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Wave induced density modification in RF sheaths and close to wave launchers

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Abstract. With the return to full metal walls - a necessary step towards viable fusion machines - and due to the high power densities of current-day ICRH (Ion Cyclotron Resonance Heating) or RF (radio frequency) antennas, there is ample renewed interest in exploring the reasons for wave-induced sputtering and formation of hot spots. Moreover, there is experimental evidence on various machines that RF waves influence the density profile close to the wave launchers so that waves indirectly influence their own coupling efficiency. The present study presents a return to first principles and describes the wave-particle interaction using a 2-time scale model involving the equation of motion, the continuity equation and the wave equation on each of the time scales. Through the changing density pattern, the fast time scale dynamics is affected by the slow time scale events. In turn, the slow time scale density and flows are modified by the presence of the RF waves through quasilinear terms. Although finite zero order flows are identified, the usual cold plasma dielectric tensor - ignoring such flows - is adopted as a first approximation to describe the wave response to the RF driver. The resulting set of equations is composed of linear and nonlinear equations and is tackled in 1D in the present paper. Whereas the former can be solved using standard numerical techniques, the latter require special handling. At the price of multiple iterations, a simple 'derivative switch-on' procedure allows to reformulate the nonlinear problem as a sequence of linear problems. Analytical expressions allow a first crude assessment - revealing that the ponderomotive potential plays a role similar to that of the electrostatic potential arising from charge separation - but numerical implementation is required to get a feeling of the full dynamics. A few tentative examples are provided to illustrate the phenomena involved.

Keywords: ICRH, ponderomotive effect, sheath, density modification
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INTRODUCTION

Although there is ample experimental evidence that radio frequency waves perturb the edge, current-day antenna designs commonly do not account for the fact that the antenna is immersed in a plasma but rather assume that it is sitting in vacuum inside the antenna box. Also the near-field dynamics just outside the antenna box is commonly oversimplified. The simultaneous presence of plasma, strong confining magnetic fields, metallic objects and intense wave fields is non-evident to account for. There e.g. is a vast amount of literature on sheaths and a somewhat more limited set of papers on sheaths in magnetised plasma, but papers addressing the dynamics of RF sheaths in magnetised plasmas are rare. The present paper aims at proposing a basic, crude set of equations allowing to assess wave induced density modifications. Standard tools are adopted: slow and fast time scale dynamics are isolated relying on linearisation and differing time scales, but coupling occurs via quasilinear/ponderomotive corrections. On the fast time scale the cold plasma dielectric tensor is adopted; on the slow time scale the limiting case $\partial/\partial t = 0$ is examined and hence Poisson's equation describes the electric field pattern.

THE RELEVANT SET OF EQUATIONS AND THEIR IMPLEMENTATION

To model the interplay of a plasma and electromagnetic waves, at least the following equations are needed: (i) The pattern of the electromagnetic waves is obtained by solving Maxwell's equations. (ii) Solving the equation of motion learns how the plasma momentum for each type of species is modified under the influence of the relevant forces. (iii) The density resulting from the flow pattern obtained by integrating the equation of motion is obtained from the continuity equation.

When describing the impact of radio frequency (RF) waves, 2 rather than 1 sets of such equations need to be studied since the total motion is the sum of a rapidly varying, direct response to the RF waves and a slowly varying response to the net forces still present after averaging over the fast time dynamics. The starting point is the single particle equation of motion under the influence of the Lorentz force. As the variation of the static magnetic field $\vec{B}_o$ is not a key player close to the antenna, $\vec{B}_o$ is taken constant. Splitting the position and the velocity into a slow and a small but fast varying contribution ($\vec{x} = \vec{x}_o + \vec{\rho}_1$, $\vec{v} = \vec{v}_o + \vec{v}_1$), neglecting higher than first order corrections of the fields and omitting the static magnetic field gradient, the slow flow is captured by solving the equation $md\vec{v}_o/dt = q[\vec{E}_o + \vec{v}_o \times \vec{B}_o] + \vec{F}_{RF}$ in which $\vec{F}_{RF}$ is the *net* force due to the rapidly varying aspects of the motion, i.e. $\vec{F}_{RF} =< q[\vec{v}_1.\nabla\vec{E}_1 + \vec{v}_1 \times \vec{B}_1] >$ where $< ... >$ denotes the smoothing time average. The driven oscillation (denoted with the subscript '1') satisfies $\vec{v}_1 = -i\omega\vec{\rho}_1$, $-i\omega m\vec{v}_1 = q[\vec{E}_1 + \vec{v}_1 \times \vec{B}_o]$ in which ω is the driver frequency and a driven response $\propto exp[-i\omega t]$ was assumed. $\vec{B}_o$ and $\vec{E}_o$ are the static magnetic and electric fields. The quasilinear $\vec{F}_{RF}$ term is known as the ponderomotive force. The corresponding acceleration can be written as $\vec{a}_{Pond} = -\nabla\Theta$ [1, 2] where

