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# Application of Dynamic Response Analysis to JET Heat Pulse Data

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## ABSTRACT

The plasma dynamic response can be used to study transport processes in a tokamak plasma. A method has been developed for the application of dynamic response analysis to study perturbations away from the plasma equilibrium. In this report perturbations on the electron temperature following a sawtooth collapse in the center of the plasma are considered. The method has been used to find a mathematical description of a series of heat pulses at JET. From the plasma dynamic response, the time constants which characterise the heat pulse are obtained. These time constants are compared to the transport coefficients found in previous analysis of the JET heat pulse data. Discussed are various methods used to apply dynamic response analysis to JET heat pulse data.

## INTRODUCTION

The understanding of transport phenomena in tokamak plasmas is of major importance for optimal design and operation of a future nuclear fusion reactor. The energy confinement has been studied by analysing the steady state of the plasma or alternatively by analysing the response to perturbations in the plasma. Until now, no suitable description has been found for the underlying physical processes of energy confinement in a tokamak.

Several methods have been used to study the evolution of perturbations in a plasma. In most studies, the experimental results are interpreted starting with the energy and particle balance equations. Assumptions have to be made on the transport coefficients used, on the initial conditions and on the boundary conditions. Widely used methods are the time-to-peak [1], Fourier analysis [2] and numerical simulations [3]. The energy and particle source terms as well as the plasma boundary conditions need to be known. The disadvantage of these methods is that assumptions on the underlying physical processes are necessary in order to obtain a suitable description. Applying these models to the experimental data may lead to systematic errors in the results.

In this report dynamic response of the plasma is used to investigate the transport mechanisms [4,5,6]. This is done by measuring the temporal evolution of the electron temperature (using ECE diagnostics) following a sawtooth collapse (heat pulse). A mathematical description of the experiment is obtained without the need to invoke a particular transport mechanism, boundary conditions or initial conditions.

First, a method is developed for the application of dynamic response analysis. This method is verified by using dynamic response analysis to describe a simulated series of sawteeth and heat pulses. Then, dynamic response analysis is applied to JET data. Two forms of JET data are considered. First, dynamic response analysis is applied to averaged data for one or more channels, i.e. different positions in the plasma. In this signal, typically twenty sawteeth are averaged which are synchronized at the time of the collapse [1]. Second, dynamic response analysis is applied to a series of sawteeth for one or more channels, using raw data. The time constants obtained from dynamic response analysis are compared to values of the heat transport coefficients obtained from previous analysis of the JET data.

## METHOD

### Description of dynamic systems

A dynamic response of the plasma can be used to investigate transport mechanisms. The method consists of measuring the temporal evolution of the plasma parameters following a perturbation, which may be either externally or internally imposed. In this approach, the plasma is represented by a dynamic system and is characterized by the time constants of the dynamic response.

If linear, the time dependent behaviour of a single-input single-output dynamic system is described by an ordinary linear differential equation:

$$\sum_{n=0}^{N} \alpha_n \frac{d^n y(t)}{dt^n} = \sum_{m=0}^{M} \beta_m \frac{d^m x(t)}{dt^m} \qquad (1)$$

where x(t) is the input signal and y(t) is the output signal. In addition, the coefficients $\alpha_n$ and $\beta_m$ are time independent. In a finite response system of order N, the additional condition $M < N$ is required. The behaviour of a dynamic system is characterized by its transfer function, which relates the input signal to the output signal. For that purpose it is necessary to apply Laplace Transform to equation 1:

$$\sum_{n=0}^{N} \alpha_n s^n Y(s) = \sum_{m=0}^{M} \beta_m s^m X(s) \qquad (2)$$

The transfer function of the dynamic system can be expressed as a rational function in the Laplace variable s:

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\sum_{m=0}^{M} \beta_m s^m}{\sum_{n=0}^{N} \alpha_n s^n} \tag{3}$$

Thus, if the input signal x(t) and the output signal y(t) are known the dynamic system can be completely described. On the other hand, if the input signal and the transfer function are known the dynamic response of the system can be modeled by applying inverse Laplace transform.

