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Kinetic resonances effect on magnetic braking in tokamaks

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The investigation of tokamak plasma kinetic effects such as the particles precession and bounce frequencies resonance effect on the dynamics of the plasma rotation under different collisionality regimes is the goal of this work.

It has been found that the kinetic processes mentioned above increase the plasma neoclassical toroidal viscosity (NTV) effect on the toroidal rotation of the plasma when peculiar conditions are fulfilled. NTV is due to the breaking of the magnetic field axisymmetry that enhances the radial nonambipolar particle transport and hence increases the NTV braking torque that finally damps the plasma rotation. When the particles electric, magnetic and bounce frequencies resonate, the nonambipolar radial particle flux increases and the plasma toroidal rotation is affected. Starting from the CGL expression of the pressure tensor, $\mathbf{p} = p_{\parallel}\mathbf{nn} + p_{\perp}(\mathbf{I} - \mathbf{nn}) = p\mathbf{I} + \pi$ (**n**=**B**/B, **B** is the equilibrium magnetic field), we have derived the 3D parallel (to **B**) perturbed pressure tensor term :

$$p_{\parallel} = \sum_{\mathrm{j=e,i}}\sum_{\mathrm{m,n}} \exp[\mathrm{i}(m\theta - n\varphi) + \gamma t] \int_0^1 (1-\Lambda B/B_{\mathrm{z0}})^{1/2} \Big\{ \alpha_{\mathrm{1j}}^{\mathrm{mn}}(1-\Lambda B/B_{\mathrm{z0}}) + \alpha_{\mathrm{2j}}^{\mathrm{mn}}(\Lambda B/B_{\mathrm{z0}}) + \sum_{\mathrm{l}} \left[\beta_{\mathrm{1j}}^{\mathrm{mn}}(1-\Lambda B/B_{\mathrm{z0}}) + \beta_{\mathrm{2j}}^{\mathrm{mn}}\Lambda B/B_{\mathrm{z0}}\right] \left[\gamma J_{5/2}^{\mathrm{lnj}} + \gamma_{\mathrm{1j}}^{\mathrm{mn}}(1-\Lambda B/B_{\mathrm{z0}})^{1/2} J_{3}^{\mathrm{lnj}}\right] \Big\} d\kappa^2 \quad (1)$$

A similar expression is obtained for the perpendicular pressure tensor, $p_{\perp}$. The α terms are pure fluid contributions, whereas the terms under the l summation are due to the particles precession and bounce oscillations. m, n and l are the harmonic numbers in the poloidal, toroidal and trapped particles bounce orbit expansion, respectively. γ is the perturbation growth rate. B_{z0} is the equilibrium toroidal magnetic field in the major axis. $\Lambda = 1/[1+\varepsilon(2\kappa^2-1)]$ is a parameter depending on the trapped bouncing particles normalized pitch angle κ [1] for a low plasma inverse aspect ratio $\varepsilon = a/R_0$. $\gamma_{\mathrm{1j}}^{\mathrm{mn}} = \mathrm{i}nB/(R_0 B_{\mathrm{z0}})\sqrt{2/M_{\mathrm{j}}}$ (M_{j} is the ion or the electron mass). $\alpha_{\mathrm{1,2j}}^{\mathrm{mn}}$ and $\beta_{\mathrm{1,2j}}^{\mathrm{mn}}$ are analytically derived expressions of plasma fluid and kinetic parameters, respectively. $J_{5/2,3}^{\mathrm{lnj}}$ are defined by:

$$J_{5/2,3}^{\mathrm{lnj}} = \int_0^{\infty} E^{\,5/2,3} \exp[-E/(K_{\mathrm{B}}T_{\mathrm{j}})] \frac{dE}{\mathrm{i}n\Omega_{\mathrm{z}} + \nu_{\mathrm{k}} + \gamma_2^{\mathrm{l}}\sqrt{\Lambda EB/B_{\mathrm{z0}}} + \gamma_3^{\mathrm{mn}}\Lambda EB/B_{\mathrm{z0}}} \quad (2)$$