$$\Theta = \frac{1}{4}\Big[\frac{1}{\omega^2 - \Omega^2}\Big]\Big[|\vec{\varepsilon}|^2 - |\frac{\vec{\varepsilon}.\vec{\Omega}}{\omega}|^2 - \frac{2\Omega}{\omega}Im[\varepsilon^*_{\perp,1}\varepsilon_{\perp,2}]\Big]. \quad (1)$$

and in which $\vec{\Omega} = q\vec{B}_o/m$ and $\vec{\varepsilon} = q\vec{E}_1/m$. Klima - who included the effect of the RF magnetic field but described the fast variation solely as a driven oscillation - extended the result found for single particles to the case of a flow, introducing the proper averaging in Lagrangian coordinates. The result is an intuitive generalisation of the single particle equation: $\frac{d\vec{v}_o}{dt} = -v_t^2\frac{\nabla N_o}{N_o} - \frac{q\nabla\Phi}{m} - \nabla\Theta + \Omega\vec{v}_o \times \vec{e}_{//} = \vec{a} + \Omega\vec{v}_o \times \vec{e}_{//}$ Here N_o is the density and $v_t^2 = kT/m$ is the square of the thermal velocity; T is the temperature. The notation $\vec{E}_o = -\nabla\Phi$ was introduced to reflect the fact that Faraday's law requires the electric field $\vec{E}_o$ to be the gradient of a function when $\partial/\partial t = 0$. Whereas the perpendicular component of this equation still forces rapid oscillations through the Lorentz term $\Omega\vec{v}_o \times \vec{e}_{//}$, the parallel component does not. For a *given* static acceleration $\vec{a}$, the solution of the perpendicular part of the equation of motion is $v_\pm = v_{\pm 0}exp[\mp i\Omega t] \mp ia_\pm/\Omega$ where $v\pm = v_{\perp,1} \pm iv_{\perp,2}$, which consists of a rapidly varying contribution oscillating at the cyclotron frequency, and of a slow drift term *across* the magnetic field lines of the usual form $\vec{v}_{drift,\perp} = m\vec{a} \times \vec{B}_o/[qB_o^2] = \vec{a} \times \vec{e}_{//}/\Omega$. Aside from the diamagnetic drift due to the $\nabla P = kT\nabla N_o$ force and the $\vec{E}_o \times \vec{B}_o$ drift due to the electrostatic field $\vec{E}_o = -\nabla\Phi$, it consists of a ponderomotive contribution due to the finite RF electric field. From this point in the paper onwards $\vec{v}$ will represent the *true slow* time scale velocity. The equation for $v_{//}$ is

$$v_{//}\frac{\partial}{\partial x_{//}}v_{//} = -\Big[\vec{v}_{drift,\perp}.\nabla v_{//} + v_t^2\frac{\partial lnN_o}{\partial x_{//}} + \frac{q}{m}\frac{\partial\Phi}{\partial x_{//}} + \frac{\partial\Theta}{\partial x_{//}}\Big]. \quad (2)$$