Discrete signals, recorded with a sampling time $T_{sam}$ can be represented by a sum of delta functions:

$$y(t) = \sum_{k=-\infty}^{\infty} y(k)\delta(t - kT_{sam}) \tag{4}$$

Due to the discretisation the linear differential equation 1 obtains the following form:

$$\sum_{n=0}^{N} \alpha_n y((k-n)T_{sam}) = \sum_{m=0}^{M} \beta_m x((k-m)T_{sar}$$

$$(t = kT_{sam}, k > N > M) \tag{5}$$

In addition, the Laplace Transform is replaced by a z-transform, which is a particular formulation of Laplace transform for discrete signals, and is defined as:

$$y(z) \equiv \sum_{k=0}^{\infty} y(k) z^{-k} \tag{6}$$

This results in a z-transfer function of the form

$$H(z) = \frac{\sum_{m=0}^{M} b_m z^{-m}}{\sum_{n=0}^{N} a_n z^{-n}} \tag{7}$$

The time evolution of the output signal is determined by the poles of the transfer function. The relation between the poles $s_n$ of the continuous transfer function (3) and the poles $z_n$ of the z-transfer function (7) follows from the mapping between the s-plane and the z-plane and is given by:

$$z_n = e^{s_n T_{sam}} \tag{8}$$

**Application of dynamic response analysis**

Using the plasma dynamic response to describe the heat pulse, the sawtooth collapse is represented by a delta function. As a result, in the right term of (5) only one term differs from zero at the sawtooth collapse. Since the plasma parameters have a finite response to the sawtooth collapse, the additional condition M < N is required. In general M = N - 1 for a finite response system. By applying fraction splitting the z-transfer function (7) can be written as

$$H(z) = \sum_{i=1}^{N} \frac{b_{0,i}}{a_{0,i} + a_{1,i} z^{-1}} \tag{9}$$

which is a sum of N z-transfer functions of a first order dynamic system. Comparing (9) to (5) reveals that a $N^{th}$ order dynamic system can be described by a parallel connection of N first order dynamic systems, which all have the same input signal. This implies that the corresponding $N^{th}$ order linear differential equation is replaced by a sum of N first order linear differential equations:

$$\begin{aligned} y(k) &= \sum_{i=1}^{N} u_i(k) \\ a_{0,i} u_i(k) &= -a_{1,i} u_i(k-1) + b_{0,i} x(k) \end{aligned} \tag{10}$$

where $u_i(k)$ is the output signal and sample time $T_{sam}$ = 1. Equation (10) shows that the output signal y(k) at time k can be derived from the preceding output signals and the input signal. The time evolution of the output signal due to the sawtooth collapse is obtained by the inverse z-transform. As a result, the output signal is represented by a sum of exponential functions:

$$y(k) = \sum_{i=1}^{N} \frac{b_{0,i}}{a_{1,i}} e^{(-\frac{a_{0,i}}{a_{1,i}} k T_{sam})} \tag{11}$$

From the coefficients $b_{0,i}$, $a_{0,i}$ and $a_{1,i}$, the amplitudes $b_{0,i}/a_{1,i}$ and the time constants $\tau_i = a_{1,i}/a_{0,i}$ can be determined. For a finite response system which describes a heat pulse, the condition $\tau_i > 0$ is required.

**Application to JET data**

The plasma dynamic response to a delta function is described by a sum of exponential functions (11) as shown in figure 2. The dynamic system can be identified using an iterative optimization of a sum of exponential functions with respect to the heat pulse. However, this optimization method is numerically unstable and strongly dependent on the initial values. Therefore, identification of the dynamic system is based on a two step optimization procedure. First an optimization with respect to the dynamic response of the plasma to a delta function is carried out.

The optimal model is defined on the basis of the error e(k) between the JET data y(k) and the modelled output y^(k), which applied to dynamic response analysis can be written as:

$$e(k) = y(k) - \hat{y}(k) = y(k) - H(z)x(k) \tag{12}$$

where H(z) is the z-transfer function defined in (9). It is more convenient to write e(k) as (using equation (10))

$$\begin{aligned} e(k) &= y(k) - \sum_{i=1}^{N} \hat{u}_i(k) \\ a_{0,i}\,\hat{u}_i(k) &= -a_{1,i}\,\hat{u}_i(k-1) + b_{0,i}x(k) \end{aligned} \tag{13}$$

where $\hat{u}_i(k)$ is the modelled output. The optimization criterion is the minimization of the error $e^2(k)$ for all values of k. The optimization is carried out using the NAG fortran routine G13BEF [8]. With this routine it is possible to determine the coefficients $a_{0,i}$, $a_{1,i}$ and $b_{0,i}$ in the set of first order differential equations (10) and (13). The white noise in the input signal is also taken into account. In order to increase the performance of the routine a constant offset can be subtracted from the data.

