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Turbulent transport in tokamak plasmas: bridging theory and experiment

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Nonlinear gyrokinetic codes allow for detailed understanding of tokamak core turbulent transport. However, their computational demand precludes their use for predictive profile modeling. An alternative approach is required to bridge the gap between theoretical understanding and prediction of experiments. A quasilinear gyrokinetic model, QuaLiKiz [C. Bourdelle, X. Garbet, et al, Phys. Plasmas 14, 112501 (2007)], is demonstrated to be rapid enough to ease systematic interface with experiments. The derivation and approximation of this approach are reviewed. The quasilinear approximation is proven valid over a wide range of core plasma parameters. Example of profile prediction using QuaLiKiz coupled to the CRONOS integrated modeling code [J.F. Artaud et al Nucl. Fusion, 50 043001 (2010)] are presented. QuaLiKiz is being coupled to other integrated modeling platforms such as ETS and JETTO. QuaLiKiz gyrokinetic quasilinear turbulent fluxes are available to all users. It allows for extensive stand-alone interpretative analysis and for first principle based integrated predictive modeling.

I. INTRODUCTION

Understanding and predicting temperature, density and rotation profiles in the confined core of tokamak plasmas requires accurate and rapid turbulent transport codes.

The continuously expanding availability of high performance computing systems facilitates the use of gyrokinetic codes, either gradient driven such as GENE [1], GKW [2], GS2 [3], and GYRO [4] or flux driven such as GYSELA [5] or GT5D [6]. Fixed gradient codes were intensively applied over the past few years for quasilinear and nonlinear investigations of experimental profiles. The role of the particle convection in determining the density profile was identified and understood [7–9]. The impurity transport level was successfully compared to quasilinear diffusive and convective coefficients [10–13], including W turbulent transport [14–16]. Angular momentum convective transport was also explored [17–19].

Despite impressive progress in comparing quasilinear gyrokinetic and measured fluxes, the main issue – predicting temperature, density, rotation in future tokamaks such as ITER – remains highly challenging. This requires the dynamic solution of the transport equations. Beyond the fluxes, this includes heat, particle, and momentum source modelling, together with a self-consistent magnetic equilibrium. This system is characterized by numerous nonlinear interactions, e.g., between temperature and current diffusion, and density and radio frequency heating. To reach a stationary state, these interactions must be modelled for a few confinement times, i.e. $\sim$1 s for JET-scale plasmas. This necessitates approximately 10^3 flux calculations. To obtain profiles in reasonable wall-clock times (e.g. < 24h) and on an easily accessible number of computing cores (e.g. $\simeq 30$), this sets a severe constraint on the transport model calculation time per call. Gradient driven gyrokinetic nonlinear simulations are too costly, taking typically 5000 to 50000 CPU hours per flux calculation for a single radial position. While optimized multiscale coupling schemes do exist [20], routine profile calculations and sensitivity studies are still out of scope. Therefore, the quasilinear approximation provides an avenue for tractable profile calculations.

In addition to the quasilinear approximation, the fluid limit is used in the Weiland model [21] (embedded in the MMM model [22, 23]), and the IFS-PPPL model [24]. Another branch of quasilinear gyrofluid models was developed, tuned to linear gyrokinetic and nonlinear gyrofluid calculations, and named GLF23 in [25] and extended to the TGLF model [26, 27]. The TGLF predicted electron and ion temperature and the density profiles are modeled with an averaged RMS error around 15%. Hybrid scenarios [28] and ITB were also modeled [29]. Despite these successes, numerous issues remain to be addressed, such as: the pedestal height prediction [30], the underestimated transport level in the deep core, the transport level in the L mode edge region [31], the combined role of MHD and turbulence on the confinement, the impact of finite β in particular in hybrid scenarios [32], etc. Nevertheless, the overall success of quasilinear turbulent transport models in reproducing tokamak plasma profiles demonstrates the validity of the quasilinear approach in multiple regimes.

QuaLiKiz is the first quasilinear gyrokinetic transport model [8] sufficiently tractable for integrated modeling. However, the quasilinear approximation alone is insufficient, providing only a factor 100 speedup compared with nonlinear, and further approximations are necessary to reach the necessary level of tractability. The axisymmetric gyrokinetic problem is further simplified thanks to the use of the lowest order ballooning transform. Some integrals are reduced by treating separately trapped and passing particles. Finally the eigenfunction is not solved self-consistently together with the eigenvalue, but rather is deduced from the fluid limit of the gyrokinetic equations. Recent efforts in simplifying the numerical schemes have optimized the dispersion relation solver [33] and the plasma dispersion functions [34]. Thanks to these further approximations, 20 radial positions and 10 wave numbers

are modeled within 5 minutes, meaning that, in an integrated framework, QuaLiKiz can model 1 s of plasma over 30 processors within 24h. QuaLiKiz is embedded in CRONOS [35], and was applied for predictive simulations of heat and particle transport for JET H-mode scenarios [36, 37].
In section 2, the linear gyrokinetic electrostatic dispersion relation is derived. The eigenfunctions derivation and validation are presented. In section 3, the validity of the quasilinear approximation is investigated by comparisons with nonlinear gyrokinetic simulations and turbulence measurements. In section 4, QuaLiKiz fluxes are compared to nonlinear gyrokinetic codes and experimental results both at a given time and integrated in CRONOS. The results are discussed in section 5. Perspectives for further speed up while enlarging the validity domain are presented.

