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REFMULF: 2D Fullwave FDTD Full Polarization Maxwell Code

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Abstract

A novel 2D full-wave FDTD code, REFMULF, includes full polarization waves, allowing the coupling of the transverse-electric mode (TE/X-mode) with the transverse-magnetic mode (TM/O-mode) via a linear vectorial differential equation for $\mathbf{J}$ with a generic external magnetic field $\mathbf{B_0}$. It extends the capabilities of microwave reflectometry simulation, and being a parallel code, is able to cope with real size problems.

Introduction

An important tool for the progress of reflectometry is numerical simulation, able to assess the measuring capabilities of existing systems and to predict the performance of future ones in machines such as ITER and DEMO. The newly developed 2D full-wave FDTD code, REFMULF, presented here allows the treatment of full polarization waves, coupling the TE/X-mode with the TM/O-mode via a linear vectorial differential equation for $\mathbf{J}$ with any kind of external magnetic field $\mathbf{B_0}$.

$$\frac{\mathrm{d}\mathbf{J}}{\mathrm{d}t} = \varepsilon_0 \omega_p^2 \mathbf{E} - \nu \mathbf{J} + \omega_c \times \mathbf{J}. \tag{1}$$

This equation further couples wave propagation, described by Maxwell curl equations to the plasma medium. The external magnetic field components of $\mathbf{B_0}$ lying on the propagation plane (B_x and B_y) are responsible for linking the TE and TM modes. For a $\mathbf{B_0}$ purely perpendicular to the propagation plane the code describes simultaneously O-mode and X-mode propagation, which enables the simulation of depolarization processes in turbulent plasmas. This offers capabilities unavailable in present day 2D reflectometry codes, closing the gap to the much sought-after usable 3D code. Being a parallel code is able to cope with real size problems.

Code description

The code considers propagation in a 2D plane (x-y plane) with no gradients along z ($\partial/\partial z = 0$). This assumption avoids a standard linear mode conversion for any arbitrary orientation of the magnetic field. Maxwell equations are solved using the classic Yee schema on two sets of staggered grids, one for the transverse magnetic mode (TMz) with field components (E_z, H_x and H_y) and another for the transverse electric mode (TEz), with field components (H_z, E_x and E_y). The TMz and TEz planes are not coupled through Maxwell equations since $\partial/\partial z = 0$ but coupling through the equation for $\mathbf{J}$ is possible through the product $\omega_c \times \mathbf{J}$. Developing equation (1) into components the coupling (emphasized in color) is clear.

$$
\begin{aligned}
\frac{\mathrm{d}J_x}{\mathrm{d}t} + \nu J_x &= \varepsilon_0 \omega_p^2 E_x + \omega_y J_z - \omega_z J_y &\quad (2)\\
\frac{\mathrm{d}J_y}{\mathrm{d}t} + \nu J_y &= \varepsilon_0 \omega_p^2 E_y + \omega_z J_x - \omega_x J_z &\quad (3)\\
\frac{\mathrm{d}J_z}{\mathrm{d}t} + \nu J_z &= \varepsilon_0 \omega_p^2 E_z + \omega_x J_y - \omega_y J_x . &\quad (4)
\end{aligned}
$$

The components B_x and B_y couple the TMz plane to the TEz (in red) and the TEz plane to the TMz (in blue). Equations (2)–(4), with $\nu = 0$, are solved using the Xu-Yuan schema [1] with the modifications proposed in [2] for extended long-run stability. The expression for J_x is given by