Next, an optimization with respect to a sum of exponential functions is carried out, using the results of the dynamic response analysis as initial values. From the coefficients $a_{0,i}$, $a_{1,i}$ and $b_{0,i}$ the coefficients in the exponential functions (11) can be determined. These values are used as initial values in an optimization with respect to a sum of exponential functions. The error (12) can now be written as

$$e(k) = y(k) - \sum_{i=1}^{N} \frac{b_{0,i}}{a_{1,i}} e^{\left(-\frac{a_{0,i}}{a_{1,i}} kT_{sam}\right)} \tag{14}$$

The NAG fortran routine E04JAF [8] is used to minimize the error $e^2(k)$ and to determine the exponential functions which describe the heat pulse.

Using this method both averaged heat pulse data and raw data are analysed as described in the introduction. When analysing the raw data, the assumption is made that each sawtooth in a series of sawteeth is represented by a delta function. To cope with the different amplitudes of the sawteeth, multiplication factors are introduced with respect to a reference sawtooth. In this way, the average time constants of a series of sawteeth at various radii are obtained, whereas the amplitudes may differ. When fitting the exponential functions to the data, the offset is determined by fitting a second order polynomial to a few data points before each sawtooth, taking the contribution of the exponential functions into account.

**RESULTS**

In this chapter the results of dynamic response analysis applied to JET data are described. An attempt is made to describe the heat pulse with two or three time constants. The fastest mode (corresponding to smallest time constant $\tau$) vanishes quickly, so the time window over which the contribution of this mode can be measured accurately is small in comparison to the time scale of the heat pulse. Also, the sample time of the signal has to be smaller than this time constant. The larger time constants can be determined over a long time interval of the heat pulse.

First, the method used to analyse JET data is tested on simulated data. A heat pulse is constructed from two or three exponential functions. The number of exponential functions used to construct the heat pulse determines the order of the dynamic system. First, a single heat pulse is simulated. Then, a series of identical heat pulses is simulated from which the exponential functions can be determined. Finally a series of identical sawteeth with a change to the background (offset) is simulated (figure 3). In all cases the time constants and amplitudes determined with the identification method are almost equal to the ones used to simulate the heat pulse. The identification of simulated data shows that the exponential functions which may describe a series of sawteeth can be extracted from the data.

Second, the analysis is applied to averaged heat pulse data. The results of dynamic response analysis turn out to depend on the initial values and accuracy of the identification procedure. In particular, the routine used for the identification of the exponential functions does not always find the minimum

$e^2(k)$ as can be defined using equation (14). The averaged sawteeth can be described by the dynamic response of a third order, or in some cases a second order dynamic system. When the time constants of different channels (different radial positions in the plasma) of one shot are identified separately, the time constants are not identical. However, under the additional constraint that the time constants are identical for each channel the efficiency of the identification is hardly affected, as can be seen from the fits in figure 4.

Other methods have been used to analyse the data in the past, yielding values for the thermal diffusivity coefficient $\chi^{hp}$ [1,2,3]. The relation between the time constants found and the thermal diffusivity coefficients previously obtained is investigated. The two largest time constants turn out to be inversely proportional to the thermal diffusivity coefficient as can be seen from figure 5.

Since the thermal diffusivity coefficient is scaled with other plasma parameters [9], the relation between the time constants and the other plasma parameters is also investigated. The best correlation is found between the second time constant $\tau_2$ and the temperature gradient of the unperturbed electron temperature $\nabla T_e$. An attempt is made to find a scaling of $\tau_2$ with the 'plasma' parameters $\chi^{hp}$ and $\nabla T_e$. The following relation is obtained:

$$\tau_2^{sc} = (93 \pm 2)\chi_{hp}^{(-0.82\pm0.35)}\nabla T_e^{(-1.09\pm0.47)}[\mathrm{ms}] \qquad (15)$$

In figure 6 the scaled time coefficient $\tau_2^{sc}$ is plotted versus the measured time constant $\tau_2$.

A disadvantage of using averaged data is that the fastest time constant, for which $\tau < 2T_{sam}$, can be filtered out. Also the largest time constant is difficult to determine since the temporal evolution of the background temperature is partly taken into account in the averaging procedure and the time window analysed is sometimes too restricted (figure 4).