II. QUALIKIZ LINEAR DISPERSION RELATION

A. The approximated QuaLiKiz linearized Vlasov equation

In the core of tokamak plasmas, the measured density fluctuation level is well below 10 % [38]. This validates assuming a background Maxwellian distribution function f_0 with a small perturbation δf. Thus, $f_s = f_{0,s} + \delta f_s$ with $f_{0,s} = n_s \frac{1}{(2\pi m_s T_s)^{3/2}} e^{-E/T_s}$. E is the kinetic energy of a particle at the thermodynamical equilibrium. When treating a plasma with rigid body rotation, $U_\parallel$, then: $E = \frac{m_s (v_\parallel - U_\parallel)^2}{2} + \frac{m_s {v_\perp}^2}{2}$. $v_\parallel$ and $v_\perp$ are respectively the velocities parallel and perpendicular to the magnetic field B. In QuaLiKiz, the low Mach number limit is assumed.
The electrostatic limit is assumed in QuaLiKiz. This limit is valid for low β plasmas. The total electrostatic potential is written as $\phi = \phi_0 + \delta\phi$.
The Vlasov equation is linearized assuming harmonic perturbations:

$$f_{\vec{n}\omega}(\vec{J}) = -\frac{f_0(\vec{J})}{T_s}(1 - \frac{\omega - \vec{n}\cdot\vec{\omega}_s^* - \vec{n}\cdot\vec{\omega}_E}{\omega - \vec{n}\cdot\vec{\Omega}_J + i0^+})e_s\phi_{\vec{n}\omega}(\vec{J}) \quad (1)$$

$\vec{J}$ are the action variables [39, 40], ω is the frequency of the pertubation δf. $\vec{n}\cdot\vec{\Omega}_J$ is the drift frequency representing the forces felt in the tokamak electromagnetic field, $\vec{n}\cdot\vec{\omega}_E = \vec{n}\cdot\frac{e_s d\phi}{T_s d\vec{J}}$ is the $E \times B$ drift and $\vec{\omega}^*$ is the diamagnetic frequency which represents the instability drive term since it expresses the departures from thermodynamical equilibrium of the quantities T_s, n_s and $U_\parallel$ such that:

$$\begin{aligned} \vec{\omega}_s^* = T_s \frac{1}{n_s}\frac{dn_s}{d\vec{J}} + \\ T_s\left(\mathcal{E} - \frac{3}{2} - \frac{U_\parallel}{v_{T_s}}\left(\frac{2v_\parallel - U_\parallel}{v_{T_s}}\right)\right)\frac{1}{T_s}\frac{dT_s}{d\vec{J}} + \\ 2T_s\left(\frac{v_\parallel - U_\parallel}{v_{T_s}}\right)\frac{dU_\parallel}{d\vec{J}}\frac{1}{v_{T_s}} \end{aligned} \quad (2)$$

with $\mathcal{E} = \frac{\frac{m_s {v_\parallel}^2}{2} + \frac{m_s {v_\perp}^2}{2}}{T_s}$ and v_{T_s} such that $T_s = \frac{1}{2}m_s v_{T_s}^2$.
In the electrostatic case, relevant wavelengths being larger than the Debye length, the dispersion relation is closed by the weak formulation of the quasineutrality condition :

$$\sum_s \mathcal{L}_s(\omega, \vec{n}) = 0 \quad (3)$$

where $\mathcal{L}_s$ is the particle Lagrangian defined for each species s by $\mathcal{L}_s(\omega,\vec{n}) = \int d^3\vec{x}\,(-\rho_s(\omega,\vec{n})\phi^*(\omega,\vec{n}))$. This quasineutrality weak formulation is exact in the case of a unique eigenfunction [41].
Combining the weak variational formulation, equation (3), to the linearized Vlasov equation (1), one obtains the dispersion relation:

$$\sum_s \frac{e_s^2 f_0^s}{T_s}[\langle\phi_{\vec{n}\omega}\phi_{\vec{n}\omega}^*\rangle - \langle\frac{\omega - \vec{n}\cdot\vec{\omega}^* - \vec{n}\cdot\vec{\omega}_E}{\omega - \vec{n}\cdot\vec{\Omega} + i0^+}\phi_{\vec{n}\omega}\phi_{\vec{n}\omega}^*\rangle] = 0 \quad (4)$$

where $\langle ... \rangle = d^3\vec{\vartheta} d^3\vec{J}...$, hence equation 4 leads to 6D integrals.
Here the abiabatic part is summed to the non-adiabatic one. An alternative formulation of the dispersion relation isolates the polarization term and the gyrocenter part [42].
The 6D dispersion relation can be reduced down to 5D by toroidal axisymmetry, and then down to 4D by the gyrokinetic approximation $\omega/\omega_c \ll 1$, where ω_c is the cyclotron frequency. This introduces a Bessel function in front of the nonadiabatic term and removes the gyroangle. Then, it reduces further down to 3 through the strong ballooning approximation. This approximation takes advantage of the fact that the dynamics along the field line is faster than the dynamics across the field line leading to $k_\parallel \ll k_\perp$ [40, 43, 44]. The fluctuating electrostatic potential is hence rewritten as the product of a slowly and rapidly varying functions: $e^{-in(\varphi - q(x)\theta)}\phi_{n\omega}(\theta)$. By utilizing the analytical Fried and Conte energy integrals the dispersion relation is further reduced to 2D. To simplify further, the trapped and passing particles are integrated separately using the simplified circular concentric large aspect ratio magnetic equilibrium $s-\alpha$. For passing particles, since $v_\parallel \gg v_\perp$, a pitch angle averaged transit frequency as well as curvature and ∇B drift frequencies are performed. For trapped particle, the bounce frequency being larger than ω, a bounced average is performed. The trapped curvature and ∇B drift frequency is averaged over θ. This approximation leads to an underestimation of Trapped Electron Modes in QuaLiKiz at intermediate wave numbers ($k_\theta \rho_s \simeq 0.5 - 10$). A Krook type collision operator is taken into account [45, 46] for trapped electrons only. Collisions on trapped ions and passing particles are neglected. QuaLiKiz dispersion relation accounts for trapped and passing ions and electrons, i.e. it covers the ITG-TEM and ETG ranges. An arbitrary number of tracers or active ions can be accounted for [33]. The detailed final dispersion relation can be found in [40].

B. Validating the fluid eigenfunctions

In the variational approach, an error ϵ in estimating the eigenfunction will lead to a smaller error, ϵ^2, on the eigen-

value, allowing for a higher tolerance in approximations for the eigenfunction determination. The eigenfunction $\phi_{n\omega}(\theta)$ is hence derived separately in the fluid limit, which corresponds to: $\omega \gg \mathbf{n} \cdot \mathbf{\Omega_J}$. The fluid limit dispersion relation is developed analytically, details can be found in [17, 39, 40].
The validity of QuaLiKiz fluid eigenfunctions was assessed by comparing to self-consistent gyrokinetic eigenfunctions. For consistency with QuaLiKiz, all direct comparisons are realized with the $s-\alpha$ equilibrium using $\alpha = 0$ in either GKW, GYRO or GENE. Without rotation and $\mathbf{E}\times\mathbf{B}$ the agreement is very good [45]. Nonetheless, wide oscillating eigenfunctions are reported in GENE/GKW for simultaneously low magnetic shear and wave number [47] and in large k_θ TEM dominated regimes [17]. The influence of $u_\parallel$ and $\nabla u_\parallel$ on the eigenfunction shift is successfully benchmarked against GKW in [17]. The impact of $\mathbf{E}\times\mathbf{B}$ shear is observed in QuaLiKiz eigenfunctions [17], but cannot be compared to self-consistent initial value gyrokinetic codes.