If the confining magnetic field is strong so that the perpendicular drift velocity is modest, the above equation of motion can readily be integrated analytically to yield $N_o = N_{o,ref}exp[-\Big(v_{//}^2/2 + q\Phi/m + \Theta\Big)/v_t^2]$. This is a generalisation of the Boltzmann expression incorporating the impact of the parallel flow and of the RF ponderomotive force aside the usual electrostatic potential. The terms $v_{//}^2/2$, $q\Phi/m$ and Θ play a similar role in the density variation and are all position-dependent. It is crucial to recall that the expression is reflecting density variations resulting from variations of zero order quantities *along* the magnetic field. For a simple plane wave solution, $\vec{E} \propto exp[ik_x x]$, the density change caused by the electric field scales as the derivative of Θ: $dlnN_o/dx|_{RF} = 2Im[k_x]\Theta/v_t^2$. This readily shows that the effect of the waves on the density is much more pronounced when waves are *evanescent* than when waves are propagative. Furthermore, Θ is proportional to the square of electric field components. For both these reasons, wave-induced density depletion is expected to be much stronger close to wave launchers than away from them, and is most pronounced in low density sheath regions.

The fast scale wave equation is solved for the reigning density. The adopted dielectric response is modeled via the usual cold plasma dielectric tensor (see e.g. [3, 4]) and hence the wave model is standard and requires no dedicated description. Four boundary conditions are required to uniquely define the solutions of the resulting 4th order differential equation. Aside from the fast time scale wave equation, the relevant equations are (i) the slow time scale continuity equation yielding $v_{//}$, (ii) the slow time scale parallel equation of motion yielding the (logarithm of the) density N_o, (iii) the fast time scale continuity equation yielding the perturbed density and (iv) the slow time scale (Poisson) wave equation yielding the electrostatic potential Φ. Various of the equations are linear equations for individual quantities but are nonlinear equations in terms of the various unknown quantities. Rather than solving all (linear and nonlinear) equations simultaneously, the set of equations can be solved *iteratively* addressing the dependence of the variables one by one and hence solving a series of linear equations at each iteration. First, the

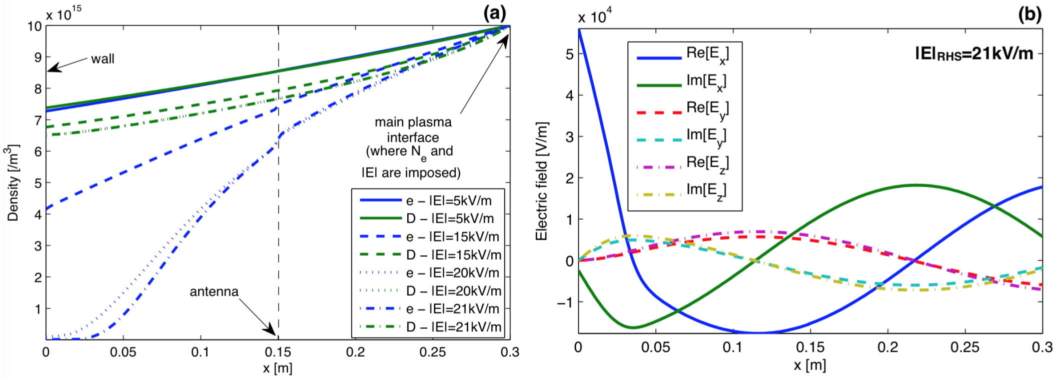

FIGURE 1. (a) Macroscopic electron and D density for typical IShTAR parameters and various electric field strengths at the location where the density is assumed known; (b) electric field structure for the highest $|E|$ considered in (a).

solution is found in absence of RF fields and from an approximate density profile N_o for the various species. Once the self-consistent solution is found in absence of RF heating, the heating is switched on and adiabatically increased. Once the desired electric field level is reached, iterations are done until convergence is reached. Only the converged solution is physically meaningful. Having this iterative procedure in mind, the first 3 of the above mentioned equations are all of the simple form $AdF/dx + BF + C = 0$. More details on the here adopted model can be found in [6].