Third, dynamic response analysis is applied to raw data. The results of the identification of one heat pulse for both the dynamic response and the exponential functions reveal that the representation of a heat pulse by a second order dynamic system is not satisfactory. As a result, the heat pulse is described with a third order dynamic system. In order to obtain more reliable information about the time constants a series of typically five sawteeth is fitted. In this way the average values of the time constants are obtained. In contrast to the use of averaged sawteeth the dynamic response is calculated for all data points. In most cases, the result of the minimization with respect to the dynamic response is better than the result of the minimization with respect to exponential functions. The time constants of different channels are identified under the additional constraint that they are identical for several channels at different radial positions. Figure 7 shows a fit of a series of sawteeth for three channels.

The values of the largest time constant obtained from raw data are much larger than the time constants obtained from averaged data. In figure 8 the values of the time constants obtained from the dynamic response are compared to the values obtained from the exponential function fit.

From the results three ranges of $\tau$ values can be distinguished. However, there is no convincing relation between the time constants and the thermal diffusivity coefficients determined from the averaged heat pulses, as can be seen from figure 9. Neither can a significant correlation of the time constants with the other plasma parameters be found.

## DISCUSSION AND CONCLUSIONS

The application of dynamic response analysis to simulated data shows that the time constants and the amplitudes of the exponential functions which describe the heat pulse can be determined successfully. However, the analysis is not applied to simulated data with a noise component. Although corrections for the white noise in the signal are carried out, the influence of noise on the identification of the heat pulse is not known.

In the analysis of the JET data two important assumptions are made. First, the sawtooth collapse is represented by a delta function. This assumption is justified, for the time scale of the sawtooth collapse is smaller than the sample time. However, the implications of using another input signal have not been tested. Second, in the identification of the dynamic system no dependency on the plasma parameters is taken into account. However, if the plasma characteristics change due to variations in the plasma parameters, the characteristics of the dynamic system may change accordingly. The assumption that the dynamic system characteristics remain unaltered for different radial positions is justified within the measurement errors.

Dynamic response analysis proves to be a reliable method to describe averaged data. In the identification procedure, the dynamic system is described by a parallel connection of three first order dynamic systems. Data reduction as described by Moret et al. [5] provided no improvement. Also, a treatment of the averaged data (calibration, filtering, Abel inversion, etc.) appeared not to be necessary.

The absence of a clear correlation of the largest and the smallest time constants with other plasma parameters can be caused by the averaging procedure. The smallest time constant is filtered out and the time window over which the largest time constant can be determined is too restricted.

The following differences between the values of the time constants obtained from raw data and the values of the time constants obtained from averaged data are observed:

- The value of the largest time constant for raw data exceeds the value of the largest time constant for averaged data several times. This is caused by the difference in the time windows in the identification procedure. In contrast to averaged data, the complete heat pulse in time is analysed for raw data. In this way a larger time constant is necessary to describe the heat pulse. On the other hand the amplitude of the exponential function related to the largest time constant (slowest mode) is smaller than the other amplitudes. This results in a smaller contribution of the slowest mode to the heat pulse.

- The radial positions over which the data are analysed are different for averaged data raw data. Due to noise on the raw data only channels close to the mixing radius can be taken into account. In the averaged data the noise is reduced, and as a result data at larger radial positions are taken into account.

- The time constants obtained from raw heat pulse data show no correlation with the plasma parameters. Some of the plasma parameters are determined at larger radial positions corresponding to the analysis of averaged heat pulse data. As there is a difference between the values of the time constant for raw data and averaged data, it is obvious that the relation with other plasma parameters is affected.

- The values of the smallest time constant are smaller for raw data than for averaged data. The lack of a correlation between the fastest time constant for averaged data and other plasma parameters and the difference with the fastest time constant of the raw data can be caused by the averaging procedure.

The results of the identification of raw data are better for the dynamic response than for the exponential functions fit. The dynamic response of the plasma is determined from each data point, whereas in the exponential functions fit the same set of functions is used for all the heat pulses. Thus the problem is better defined for the dynamic response. Hence, the identification of the dynamic response is necessary to analyse the raw data. Also, the number of heat pulses taken into account is limited. An attempt has been made to fit twenty heat pulses of one shot, which turned out not to be possible (no stable solution could be found).

In conclusion, dynamic response analysis is a powerful method to describe heat pulses. In this paper the foundation is laid for the analysis of JET heat pulse data using the plasma dynamic response. This method can be easily applied to other perturbation methods, since from the analysis of the heat pulse it is found that no assumptions need to be made on the boundary conditions or initial conditions, or on the transport phenomena involved.

