, height and current in each of the wires is shown. Not shown on the list is the central solenoid, which for this calculation consists of a set of 29 wires, located at R = 1.2m, z from −2.8m to 2.8m, every 0.2m, carrying 1.2MA each.

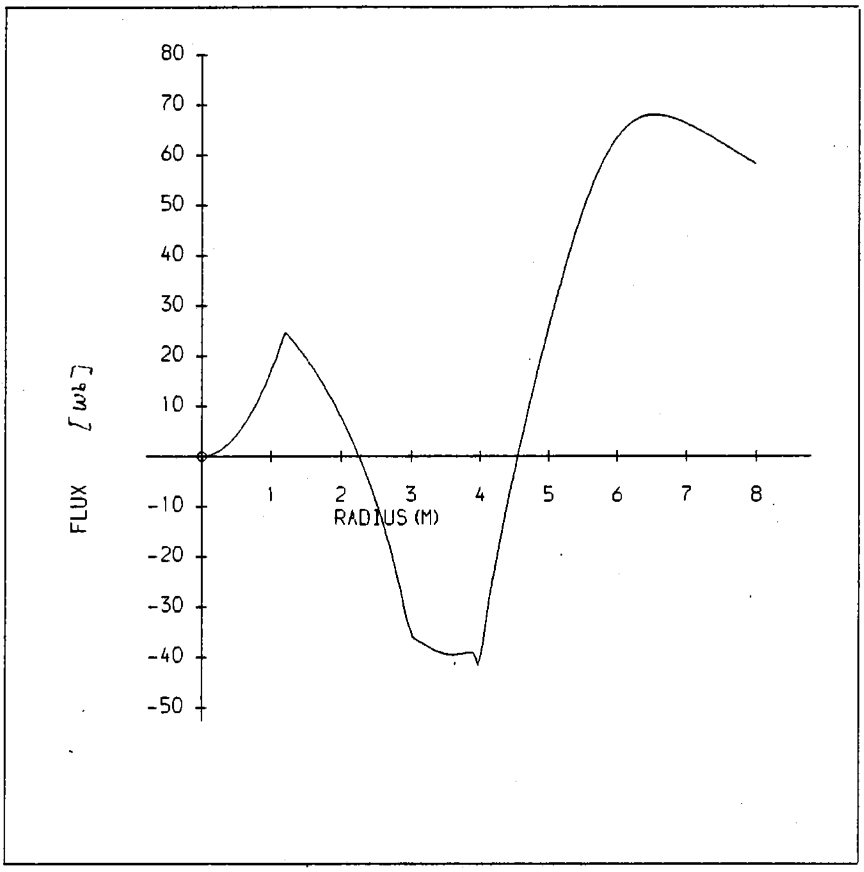

Figure 3. The flux in the midplane z = 0 as a function of major radius. Note that the flux is meaningless inside the plasma.

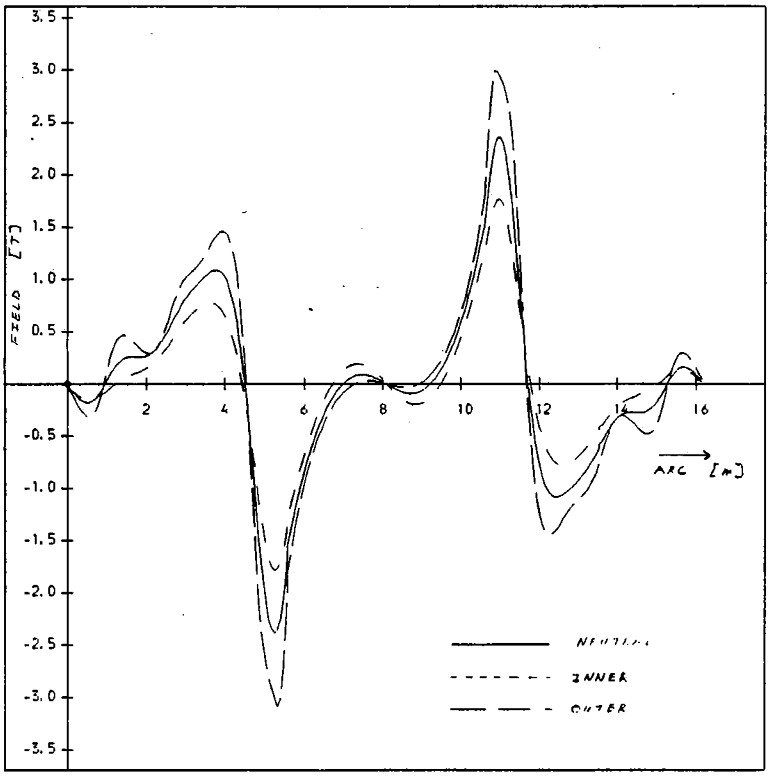

Figure 4. The component of the poloidal field perpendicular to the toroidal field coil, as a function of arc length along the coil. This is given for the neutral fibre of the coil and for the inner and outer contour.