C. Searching for the eigenvalues

QuaLiKiz is written in Fortran 90. Open source modules are used for the Fried and Conte integrals, the Bessel functions, the elliptical integrals and the adaptive 1D and 2D integration methods.
To find the eigenvalues of the QuaLiKiz dispersion relation, the Davies method is used [48]. This method can determine the number of eigenvalues within a given contour, as well as an estimate of their values. Their precise localization is then refined using a standard Newton's method. This numerical scheme was recently further optimized [33].

D. Linear benchmark of the growth rates

The first benchmark was presented in [39] and showed that the stabilizing impact of Z_{eff} was found in both QuaLiKiz and GS2. Following the implementation of collisions on trapped electrons in Kinezero [45], their stabilizing impact on TEM was tested against GS2 [49]. More recently, QuaLiKiz growth rates were compared to GENE over a magnetic shear (s) scan [47]. In QuaLiKiz the stabilization for negative values of s is overestimated. It could be due to the pitch angle averaged transit frequency for passing particles leading to an underestimation of the slab branch. In the presence of finite rotation and/or $E\times B$ shear QuaLiKiz was extensively benchmarked against GKW [17]. In most cases the order of magnitude of the growth rate is correct within a few tens of %. QuaLiKiz tends to underestimate the growth rates due to narrower eigenfunctions in θ.

III. THE QUASILINEAR FORMULATION AND ITS VALIDITY

A. The quasilinear flux formulation

The quasilinear approach can be used if a time scale, τ, exists such that $1/\gamma < \tau < T_0$, where T_0 is the equilibrium time scale. The quasilinear flux, e.g. for particle flux, is defined as $\Gamma = \langle \delta V_{E\times B} \delta n \rangle$, where $\langle \dots \rangle$ is a flux surface average and time average aver τ. For electrostatic turbulence using the linearized Vlasov equation (1) it becomes:

$$\vec{\Gamma_s} = \sum_{\vec{n},\omega} \vec{n} \mathrm{Im} \left(\vec{n} \cdot \frac{df_{0,s}(\vec{J})}{d\vec{J}} \frac{1}{\omega - \vec{n}\cdot\vec{\Omega_J} + i0^+} e_s^2 |\phi_{\vec{n}\omega}(\vec{J})|^2 \right) \tag{5}$$

Note that energy, and momentum transport fluxes are similarly derived [17, 40]. Here the time averaging was carried out and the frequency ω is taken as the real part of the solution of the dispersion relation equation (1). The finite positive imaginary part 0^+ is needed to insure the causality property such that the fluctuations cancel out for a time t tending towards $-\infty$. However, nonlinear effects tend to broaden the frequency spectrum, hence a finite value will be used instead of 0^+, as discussed below. Equation 5 is valid if the two following linked assumptions hold: i) the linear properties are maintained in the saturated phase and ii) the random-walk assumption holds. In the following, both assumptions are shown to be valid over a large range of parameters. Nonetheless, the amplitude and spectral shape of the saturated potential equation 5 is not set by linear physics. This needs to be tuned by nonlinear simulations, turbulence measurements, and physical intuition. This is a key attribute of all quasilinear transport models.

B. Testing the quasilinear approximation validity

1. The Kubo number

Historically, the standard quasilinear theory was elaborated for test particles [50]. A condition of validity for this quasilinear framework is that the particles should not be trapped in the field. This means that the decorrelation time of the particle in the potential field should be shorter than the eddy turn-over time. The ratio of these two times is known as the Kubo number [51, 52]. Using gyrokinetic nonlinear codes, these times were derived and compared to each other for ETG turbulence [53] and for ITG-TEM turbulence [47, 54]. For all cases, no field trapping is reported, hence a Kubo number below 1. Therefore, it is demonstrated that well developed turbulence for typical tokamak plasma parameters is such that random walk processes do take place and can be modeled by the quasilinear theory. This is an important feature of tokamak plasma turbulence. It is coherent with results reported in the introduction regarding the overall success of quasilinear models in reproducing temperature and density profiles within around 15% averaged RMS error [55].

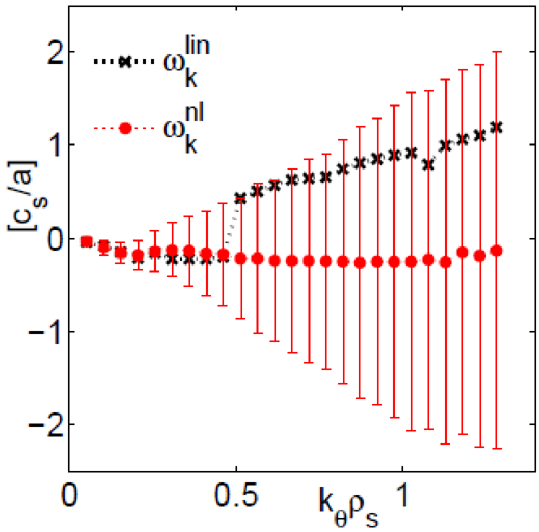

FIG. 1: k_θ spectrum of the frequencies of the linear most unstable mode and of the nonlinear frequencies (the bars indicate the statistical variance from the mean value). The results refer to nonlinear and linear (most unstable mode) simulations using GYRO on TS39596 at $r/a = 0.7$ [58].