The time constants obtained from the analysis can be used to characterise the heat pulse and can be used as heat pulse parameters. A comparison of the time constants of the dynamic response and the thermal diffusivity, obtained from previous analysis of the data, proves to be difficult at this stage. A possible explanation for this apparent lack of correlation lies in the data (averaging procedure and noise on the raw data), not in the method itself. Development of the method to analyse the raw data and study of the results is necessary to make the relation with transport phenomena.

At this stage the dynamic response analysis will be a useful tool to study the plasma dynamic response to various different type of perturbations in the plasma, or to study the changes of the plasma response to any type of perturbation experiment in which the plasma parameters are scanned systematically.

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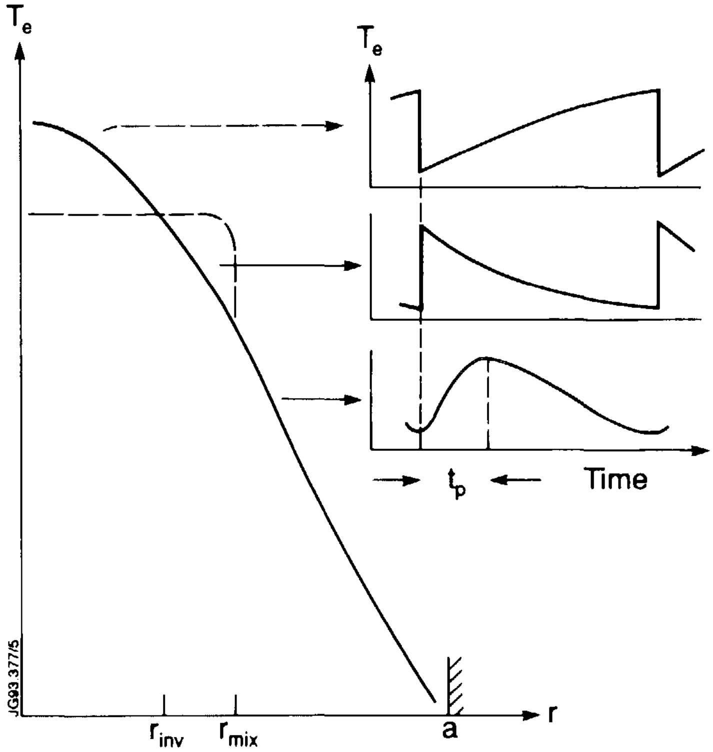



Figure 1: The change of the electron temperature profile due to a sawtooth collapse. Inside the mixing radius ($r_{mix}$) the electron temperature profile flattens during a sawtooth collapse (dashed line). Due to the flattening the electron temperature decreases inside the inversion radius ($r_{inv}$) and increases between the mixing radius and the inversion radius. Outside the mixing radius the electron temperature perturbation spreads towards the edge. The inset shows the electron temperature evolution at different positions in the plasma (heat pulse).

