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ABSTRACT

The poloidal variation of impurity densities over magnetic surfaces brings about an enhancement of neoclasical transport coefficients, as shown by Romanelli and Ottaviani for impurities in the Pfirsch Schlüter regime and by Helander for particles in the banana-plateau regime, both in a large aspect ratio tokamak.

The same effect will occur in a finite aspect ratio tokamak and therefore it is considered to be relevant for inclusion in transport codes [6] for comparison with the experimental measurements of impurity transport.

Here an expression for the impurity-density poloidal-variation generated by the fast toroidal rotation of the plasma column is presented in general coordinates.

INTRODUCTION

When the flow velocity of a species becomes of the same order of its thermal velocity (Mach number of order one) centrifugal effects start to play a role in the momentum balance equation [1].

This condition is easily achieved in modern tokamaks and heavy impurities when neutral beam heating is applied. The centrifugal force along the magnetic field lines is balanced by a parallel pressure gradient which features as a poloidal asymmetry in the species density. This poloidal asymmetry is difficult to see in the experimental data (where only radial profiles of physical quantities are measured) unless specific experiments are carried out. The JET Soft X-ray tomography revealed this feature in impurity injection experiments where nickel and other impurities were introduced into the plasma [2]. The same feature has also been seen in other tokamaks as well.

As shown later in this report any poloidal variation of physical quantities brings about an enhancement of the neoclassical transport by modifying the surface averages and this effect should be taken into account when comparing the neoclassical predictions with experimental data.

ANALYSIS

In what follows the impurity species is taken to be a trace impurity (no effect on the background magnetic field and electric field can be attributed to the presence of the impurity itself). The momentum equation of the impurity is [1]

$$-m_I n_I \Omega^2 R \mathbf{e}_R = -\nabla p_I + n_I Ze\left(\mathbf{E} + \mathbf{V}_I \times \mathbf{B}\right) \tag{1}$$

The term $mn\Omega^2 R\boldsymbol{e}_R$ is the centrifugal force, Ω is the toroidal rotation frequency (assumed to be the same as that of the background plasma) and $\boldsymbol{e}_R$ is the unit vector in the direction of the torus major radius R.

Using the toroidal symmetry the unit vector in the direction of the magnetic field can be taken to be of the form

$$\mathbf{B} = \mathbf{e}_\phi + I\mathbf{e}_\phi \times \nabla\Psi$$

$$\mathbf{b} = \frac{B_\phi}{B}\mathbf{e}_\phi + \frac{B_\theta}{B}\mathbf{e}_\theta$$

where ψ is the flux coordinate. The component of equation (1) parallel to the magnetic field is

$$-m_I n_I \Omega^2 R \mathrm{e}_R \cdot \mathbf{b} = -\boldsymbol{\nabla} p_I \cdot \mathbf{b} + n_I Ze(\mathbf{E}\cdot\mathbf{b}) \tag{2}$$

The parallel electric field is that generated by the background electron and ion momentum balance:

$$-m_i n_i \Omega^2 R \mathrm{e}_R \cdot \mathbf{b} = -\boldsymbol{\nabla} p_i \cdot \mathbf{b} + n_i e(\mathbf{E}\cdot\mathbf{b})$$

$$0 = -\boldsymbol{\nabla} p_e \cdot \mathbf{b} - n_e e(\mathbf{E}\cdot\mathbf{b})$$

from which, assuming quasineutrality ($n_e = n_i$) we find

$$\mathbf{E}\cdot\mathbf{b} = -\frac{m_i \Omega^2 R T_e}{e(T_i + T_e)}\mathbf{e}_R \cdot \mathbf{b}$$

which substituted in equation (2) gives

$$-m^* n_I \Omega^2 R \mathrm{e}_R \cdot \mathbf{b} = -\boldsymbol{\nabla} p_I \cdot \mathbf{b} \tag{3}$$

Here we have introduced the reduced mass $m^* = m_I - \frac{Z m_i T_e}{T_e + T_i}$, which is of the order of the impurity mass for $T_e \le T_i$.

Now we note that both the pressure gradient and the radial unit vector in equation (3) have only a poloidal component parallel to the magnetic field,

$$\mathbf{e_R}\cdot\mathbf{b} = \mathbf{e_R}\cdot\mathbf{b}_\theta, \qquad \nabla p \cdot \mathbf{b} = \nabla p \cdot \mathbf{b}_\theta$$

where we adopt the reference system (Ψ,θ,ϕ) with

$$\nabla\theta = I\mathbf{e}_\phi \times \nabla\Psi$$

ψ and θ are functions of the usual coordinates R and Z (and viceversa), where R is parallel to the torus major radius and Z parallel to the torus symmetry axis.