2. *The nonlinear phase*

In a nonlinear simulation, the cross phase between the saturated potential and the transported quantities such as density or temperature, can be compared to the linear phase ω_r for a series of k values. This test of validity of the quasilinear approach, introduced by [56] for pure TEM turbulence was extended to coupled ITG–TEM turbulence in [54] using the initial value code GYRO. Similarly to formerly published results, figure 1 shows a very good agreement between the nonlinear and the linear phases [57] for $k_\theta\rho_s \leq 0.4$. At lower spatial scales, or higher $k_\theta\rho_s$, the cross phase cannot be interpreted in terms of remnants of linear modes. This expected departure happens at wave-numbers which do not dominate the overall transport.

3. *The frequency broadening*

As mentioned in equation 5, a finite positive imaginary part at the denominator guarantees causality. In the case of $0^+ \to 0$, the term $\mathrm{Im}\left(\frac{1}{\omega - \vec{n}\cdot\vec{\Omega_J} + i0^+}\right)$ tends towards a Dirac function $-\pi\delta(\omega - \vec{n}\cdot\vec{\Omega_J})$. To avoid these singularities and obtain continuous wave-number and frequency spectra, a finite positive imaginary part is required. This is often referred as the renormalized quasilinear theory, also called the Resonance Broadening Theory [59]. Indeed, if a finite imaginary part ν_{QL} is assumed instead of 0^+, one obtains:

$$\mathrm{Im}\frac{1}{\omega - \vec{n}\cdot\vec{\Omega_J} + i\nu_{QL}} = \frac{\nu_{QL}}{(\omega - \vec{n}\cdot\vec{\Omega_J})^2 + \nu_{QL}^2} \quad (6)$$

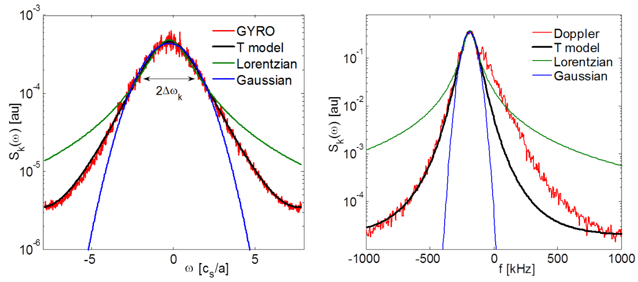

FIG. 2: At $\rho = 0.7$, TS39596. Experimental density fluctuation frequency spectrum from Doppler reflectometry at $k_\theta\rho_s = 0.82$ on the right hand side and comparison with GYRO nonlinear predictions at the same k_θ value on the left hand side. From [57].

which is a Lorentzian of width ν_{QL}. Two questions have now to be answered: is the Lorentzian an appropriate shape, and what is the value of ν_{QL}.

In tokamak plasmas, the frequency spectrum is measured by Doppler reflectometry and its shape was compared to Lorentzian as well as Gaussian fits. The same procedure was carried out for a GYRO nonlinear frequency spectrum [57]. Neither of the two shapes are correct, as illustrated by figure 2. The turbulence cannot be approximated by either a purely diffusive (Lorentzian) or convective (Gaussian) model. Nevertheless, both fits capture the majority of the spectrum (note that the figure is in a semi-log scale), and although not exact, a Lorentzian fit, as chosen in QuaLiKiz is acceptable.

The width ν_{QL} can be estimated from a crude approximation assuming that for each mode, the linear growth rate competes with the nonlinear damping. Within this picture, the nonlinear decorrelation takes place at the same rate that the free energy is replenished [**?**], leading to:

$$\nu_{QL} = \gamma \quad (7)$$

This hypothesis was tested in nonlinear simulations [47, 57]. Figure 3 shows that for $k_y \simeq k_\theta\rho_s < 0.2$, the linear growth rate of the most unstable mode represents rather well the nonlinear width. Since most of the transport occurs at low k_θ, the Lorentzian of width γ is a reasonable assumption for most cases.

A disagreement remains which does not concern the saturated potential but rather the nonlinear weight when compared to the linear response. A nonlinear transport weight, w_k^{NL} is the ratio of the nonlinear flux $< \Gamma_k(x,y,t) >_{x,y}$ to the saturated potential $|\phi_k(x,y,t)|^2$, averaged over a significant time window of the saturated phase :

$$w_k^{NL} = \left\langle \frac{< \Gamma_k(x,y,t) >_{x,y}}{< |\phi_k(x,y,t)|^2 >_{x,y}} \right\rangle_t \quad (8)$$

To compare w_k^{NL} to the linear response, the saturated potential is set identical in the quasilinear and nonlinear fluxe estimates within the GYRO (or GENE) framework [60]. It is found that the ratio of the quasilinear to the nonlinear transport weights is around 1.4. This value is independent of $k_\theta\rho_s$ in the low k range dominating the transport [47, 54]. The origin of this 1.4 factor could be linked to nonlinear transfer of

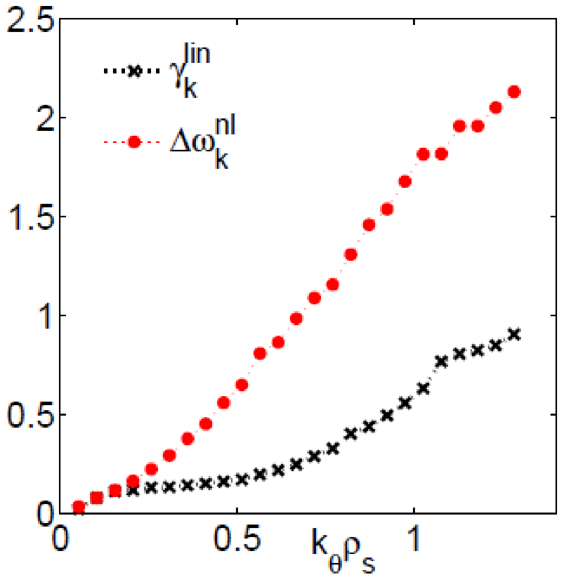

FIG. 3: k_θ spectrum of the linear growth rates and of the nonlinear frequency widths. The results refer to nonlinear and linear (most unstable mode) simulations using GYRO on TS39596 at $r/a = 0.7$ [58].