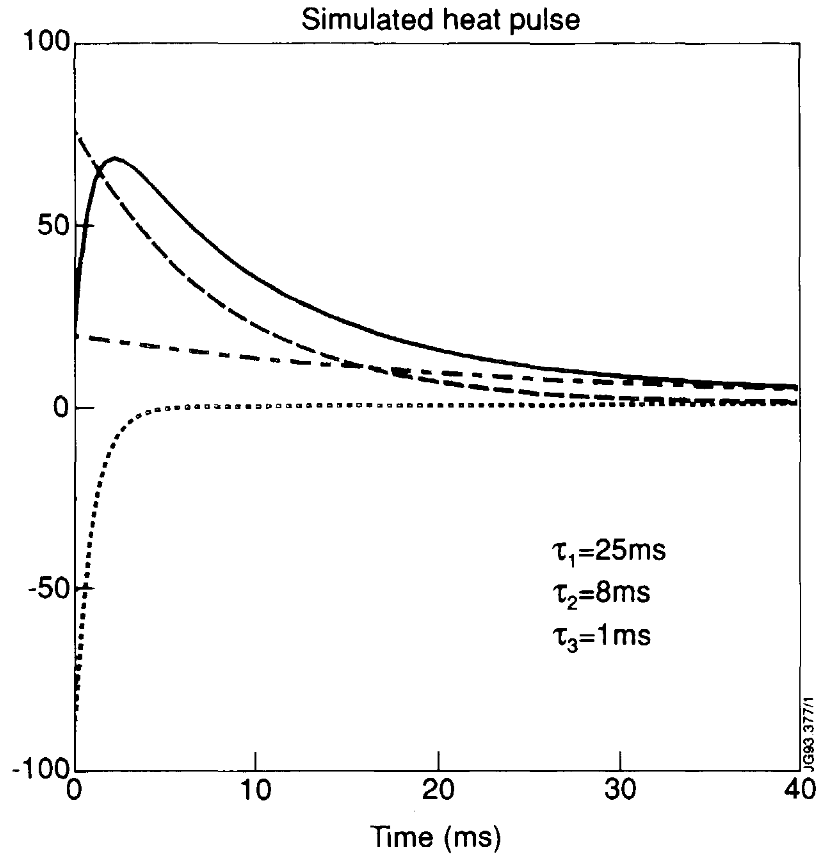



Figure 2: A heat pulse composed of three exponential functions (N=3). Note the difference in the amplitudes and time constants of the exponential functions.

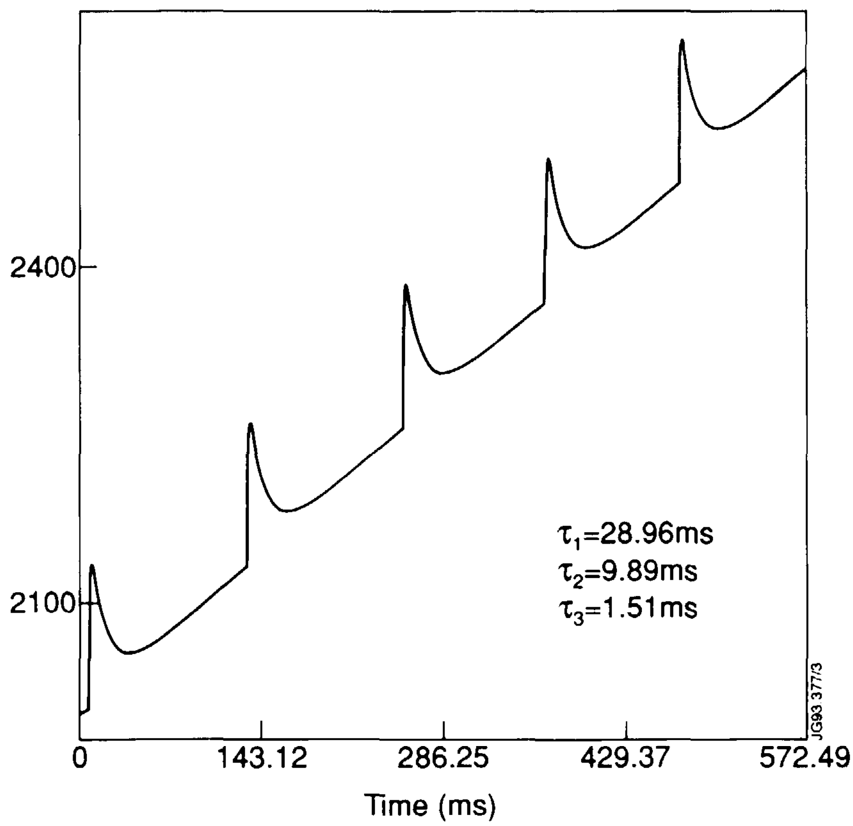



Figure 3: A simulated series of sawteeth with offset and corresponding fit. The fit can not be distinguished from the data. The imput data for the simulation are: $\tau_1$=30 ms, $\tau_2$=10 ms and $\tau_3$=1.5 ms.