K_{B} is the Boltzmann constant, T_{j} the j-particle temperature and E the particle total energy. Ω_{z} is the plasma toroidal rotation frequency. $\omega_{\mathrm{b}} = -(\mathrm{i}\gamma_2^{\mathrm{l}}/l)\sqrt{\Lambda EB/B_{\mathrm{z0}}}$ and $\omega_{\mathrm{D}} = \mathrm{i}\gamma_3^{\mathrm{mn}}\Lambda EB/(nB_{\mathrm{z0}})$ are the 3D derived particle bounce frequency and bounce averaged toroidal precession frequency, respectively, where γ_2^{l} and γ_3^{mn} are analytically obtained parameters. $\nu_{\mathrm{k}} \sim \nu E^{-3/2}$ is the Krook collisional parameter [2], with ν the particle collision frequency. Depending on the range of the plasma rotation frequency, the above integral is different for specific collisional regimes. In the non-resonant regimes ($1/\nu$ and $\nu - \sqrt{\nu}$) [3] the particles collision frequency influences the NTV magnitude. The $1/\nu$ regime also includes the trapped-circulating particle boundary layer physics [4]. Park et al. model [5] that involves the Krook collision operator, does not consider any $1/\nu$ regime. The above mentioned model has been recently validated [6] for the weak collisional regime close to the dangerous superbanana plateau regime ($SB-P$). We are interested to describe this regime under kinetic resonance conditions. In a lower collisionality case, the main contribution for the NTV magnitude is due to the resonant particles. For very low collisionality, the integral (2) over the particle energy becomes improper. For the case we are interested in, the $l \neq 0$ case, the toroidal precession frequency is significantly lower compared to the trapped particle bounce frequency and can be neglected. On the other hand, at usually high plasma rotation levels, no resonance occurs in the absence of the bounce oscillations, the precession rotation level being too low to resonate. The energy integral that measures the resonance effect is derived at different collisional regimes: a high collisional regime, a low collisional near the superbanana regime and an intermediate regime of collisionality. It should be noted that the use of a Krook collisional operator cannot include the collisional boundary layer introduced by Shaing et al. [4] to overcome the singularity of a pitch angle integral at the trapped-circulating particles boundary limit. However, at very low collisionality regimes we are interested in, the results using the Krook operator are valid. For the $1/\nu$ collisional, $SB-P$ and $1/\nu \to SB-P$ regimes we have analitically obtained specific Cauchy principal values for the integral quantities given by (2). With the aid of it we get a final analytic form of the NTV torque

$$\langle \mathbf{B}_{\text{z-total}} \cdot \nabla \pi \rangle = \sum_{\mathrm{m,n}} \Big\{ \left[f_{\mathrm{mn}}(r)\Phi^{\mathrm{mn}} + g_{\mathrm{mn}}(r)\Phi'^{\mathrm{mn}} \right] S_{\mathrm{n}}^{0} \left[(2-s)\Phi^{\mathrm{mn}*} + r\Phi'^{\mathrm{mn}*} \right]$$
$$+ \sum_{\substack{\mathrm{j=-3} \\ j \neq 0}}^{3} h_{\mathrm{mnj}}(r, S_{\mathrm{n}}^{0}, S_{\mathrm{n}}^{2})\Phi'^{\mathrm{mn}} \left[(2-s)\Phi^{\mathrm{m+j,n}*} + r\Phi'^{\mathrm{m+j,n}*} \right] + \overline{h}_{\mathrm{mnj}}(r, \Lambda_{\mathrm{j}}, \Lambda'_{\mathrm{j}}, \Lambda''_{\mathrm{j}}) S_{\mathrm{n}}^{0} \Phi'^{\mathrm{mn}} \Phi^{\mathrm{m+j,n}*} \Big\} \quad (3)$$