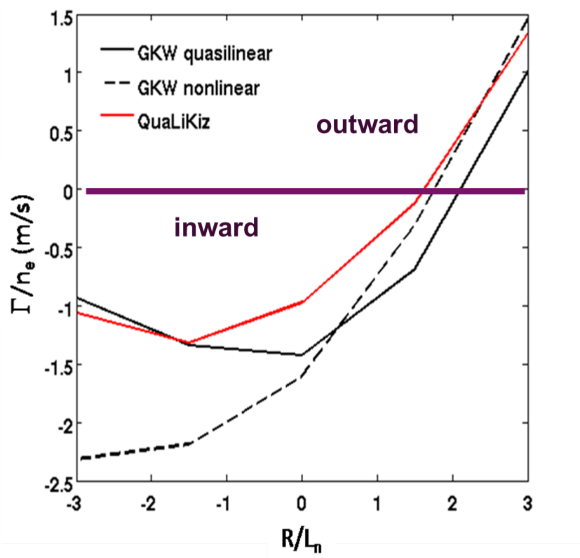

FIG. 4: Particle flux divided by the density as a function of R/Ln, calculated by the gyrokinetic codes QuaLiKiz and GKW [37]. $R/L_T = 9$, $q = 1.2$, $s = 0.58$, $\alpha = 0$, with collisions, $r/a = 0.5$.

energy through zonal flows or through transfer towards stable eigenmodes [61]. These hypotheses point towards the need for a more theoretically robust renormalized quasilinear theory as the behavior at low s further illustrates. Indeed, as s is reduced from 1 down to 0.1, the transport weight overage is increased from 1.4 up to larger values, even at low k_θ where the transport takes place [47]. In the absence of more rigorous theory, an ad-hoc additional shear dependent normalization factor was included on top of the 1.4 factor for $|s| < 0.6$, in the form of:

$$2.5(1 - |s|) \tag{9}$$

Another departure from 1.4 is observed in the case of strong inward fluxes[60]. A similar trend was found using GKW in a more recent work investigating hollow density profiles [37] as illustrated by figure 4. This remains to be understood.

Moreover, recently quasi-coherent modes were observed experimentally in reflectometry spectra [62] and were successfully modeled in GENE nonlinear simulation [63]. Such modified frequency spectra might impact the renormalized quasilinear fluxes. The renormalized quasilinear theory need to be revisited in order to provide robust first-principle saturated potentials required in quasilinear models such as QuaLiKiz.

4. *The saturated potential level and k spectrum in QuaLiKiz*

On most experiments [64–66], the density fluctuation wavenumber spectrum : $S(k_\perp) = \left|\frac{\delta n(k_\perp)}{n}\right|^2$ shows a decay for $k_\perp\rho_s \geq 0.3$ such that: $S(k_\perp) \simeq k_\perp^{\alpha_\perp}$ with $\alpha_\perp = -3.5 \pm 0.5$. This was also observed on a Tore Supra ohmic L mode pulse using two reflectometers which agreed well with nonlinear GYRO simulations confirming isotropic k spectra with a decay of around -3 [58]. Moreover this decay is successfully recovered by the shell model [67]. This choice is hence retained in QuaLiKiz.
Now that the slope above and below the maximum potential are determined, it is necessary to set the k value of the maximum potential: k_{max}. It is chosen such that the effective diffusivity, D_{eff}, follows the mixing length rule:

$$max\left(D_{eff} \simeq \frac{R\Gamma_s}{n_s}\right)_{k_{max}} = \frac{R}{n_s}\frac{k_\theta}{B}\frac{n_s e_s}{T_s}|\tilde{\phi}^2_{n\omega}|_{k_{max}} = \frac{\gamma}{\langle k_\perp^2\rangle_{k_{max}}} \tag{10}$$

By making use of the mixing length rule, a random Gaussian statistics for the saturated electrostatic potential is assumed [52]. This result is based on the picture that the nonlinear damping rate, $D_{eff}\langle k_\perp^2\rangle_{k_{max}}$ balances the linear growth rate γ [68]. Hence $D_{eff}\langle k_\perp^2\rangle_{k_{max}}$ is equal to the previously introduced ν_{QL} and is broadened in presence of zonal flows for example [69].
The choice for $\langle k_\perp^2\rangle$ is based on both experimental observations and nonlinear simulation results. It should lead to a maximum $|\phi^2_{n\omega}|$ around $k_\theta\rho_s \simeq 0.2$ as reported by numerous nonlinear simulations [58], and as observed with Beam Emission Spectroscopy [70]. It should also depend on q as observed in nonlinear simulations [56, 71]. A pertinent choice for $\langle k_\perp^2\rangle$ combining these two aspects was proposed by [7, 56, 72], extended to account for the MHD α parameter on the curvature drift as well as on the magnetic shear s in [8] such that:

$$\langle k_\perp^2\rangle = k_\theta^2\left[1 + (s-\alpha)^2 < \theta^2 >\right] \tag{11}$$

with:

$$< \theta^2 >= \frac{\int \theta^2 |\phi_{n\omega}(\theta)|^2 d\theta}{\int |\phi_{n\omega}(\theta)|^2 d\theta} \tag{12}$$

With the eigenfunction $|\phi_{n\omega}(\theta)|^2$ being a shifted Gaussian, accounting for finite $\mathbf{E} \times \mathbf{B}$ and rotation. Hence, symmetry breaking in $\langle k_\perp^2 \rangle$ is enacted through the imaginary part of the eigenfunction shift and the real part of the mode width, the latter being proportional to the growth rate found in the fluid model.
The expression for $\langle k_\perp^2 \rangle$, proposed in [8], was revisited in [47] to improve QuaLiKiz fluxes estimation at low magnetic shear. It reads:

$$\langle k_\perp^2 \rangle = k_\theta^2 + k_r^2$$
$$k_r =$$
$$\sqrt{k_\theta^2 \hat{s}^2 \langle \theta^2 \rangle} + \frac{0.4\exp(-2\hat{s})}{\sqrt{q}} + 1.5(k_\theta - 0.2/\rho_s)H(k_\theta - 0.2/\rho_s) \tag{13}$$