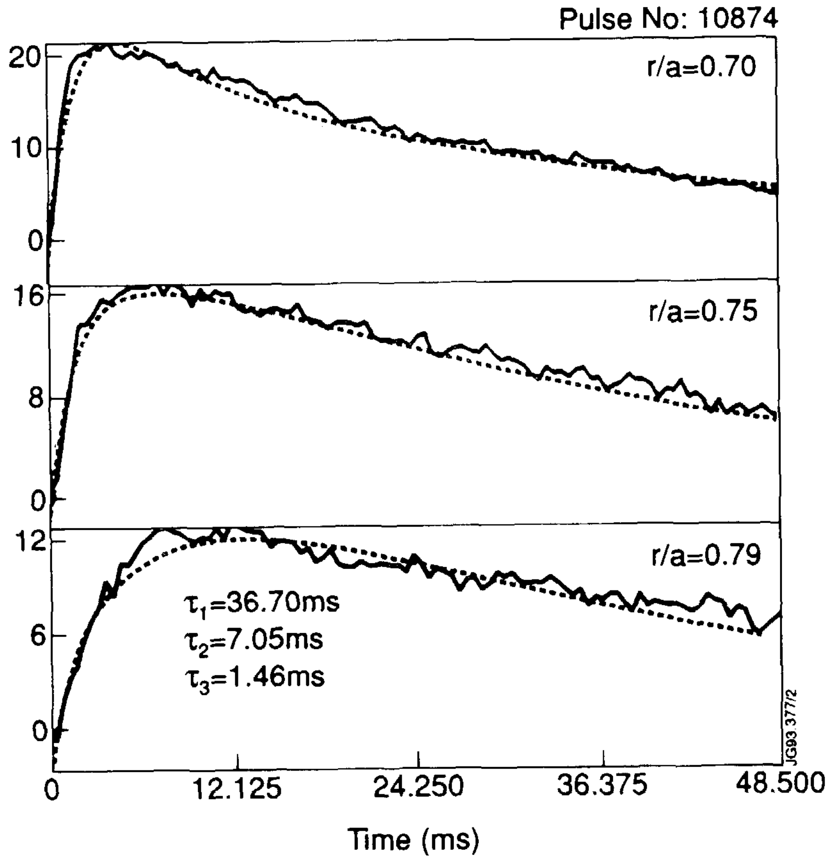
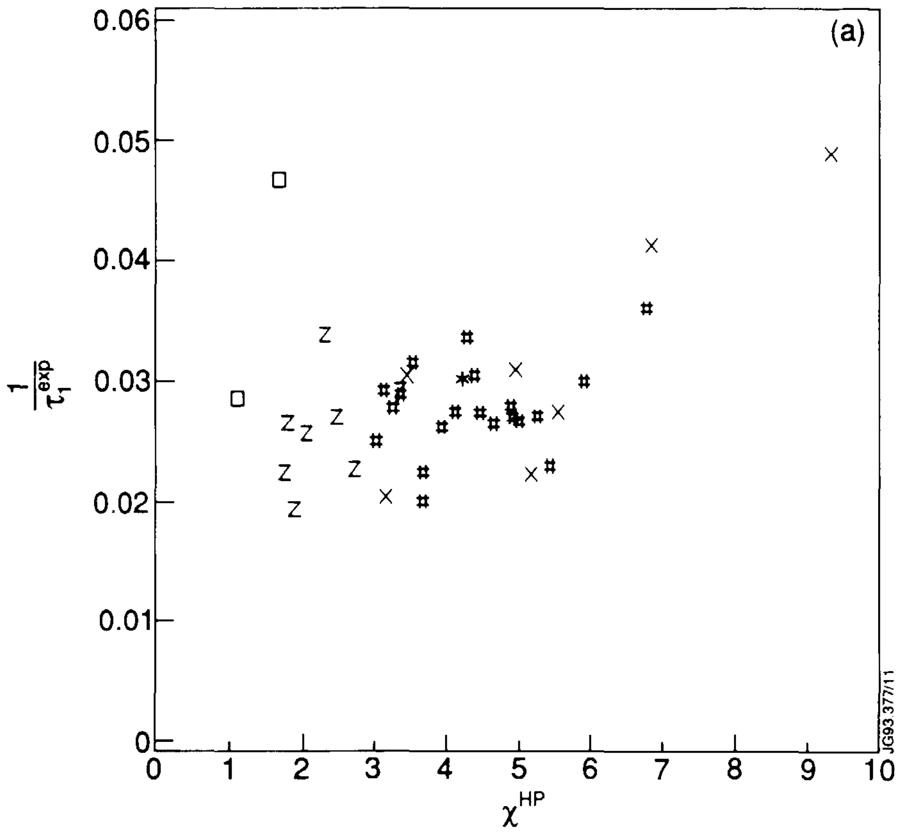



Figure 4: Averaged sawteeth for different channels of one shot and the corresponding fit. The radius to which the $\tau$ values are attributed is the averaged radius of the three channels.