$$
\begin{aligned}
C_{0x} &= 1 + \frac{\omega_x^2 \Delta t^2}{4} &\quad (5)\\
C_{1x} &= 1 + \frac{\omega_y^2 \Delta t^2}{4} - \left(\frac{\omega_x^2 \omega_y^2 \Delta t^4}{16 C_{0x}} - \frac{\Delta t^2 \omega_z^2}{4 C_{0x}} \right) \\
J_x^{n+1/2}(i, j+1/2) &= \frac{1}{C_{1x}} \left[1 - \frac{\omega_y^2 \Delta t^2}{4} + \left(\frac{\omega_x^2 \omega_y^2 \Delta t^4}{16 C_{0x}} - \frac{\Delta t^2 \omega_z^2}{4 C_{0x}} \right) \right] J_x^{n-1/2}(i, j+1/2) + \\
&+ \frac{\varepsilon_0 \omega_p^2 \Delta t}{C_{1x}} \left[E_x^n(i, j+1/2) + \left(\frac{\omega_x \omega_y \Delta t^2}{4 C_{0x}} - \frac{\Delta t \omega_z}{2 C_{0x}} \right) E_y^n(i+1/2, j) \right] + \\
&+ \frac{\varepsilon_0 \omega_p^2 \Delta t^2}{2 C_{1x}} \left[\omega_y - \left(\frac{\omega_x^2 \omega_y \Delta t^2}{4 C_{0x}} - \frac{\Delta t \omega_z \omega_x}{2 C_{0x}} \right) \right] E_z^n(i-1/2, j-1/2) + \\
&+ \left(\frac{\omega_x \omega_y \Delta t^2}{4 C_{1x}} - \frac{\Delta t \omega_z}{2 C_{1x}} \right) \left[1 + \left(\frac{1}{C_{0x}} - \frac{\omega_x^2 \Delta t^2}{4 C_{0x}} \right) \right] J_y^{n-1/2}(i+1/2, j) + \\
&+ \frac{1}{C_{1x}} \left[\omega_y \Delta t - \frac{\omega_x \Delta t}{C_{0x}} \left(\frac{\omega_x \omega_y \Delta t^2}{4} - \frac{\Delta t \omega_z}{2} \right) \right] J_z^{n-1/2}(i-1/2, j-1/2).
\end{aligned}
$$

Expressions for J_y and J_z can be easily obtained by inspection considering the positive circulation $x \to y \to z$. Note that if $\mathbf{B} = \mathbf{B_z}$, frequencies ω_x and ω_y are null and the equations (2)–(3) describe the X-mode propagation as used in REFMULX [2] code while equation (4) describes O-mode propagation as used in REFMUL code [3]. In this case the code can be used within the same scenario to simultaneously run both X- and O-mode cases in order to compare them. To run just a X- or an O-mode case is also possible by keeping the unwanted mode unexcited. Nevertheless that would result in a waste of resources, therefore the code can be compiled not only with the full polarization (default) but also with just X- or O-mode

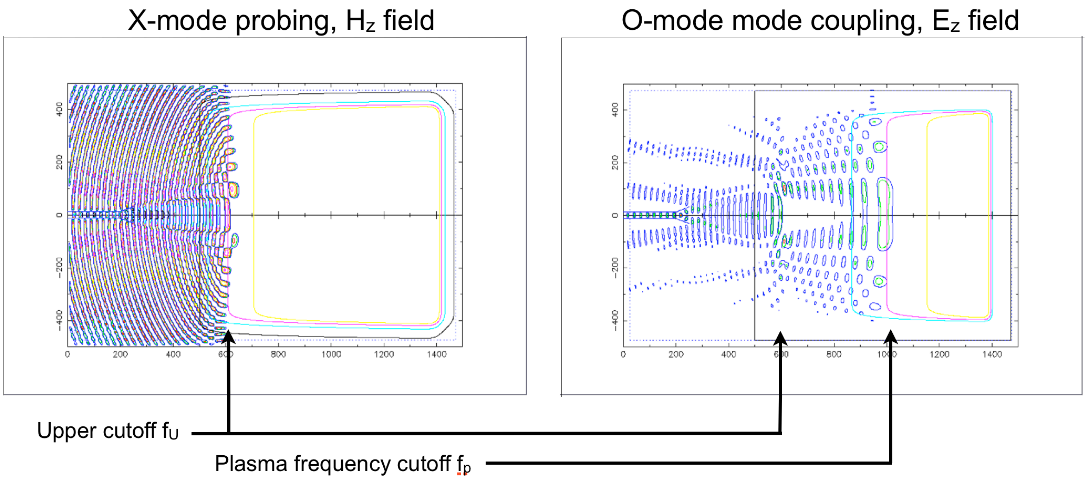

Figure 1: *X-mode is excited on the TEz plane, being reflected at the upper branch cutoff frequency f_U. The signal couples to O-mode which propagates until the plasma frequency f_p.*