At low magnetic shear, the $k_r^2 = k_\theta^2 \hat{s}^2 \langle \theta^2 \rangle$ resulting from magnetic field line shearing, is underestimated with respect to nonlinear k_r [47]. The factor $\frac{0.4\exp(-2\hat{s})}{\sqrt{q}}$ was found to represent well the non linear isotropization at low magnetic shear. Finally, the term $1.5(k_\theta - 0.2/\rho_s)H(k_\theta - 0.2/\rho_s)^2$ (H is the Heaviside function) is present for completeness, to correctly capture the approximate isotropy at high wave-numbers, but typically plays a negligible role is setting the maximum value of $\gamma/\langle k_\perp^2 \rangle$.
From the above equation (13), it is clear that the mixing length needed to estimate the saturated level is the Achilles heel of the quasilinear flux construction. Some models, such as GLF23 [25], have attempted to include Zonal Flows and Geodesic Acoustic Modes impact in an ad-hoc manner. The interplay between the waves with themselves, with Zonal Flows and with Geodesic Acoustic Modes was studied using extensive nonlinear GYRO simulations in [73] showing that all saturation mechanisms co-exist.
To conclude, it should be stressed that the amplitude of the saturated potential peak is normalized in QuaLiKiz by a unique constant, such that the quasilinear ion heat flux for the GA-standard case reproduces the nonlinear result. This QuaLiKiz saturated potential level and k spectrum was successfully compared to GKWfor three values of the $\mathbf{E} \times \mathbf{B}$ shear. Despite some quantitative differences, the nonlinear flux quenching with $\mathbf{E} \times \mathbf{B}$ shear is captured qualitatively with a shifted eigenfunction calculated in the fluid limit. Moreover the linear k_θ dependence up to the maximum, the value of k_{max} and the slope in k_θ^{-3} above k_{max} all reproduce fairly well the nonlinear simulation saturated potential. In particular, the k_θ^{-3} spectrum was reported from reflectometry measurements as well as from nonlinear GYRO and GYSELA modeling [58]. This validates QuaLiKiz choices for the k spectrum and the maximum saturated potential.

IV. COMPARING QUALIKIZ QUASILINEAR FLUXES

QuaLiKiz fluxes are constructed following the expression given in equation 5. The sum over k is typically done over 20 values from $k_\theta \rho_s = 0.05$ up to 2. For detailed derivation of the quasilinear fluxes see [8] for particle, [17] for angular momentum and [40] for on overview.

A. QuaLiKiz fluxes compared to nonlinearly computed fluxes

QuaLiKiz ion and electron energy and particle fluxes without rotation nor $\mathbf{E} \times \mathbf{B}$ shear were extensively compared to nonlinear GYRO simulations [54, 57]. Over a wide range of normalized temperature gradients: $4.5 < R/L_{Ti} = R/L_{Te} < 13.5$ QuaLiKiz and GYRO agree within 15%. Both the ratio between the transport channels and the parametric dependence are well captured by the quasilinear approach.
The collisionality impact on transport is a crucial test for quasilinear models. Indeed two effects are potentially at play: a collisional damping of zonal flows [74] not included in QuaLiKiz and a linear collisional TEM damping. Over an experimentally relevant range of collisionality [54], the linear TEM damping is found to be dominant and QuaLiKiz fluxes agree well with GYRO. In particular, the particle flux reverses direction as ν_* increases as expected [7]. A transition from TEM dominated to ITG dominated domains is realized by keeping fixed $R/L_{Te} = 9.0$ and varying only the ion gradient R/L_{Ti} from 1 to 18 [54]. The electron energy fluxes are well matched; discrepancies are instead observed on the particle fluxes for strong ITG turbulence and for the ion energy flux both in the deep TEM and deep ITG regimes. Above $R/L_{Ti} = 13$, the QuaLiKiz overestimation can be ascribed to a more pronounced effect of zonal flows in the nonlinear saturation. When approaching the critical temperature threshold, the impact of zonal flows in pure ITG turbulence was shown to lead to an upshift of the threshold referred to as the "Dimits upshift" [75]. It was, for the Cyclone base case and with adiabatic electrons, a shift in normalized temperature gradient units such that: $\Delta(R/L_T) \simeq 2$. In presence of kinetic electrons, it was reduced to $\Delta(R/L_T) \simeq 0.8$ [76]. To explore this region, the $R/L_T = R/L_{Te} = R/L_{Ti}$ scan down to the linear threshold is performed illustrated by figure 5. The electrons are kinetic electrons and the collisionality is finite as detailed in table I. Close to the stability threshold the nonlinear runs are more demanding in term of resolution and their convergence should be carefully checked. Therefore, from figure 5, only an upper limit to the Dimits shift can be set such that $\Delta(R/L_T) \leq 0.7$. The reduced Dimits shift observed with kinetic electrons and finite collisionality is hence coherent with the overall success of quasilinear models in reproducing profiles within around 15% averaged RMS error [36, 55].

In [47], the impact of q and more specifically of magnetic shear s was extensively investigated. The RMS error on the particle and heat fluxes between QuaLiKiz and the nonlin-

R_0/a	r/a	R/L_n	q	s	T_i/T_e	Z_{eff}	ρ_*	β	ν_{ei} in units of c_s/a
2.8	0.5	2.24	1.4	0.78	1.0	1.0	0.006	0	0.03

TABLE I: Plasma parameters of $R/L_T = R/L_{Te} = R/L_{Ti}$ scan of figure 5.

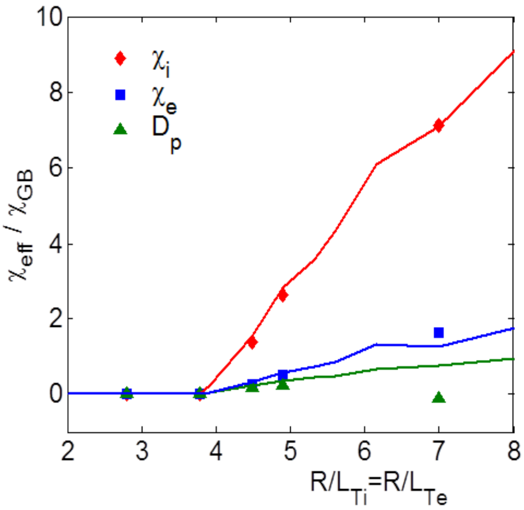

FIG. 5: Ion energy, electron energy and particle effective diffusivities from GYRO (points) and QuaLiKiz (lines) for the parameters of table I plotted versus $R/L_T = R/L_{Te} = R/L_{Ti}$.

ear GENE and GYRO simulations are for the ion heat flux $\sigma = 0.26$, for electron heat flux $\sigma = 0.33$ and particle flux $\sigma = 1$.