f_{mn}, g_{mn} and $\overline{h}_{\mathrm{mnj}}$ are exactly derived coefficients. π is the traceless stress tensor and $\mathbf{B}_{\text{total}} = \mathbf{B} + \mathbf{b}$, where the perturbed magnetic field is parameterized as $\mathbf{b} = \nabla \times [(1/B)\nabla\Phi \times \mathbf{B}] \times \mathbf{B}$. Φ is the perturbed normalized scalar electric potential. s is the local magnetic shear. $'$ means radial (flux) derivative and $*$ indicates the hermitian conjugation. $\Lambda_{\mathrm{j}} = \Delta\delta_{|\mathrm{j}|,1} + E\delta_{|\mathrm{j}|,2} + T\delta_{|\mathrm{j}|,3}$ is the plasma shaping parameter (depending on local Shafranov shift Δ, ellipticity E and triangularity T) and $\delta_{|\mathrm{j}|,\mathrm{i}}$ is the Kronecker delta. S_{n}^{0}, S_{n}^{2} and c_{ln} are exactly derived kinetic coefficients. To estimate the NTV torque quantity, the Fourier components of the perturbed magnetic field are found. In terms of Φ^{mn} and Φ'^{mn} we have analytically solved the perturbed magnetic field equations (perturbed plasma, vacuum and feedback circuit equations) obtaining

$$\Phi^{l_i} = \frac{1}{\Delta} \sum_{\substack{l_1,..,l_{2L}=1 \\ \text{distinct}}}^{2L} sgn(l_1,..,l_{2L}) \prod_{p=1}^{2L} \Gamma^{p}_{l_p l_i}, \quad \Phi'^{l_i} = \frac{1}{\Delta} \sum_{\substack{l_1,..,l_{2L}=1 \\ \text{distinct}}}^{2L} sgn(l_1,..,l_{2L}) \prod_{p=1}^{2L} \tilde{\Gamma}^{p}_{l_p l_i} \tag{4}$$

They already depend on the kinetic effects. The new index ordering is explained in [7]. The $\Gamma^p_{l_p l_i}$, $\tilde{\Gamma}^p_{l_p l_i}$ and $\overline{\Delta}$ coefficients depend on plasma, wall and feedback circuit parameters.

In Fig. 1 the calculated normalized NTV for $1/\nu$ and the transition regime to superbanana plateau regime are plotted. The dependence of ion-ion collisional operator is showed. The profiles agree with those obtained with the Park-Boozer formula, for the same collisionality regimes [8]. No assumption about the trapped-circulating particles boundary layer regime is made.

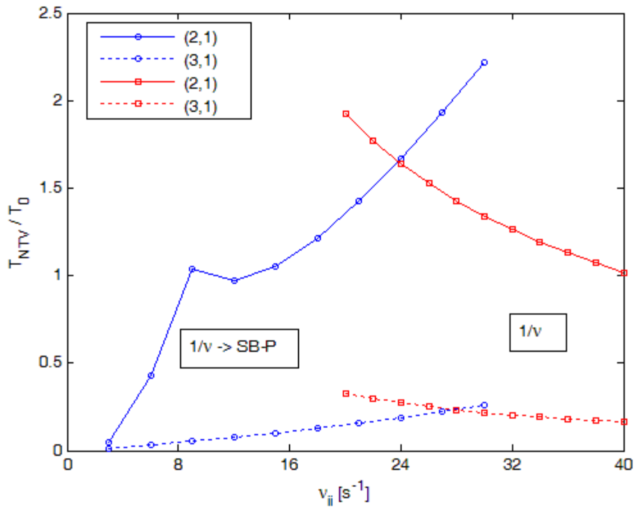

Figure 1: *NTV torque for non-resonant collisional regimes as a function of ion-ion collisional frequency.*

Fig. 2 shows the dependence of the NTV torque at resonance conditions on the error field components amplitude. A higher error field drives a higher NTV quantity. The NTV is calculated at the level of two magnetic surfaces: inside and close to plasma boundary. As expected, the NTV close to the plasma surface is lower. Being calculated at resonance, at very low collisionality, it is clear that a higher plasma rotation is a condition to avoid the resonance.