executables. With this compile options, REFMULF is intended to replace REFMUL (the O-mode code) and REFMULX (the X-mode code). REFMULF uses Berenger's perfectly matched layer (PML) boundary conditions [4] and an unidirectional transparent source (UTS) [3]. The code is parallelized in the sense that its computational domain is decomposed and distributed among different computer cores using the message-passing interface (MPI) standard [5]. Within each MPI sub-domain, the OpenMP thread-parallelism [6] is further invoked to take advantage of the current trend in modern hierarchical architectures, on which several computer cores are grouped together and locally share memory. Such hybrid parallelization paradigm allows for an efficient scaling up to several hundreds of cores, which is a necessary condition for an extension of the model to three spatial dimensions.

Full polarization examples

We can observe in Fig. 1 snapshots of the H_z (left) and of the E_z (right) fields for a simulation with a linear density profile and an external magnetic field $B_0 = 0.1\mathbf{e_y} + 0.9\mathbf{e_z}$ T. A $f_c = 35$ GHz fixed frequency signal is excited in the TEz plane (*X-mode*, on the left), where it propagates until it is reflected at the upper branch cutoff frequency f_U. Due to the B_y component the wave energy is coupled to the TMz plane (*O-mode*, on the right) and can propagate further into the plasma up to the plasma frequency cutoff f_p position, where it is reflected before returning to the antenna. In the complementary situation (Fig. 2), with the same plasma and magnetic field B_0, in the TMz plane (*O-mode*), the same frequency f_c is excited and the wave propagates until it reaches the plasma frequency f_p cut-off. There is coupling to the *X-mode* in the TEz plane and reflection at at the upper branch frequency cutoff f_U position. In each of these two examples the wave was only excited on one of the

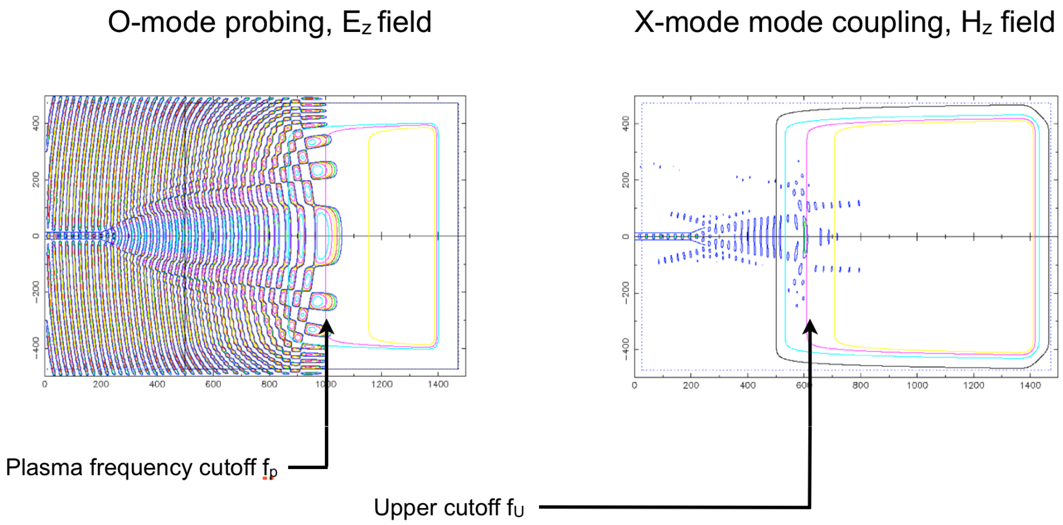

Figure 2: *O-mode is excited on the TMz plane, being reflected at the plasma frequency* f_p*. Signal is coupled to X-mode propagating until reaching the upper branch frequency cutoff* f_U*.*

planes, but more general cases can be treated with excitation on both planes with different frequencies, as well as using different techniques (e.g. pulse, CWFM, etc.).

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