EXAMPLES

Two preliminary illustrations of the physics described by the equations are provided. Parameters characteristic for the IShTAR test stand (Garching, Germany) have been chosen [7, 8]: the magnetic field strength is $B_o = 0.1T$, the density at the rightmost point of the integration interval is imposed to be $N_e = 10^{16}/m^3$. In this IPP test stand, near-antenna-field and RF sheath physics will be studied as soon as a dedicated ICRH antenna - designed at LPP-ERM/KMS (Brussels, Belgium) [9] - will be installed in it. Recently, a frequency window compatible with the engineering constraints of the machine has been identified [10]. To highlight the interplay of the fast and slow waves close to launchers, and to assess the impact of the fast and slow wave cutoff and the lower hybrid resonance on the electric field pattern close to the antenna straps and on the coupled power, a frequency of $f \approx 10MHz$ should be adopted. This frequency was chosen for the examples. The left subplot of Fig.1 illustrates the macroscopic density profile set up for a D plasma in presence of ICRH waves. Different electric field values are chosen; they are loosely based on the fact that voltages of the order of a kV will be imposed on the antenna while the grounded antenna box is only a few cm away from the strap. A parallel flow of 1.1 times the ion thermal velocity was imposed at the right of the integration interval. A zero electrostatic potential and negligible derivative were assumed (i.e. charge neutrality is assumed for this example; when modeling sheaths, Poisson's equation becomes a key tool). In agreement with the analytical expression for the density given earlier, the RF waves chase the electrons out of the antenna region. The ion density is less changed but also affected. A key difference between the dynamics of the 2 species is the (non-)importance of the $v_{//}$ variations. The density depletion is a nonlinear function of the wave amplitude: at modest field values the density is almost unaffected; for $|E| = 0$ - not shown - the density is essentially identical to that for $5kV/m$. For high electric field values the electron density drops to zero at the wall. The right subplot depicts the electric field for $|E| = 21kV/m$.

CONCLUSIONS AND DISCUSSION

In the present paper a model is presented to study the interaction of RF waves and a plasma. It relies on 'classical' ingredients: In view of the vastly different time scales involved, a 2-time scale model has been set up. On both time scales the model consists of a wave equation, an equation of motion, and the continuity equation. The slow time scale dynamics is influenced by the fast time scale physics through quasilinear modifications. The fast time scale equation of motion is solved analytically as a driven problem, and the corresponding wave fields and fast time scale density modification are computed starting from this same assumption. The slow time scale equation of motion is equally solved analytically for what concerns the perpendicular dynamics while the parallel equation of motion needs to be solved numerically, except for the case when drifts can be neglected. The wave polarisation has repercussions on which species are affected: For frequencies in the ion cyclotron frequency range, the electron dynamics is dominantly controlled by the parallel electric field while the ions are mainly affected by the perpendicular components. The adopted fast time scale wave equation relies on the cold plasma dielectric tensor. For the slow time scale wave equation it is assumed that a steady state has been reached so that only Poisson's equation consistent with a charge separation needs to be solved.

The resulting set of equations is a mix of linear and non-linear equations. Rather than solving the actual nonlinear problem, an iterative scheme was set up to solve the non-linear problem as a series of linear ones. Upgrading the model is a must. The here presented set of equations has been applied in 1D. Retaining rather than neglecting the omitted derivatives and adding proper 'source' terms would allow to assess the dynamics in 2D or 3D with sufficient rigour; Lu [11] works on incorporating the proper physics into a 2D model due to Jacquot and Colas [12, 13]. But the nonlinear equations that need to be solved are numerically touchy. In their barest form, this was already realised by people implementing the sheath boundary condition due to Myra and D'Ippolito [14, 15] (see e.g. [16]). In earlier work of the authors [2], the accent was on the perpendicular drifts brought about by the ponderomotive force. It allowed to explain that variations of the electric field amplitude perpendicular to the antenna straps cause poloidal drifts along it and that the strong gradients at the antenna strap tips cause radial flows towards and away from the antenna, highlighting the physics at hand is basically *multidimensional*.

The present paper proposes a possible path towards better understanding of wave-plasma cross-talk close to metallic walls and close to wave launchers. The adopted simplified multi-fluid method clearly has a number of shortcomings, the impact of which needs to be assessed once the necessary pieces of the model have been duly assembled and tested. Kinetic effects and full non-linearity (turbulence) are clearly far out of its reach. Likewise, situations where the time scales involved are not well separated (as e.g. occurs very close to the cyclotron resonance) or where the linearisation breaks down need special attention. The model's capacity - or incapacity ... - to capture key aspects of experiments (see e.g. [8]) will be the ultimate judge of its usefulness.

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