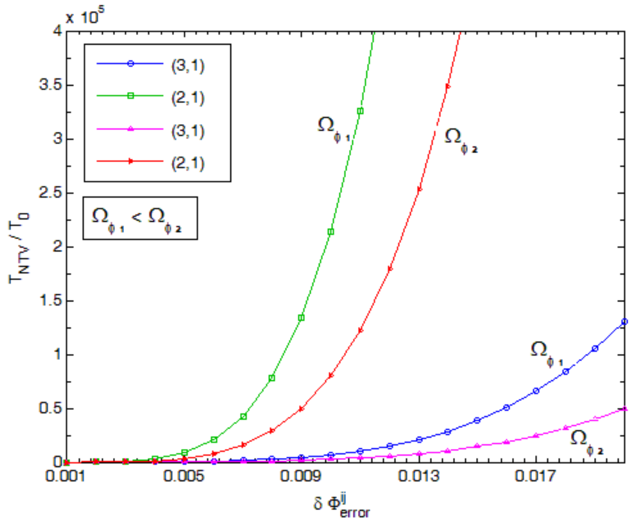

Figure 2: *Error field dependence of NTV at resonance for different plasma toroidal rotations.*

Fig. 3 shows the bounce harmonic number resonant contributions to the calculated NTV for the resonant regime, at very low collisional frequency. It is obvious the significant increase of the NTV at rotation levels that satisfy the resonance condition.

For each l harmonic number in the trapped particles bounce orbit expansion, there is a peak of the NTV torque, at a different plasma rotation. The higher l is, the lower NTV peak is obtained at a higher corresponding plasma toroidal rotation.

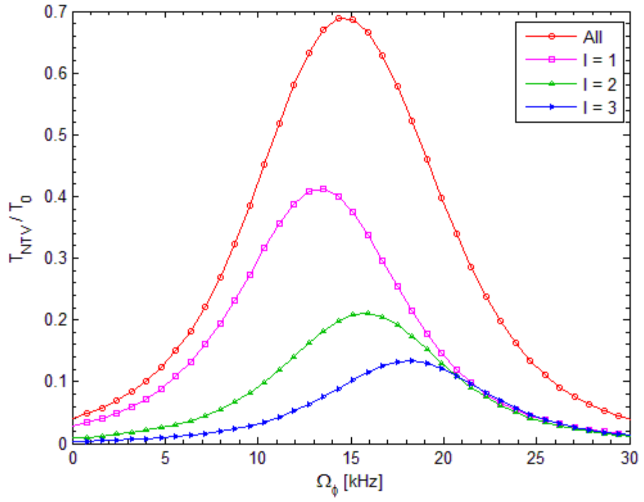

Figure 3: *Calculated NTV torque as a function of plasma toroidal rotation. The l harmonic number contributions.*

To resume, a 3D perturbed particle distribution function has been obtained as a function of the magnetic field perturbations. The latter are derived by solving the linearized perturbed complete system of the multi-mode magnetic field equations. The magnetic perturbations are not inserted parametrically into the calculated NTV but they are calculated (4), already enclosing the kinetic resonance influence.

In summary, a model to provide an analytic form of the NTV torque (3) is built. It describes and proves the destabilizing influence of the particles kinetic resonances effect on magnetic braking of plasma rotation in tokamak plasmas of specific collisionality regimes.

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References

[1] K.C. Shaing, Phys. Plasmas **10**, 1443 (2003)

[2] H. E. Mynick, Nucl. Fusion **26**, 491 (1986).

[3] Y. Sun, Y. Liang, K.C. Shaing et al., Nucl. Fusion **51**, 053015 (2011)

[4] K.C. Shaing et al., Phys. Plasmas **15**, 082506 (2008)

[5] J-k. Park, A.H. Boozer and J.E. Menard, Phys. Rev. Letters **102**, 065002 (2009)

[6] J-k. Park, Y.M. Jeon, J.E. Menard et al., Phys. Rev. Letters **111**, 095002 (2013)

[7] I.G. Miron, Plasma Phys. Control. Fusion **50**, 095003 (2008)

[8] S. Satake, J.-K. Park, H. Sugama, and R. Kanno, Phys. Rev. Letters **107**, 055001 (2011)