We can write the poloidal component of the parallel impurity pressure gradient as

$$\nabla p_I = \frac{1}{\sqrt{g_{\psi\psi}}}\frac{\partial p_I}{\partial\psi}\mathbf{e}_\psi + \frac{T_i}{\sqrt{g_{\theta\theta}}}\frac{\partial n_I}{\partial\theta}\mathbf{e}_\theta$$

$$\nabla p_I \cdot \mathbf{b}_\theta = \frac{T_i}{\sqrt{g_{\theta\theta}}}\frac{\partial n_I}{\partial\theta}\frac{B_\theta}{B}$$

where the metric elements of the coordinate system are indicated with g, in particular

$$g_{\theta\theta} = \left(\frac{\partial R}{\partial \theta}\right)^2 + \left(\frac{\partial Z}{\partial \theta}\right)^2$$

The impurity temperature is assumed to be constant along the magnetic field line and equal to the hydrogen temperature.

Equation (3) can be written as

$$\frac{1}{n_I}\frac{\partial n_I}{\partial \theta} = \frac{m^* \Omega^2 R}{T_i}\frac{B}{B_\theta}\sqrt{g_{\theta\theta}}\,\mathbf{e}_R \cdot \mathbf{b}_\theta$$

and integrated between 0 and θ to give

$$n_I(\Psi,\theta) = n_I(\Psi,0) exp\left[\frac{m^* \Omega^2(\Psi)}{T_i(\Psi)}\int_0^\theta R(\Psi,\theta)\frac{B}{B_\theta}\mathbf{e}_R \cdot \mathbf{b}\sqrt{g_{\theta\theta}}\,\mathrm{d}\theta\right] \qquad (4)$$

The integral of equation (4) can be explicitly calculated for any flux function ψ by noticing that

$$\sqrt{g_{\theta\theta}}\,d\theta = ds$$

where s is the length of the magnetic field line projected on the poloidal cross section, and that for any function $f(R,Z)$ and any curve γ of the (R,Z) plane the following relation holds

$$\nabla f \cdot \mathbf{t}_\gamma = \frac{df}{ds}$$

here **t** denotes the unit vector tangent to the curve γ, follows that

$$\nabla R \cdot \mathbf{e}_\theta = \mathbf{e_R} \cdot \mathbf{e}_\theta = \frac{dR}{ds}$$

where $\mathbf{e}_\theta$ is the unit vector tangent to the magnetic field line (magnetic surface) projected on the torus poloidal cross section. Using the above relations we can write the integral

$$\int_0^\theta R(\Psi,\theta)\mathrm{e}_R \cdot \mathbf{b}\sqrt{g_{\theta\theta}}\,\mathrm{d}\theta = \int_0^\theta R(\Psi,\theta)\frac{dR}{ds}ds =$$

$$= \int_{R(\Psi,0)}^{R(\Psi,\theta)} R dR = \frac{1}{2}\left(R^2(\Psi,\theta) - R^2(\Psi,0)\right)$$

and so

$$n_I(\Psi,\theta) = n_I(\Psi,0) exp\ \frac{m^* \Omega^2(\Psi)}{2T_i(\Psi)}\left(R^2(\Psi,\theta) - R^2(\Psi,0)\right)$$

The same result was obtained by Wesson [3] where he takes into account also the effect of an impure plasma. Within reference [3], in the expressions (10) and (12) the quantities R and n_z are functions of Ψ and θ while all other quantities are function of ψ only; both n_{z0} and R_0 are taken at θ=0.

The flux surface average <> of the impurity density will be

$$\langle n_I \rangle = n_I(\Psi,0) exp \frac{m^* \Omega^2(\Psi)}{2T_i(\Psi)} \left(\left\langle R^2(\Psi,\theta) \right\rangle - R^2(\Psi,0) \right)$$

and it can prove to be very different from the previously assumed $\langle n_I \rangle = n_I(\Psi,0)$.

EFFECT OF DENSITY ASYMMETRY ON TRANSPORT

The neoclassical expression of the average flux for the species *a* is [4]

$$\langle \Gamma_a \cdot \nabla\Psi \rangle = \sum_b \frac{T_b}{e_a e_b} \left\langle \left(l_{11}^{ab} \frac{d\,ln\, p_b}{d\Psi} + l_{12}^{ab} \frac{d\,ln\, T_b}{d\Psi} \right) \left(\frac{1}{B^2} - \frac{1}{\langle B^2 \rangle} \right) \right\rangle \tag{5}$$

where the *l* 's are the transport coefficients proportional to the collision frequency, and hence to the density of the species *a*. When the density and temperature of each species are assumed constant over the magnetic surface the first parenthesis in the average of equation (5) can be pulled out of the average leaving the magnetic field non uniformity the only contribution to the neoclassical enhancement of collisional transport. On the other hand, when the density is not uniform over the magnetic surfaces the proper flux surface average of the first parenthesis must be calculated. One can show with a simple analysis that while the Pfirsh-Schlüter contribution due to the magnetic field only in a large aspect ratio tokamak is of order ε^2 (the Pfirsh-Schlüter factor $2q^2$ which multiplies the classical diffusion coefficient) the contribution coming from an outboard peaking of particle densities would be of order ε and therefore one order of magnitude larger. The above result was calculated in reference [1]. A less strong effect is found for particles in the banana regime [5].

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