Quasilinear angular momentum fluxes were also validated [17]. For an $\mathbf{E} \times \mathbf{B}$ scan at zero $U_{\|}$, the angular momentum flux corresponds to the residual stress. In absolute value, the momentum flux increases at first with γ_E due the $\mathbf{E} \times \mathbf{B}$ shear asymmetrization of the eigenfunction. Then, the momentum flux is slowly reduced due to the turbulence quenching by the $\mathbf{E} \times \mathbf{B}$ shear. This qualitative trend is found for GYRO, GKW and QuaLiKiz [17]. Quantitatively, QuaLiKiz overestimates the momentum flux found with GKW by $\sim 50\%$. The discrepancy between QuaLiKiz and GKW is related to the overestimation of the saturated potential amplitude at lower $k_\theta \rho_s$ and intermediate values of γ_E in QuaLiKiz as detailed in the previous section. In QuaLiKiz the $\mathbf{E} \times \mathbf{B}$ shear is estimated assuming that the $\nabla_r(\nabla_r(P_s))$ terms are negligible, hence assuming that $\nabla_r U_{\|}$ is the dominant contribution. This approximation is not always justified, as in [77], nonlinear GYRO simulations show that the neglected terms can provide a residual stress comparable to the profile velocity shear contribution. Depending on the respective signs, the two contributions can be additive or oppose each other.

The effect of $\nabla u_{\|}$ and $u_{\|}$ on the momentum flux are analyzed with the aid of dimensionless quantities: the Prandtl number $\frac{\chi_{\|}}{\chi_i}$ and the pinch number $\frac{RV_{\|}}{\chi_{\|}}$. A normalized density gradient R/L_n scan is chosen to illustrate this comparison [78, 79]. In [17], the QuaLiKiz Prandtl number is found to be close to 0.7

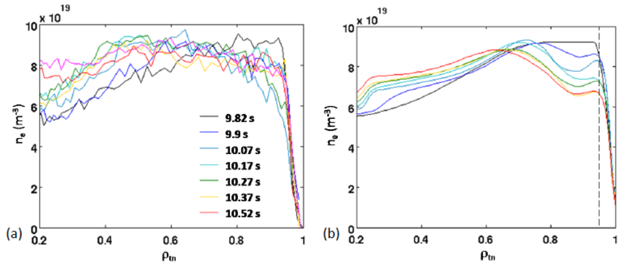

FIG. 6: Evolution of the density profile (a) from HRTS in the experimental JET pulse 79676 and (b) as obtained by the QuaLiKiz simulation in CRONOS. Different colors correspond to the times of the legend of fig. (a). The experimental density profile at 10.6 s, after the first sawtooth, is included in the graph (a) in purple. The dashed line of (b) corresponds to the radial location of the imposed boundary conditions. From [37].

agreeing with quasilinear [80] and nonlinear GKWsimulations [79]. The QuaLiKiz pinch number $\frac{RV_{\|}}{\chi_{\|}}$ is found to vary from -2 to -5, with a strong correlation with R/L_n, as in [79]. This illustrates that the eigenfunction shift is a key element and that QuaLiKiz models correctly its impact on the momentum flux convective and diffusive parts.

B. quasilinear fluxes vs experiments

1. *Electron particle flux*

Following the overall success of quasilinear models in reproducing numerous features of the electron particle transport at a given time slice [8, 18, 81, 82], it is natural to now use such models in an integrated modeling framework to model density profile evolution. The experimental case on which the focus is made concerns high plasma current JET H-mode discharges ($I_p > 2.0 - 2.5$ MA, depending on plasma shape), with naturally higher plasma densities for which NBI penetration is poorer [83]. It leads to hollow density profiles persisting for timescales of several energy confinement times. When extrapolated towards ITER in D-T operation, such transient hollow density profiles lower the L to H power threshold and hence the α central heating necessary to maintain the H mode. To improve the accuracy of ITER extrapolation, the modeling of JET pulse by applying QuaLiKiz embedded in CRONOS was carried out [37]. Provided that the density pedestal is recovered, QuaLiKiz predicted hollow density profiles consistent with the experiment as illustrated by figure 6. The hollow profile is maintained since the diffusion – initially directed outward – is not counterbalanced by sufficient inward convection.

2. *Impurity particle transport*

Impurity particle transport can be explored experimentally thanks to perturbation techniques [12, 84–86]. The impact of

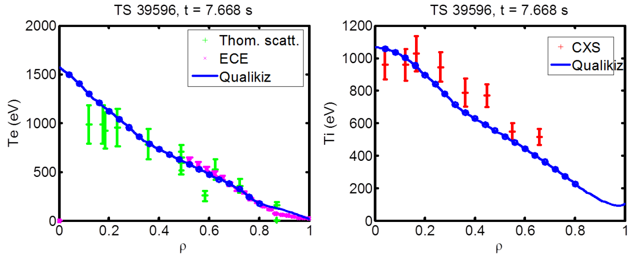

FIG. 7: Left hand side: Electron temperature profile measured by ECE and Thomson Scattering and QuaLiKiz prediction. Right hand side: Charge Exchange measured ion temperature and QuaLiKiz prediction. Tore Supra pulse 39596 [58].

the impurity charge Z was studied [8, 11, 87, 88]. In Tore Supra, no dependence in Z was found on the diffusivities and the inferred convective velocity was found directed inward [85],in agreement with QuaLiKiz. Nonetheless, the observed Z impact on the modeled gyrokinetic convection was not experimentally reported.
For impurity transport dominated by TEM turbulent transport, an influence of R/L_{Te} is expected. This was studied in [12]. Within the uncertainties, QuaLiKiz reproduces the diffusion and convection of Nickel reasonably well as well as the increase of the diffusion with larger R/L_{Te}.
In ASDEX Upgrade, in the presence of central electron heating, average Z impurities had either a weak or even a positive convection. This leads to weaker core impurity accumulation. The theoretical understanding of this mechanism is thought to be due to the passing electron compressibility term, leading to an outward convection in case of dominant TEM as reported in [9, 89]. It is shown that an outward convection is obtained only if both R/L_{Ti} is low and R/L_{Te} is high. To reproduce ASDEX Upgrade's result R/L_{Te} has to be increased above its experimental value. This mechanism cannot explain results obtained in JET on Nickel transport [10]. It is important to note that this contribution is not modeled properly in the QuaLiKiz framework where extended eigenfunctions along the field line are not compatible with the lowest order ballooning representation. Therefore complementary gyrokinetic simulations have to be carried out when investigating central electron heating cases leading to outward convection of impurities. It is also to note that, in Tore Supra dominantly electron heated pulses, the measured and modeled impurity convection was never found directed outward.

3. *Heat flux*

For a wide range of parameter space the QuaLiKiz electrostatic assumption for ion heat flux is appropriate, including baseline H-modes [36] and L-modes as illustrated by figure 7. On this figure, the temperature profiles are predicted by QuaLiKiz in CRONOS from the core to a normalized radius of 0.8 for an ohmic Tore Supra L mode. In other cases, with high β and/or significant fast ion pressure, such as in the hybrid H modes, the plasma conditions are close to the Kinetic Ballooning Modes limit. Under such condition, the nonlinear electromagnetic stabilization will be important [17] and QuaLiKiz will overpredict the fluxes [36]. This can be accounted for by changing the saturated potential rather than the linear part, since it is mostly a nonlinear effect.
The L mode edge was also reported as a region where both nonlinear and quasilinear gyrokinetic models failed in reproducing systematically measured turbulent transport levels [90–92]. Resistive Ballooning Modes were proposed as potential key players [23, 93], more recently the role of the bulk rotation direction was pointed out [94].
Towards the magnetic axis, experimentally an electron heat diffusivity larger than the neoclassical predictions is reported whereas gyrokinetic codes are often below the linear stability threshold. To reconcile the modeling with the experimental observations, nonlinear turbulence spreading [95] should be investigated. Also recent multi-scale simulations show that high k ETG can nonlinearly be responsible for large increase of high k turbulent fluxes [96].

4. *Angular momentum transport*

To determine experimentally the momentum convective term, modulations of the momentum sources are necessary. Such experiments were carried out in JET H-mode pulses, where the momentum source was modulated by modulating the Neutral Beam Injection [97]. It was demonstrated that both the amplitude and phase of the modulated toroidal velocity from the experiment are best reproduced when a momentum pinch is taken into account. In this analysis, the residual stress contribution was neglected. This JET pulse was analyzed by QuaLiKiz. The estimated Prandtl number lies within 0.8 and 1.4, close to both GKWand experimental values [97]. The pinch number calculated with QuaLiKiz ranges from -3 to -7, in good agreement with the experimental values. However a definitive conclusion would require smaller error bars. Moreover some significant contributions to the residual stress reviewed in [77] are not taken into account in local models such as QuaLiKiz.

V. CONCLUSION AND PERSPECTIVES

The quasilinear approximation was demonstrated to be valid over a wide range of core tokamak parameters using both nonlinear gyrokinetic simulations and turbulence measurements [17, 40, 47, 54]. However, the construction of the saturated potential through nonlinear mechanisms needs to be further improved by revisiting renormalized quasilinear theory. Indeed for low magnetic shear, $|s| < 0.6$, ad-hoc formation had to be used. The QuaLiKiz particle fluxes for strongly inward fluxes do not reproduce well nonlinear fluxes. Recent observation of a quasi-coherent structure in measured frequency spectra linked to unstable TEM [62] is also pointing towards the need of a revisited renormalized quasilinear theory. Moreover, now that QuaLiKiz can be used in an integrated framework, hence flux driven, its predicted profiles can

be compared to nonlinear gyrokinetic flux driven codes such as GYSELA [98]. Such investigations should allow exploring further the validity domain of the quasilinear approximation.
Nonetheless, over a wide range of parameters QuaLiKiz heat, particle and momentum fluxes reproduce well nonlinear gyrokinetic fluxes while gaining a factor 10^6 on the computing time. QuaLiKiz hence computes turbulent fluxes for 20 radial positions and 10 wave numbers in less than 5 minutes. With such a code, 1 to 10 s of plasma evolution can be modelled within a 24h using an easily accessible number of 30 CPUs. Note that such time scales are similar to TGLF [27]. QuaLiKiz imbedded in CRONOS [35] reproduces well temperature profiles of in JET baseline [36], ohmic L mode temperature in Tore Supra up to $r/a = 0.8$ as well as slowly evolving hollow density profiles [37]. The L mode edge issue encounter on DIII-D cases [90, 92] needs to be investigated with QuaLiKiz, as well as current ramp-up modeling which is challenging edge turbulent electron heat transport [99, 100]. To model high β hybrid H modes[36], QuaLiKiz which is presently electrostatic needs to be extended to included nonlinear electromagnetic stabilization [32]. Accounting for electromagnetic effects and general geometry is expected to slow down QuaLiKiz. It is presently marginally fast enough, therefore an alternative route is being explored to include richer physical effects while speeding up the calculation. This is based on using a Neural Network emulation on a large QuaLiKiz database at first, then a database including results from GKW, GENE, etc. The proof of principle of this idea was successfully demonstrated on 5D QuaLiKiz database with adiabatic electrons [101]. It is presently being extended to kinetic electrons and up to 12D.
Despite various open issues to be addressed, QuaLiKiz is an adequate quasilinear gyrokinetic turbulent transport core for integrated modeling of H mode baseline and L mode core. It is very rapid and, if used in stand alone, it can allow for extensive input scans which are essential when comparing the results to experimental fluxes. Indeed fixed gradient codes are highly sensitive to the uncertainties on its inputs such as the normalized temperature gradient for example. When used in a source driven integrated framework such as CRONOS, as TGLF, QuaLiKiz models temperatures, densities and rotation profiles of few seconds of plasma. Hence now, two first principle based turbulent transport codes, bridging theory and experiments are available to all interested users. QuaLiKiz will be coupled in the European Transport Solver and JETTO by end of 2015. Documentation on code derivation details are available online [40]. For more information and to download the code, contact the two main authors of this article: clarisse.bourdelle@cea.fr or j.citrin@differ.nl.

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