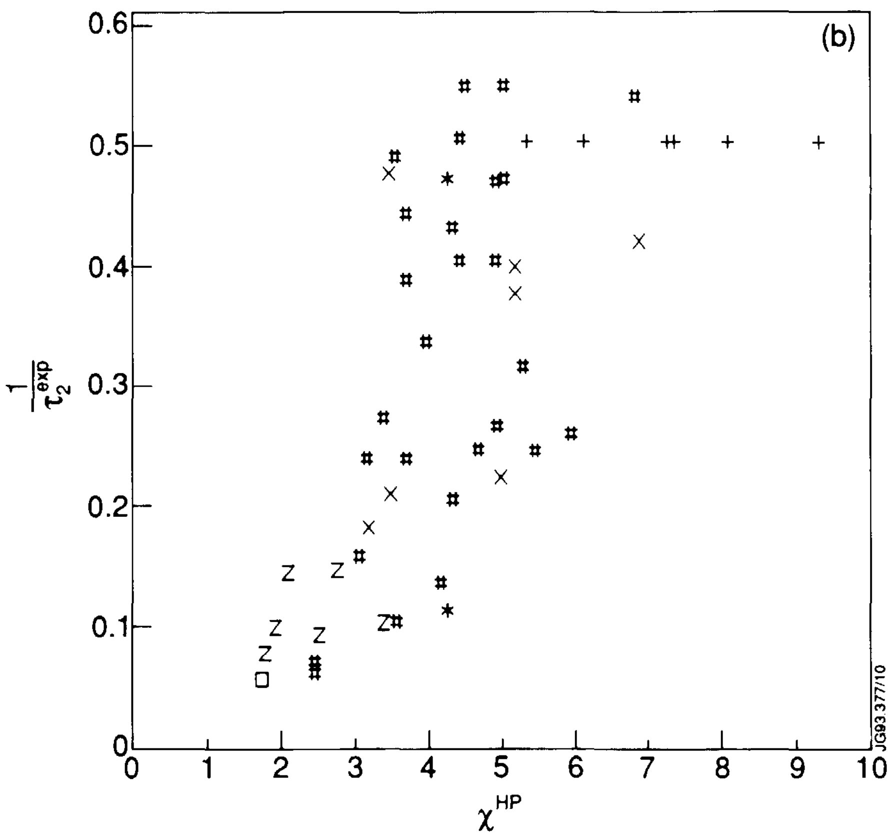




Figure 5: Plot of $1/\tau_i$ versus $\chi^{hp}$ for averaged sawteeth. Figure (a) shows $1/\tau_1$ versus $\chi^{hp}$ and figure (b) shows $1/\tau_2$ versus $\chi^{hp}$.

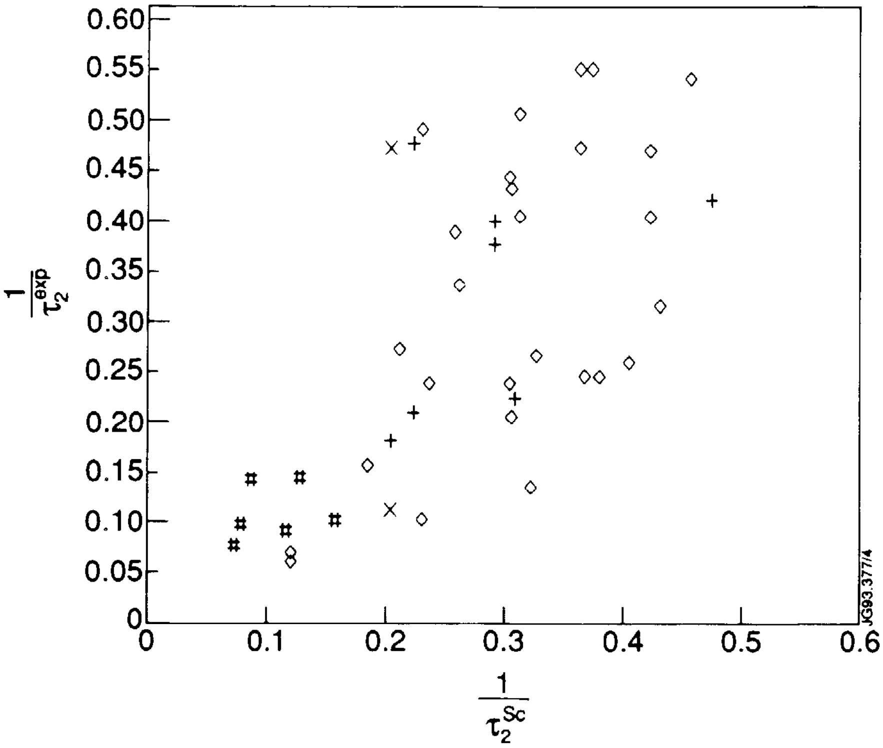



Figure 6: Plot of the scaled time coefficient $1/\tau_2^{sc}$ versus the measured time constant $1/\tau_2$.

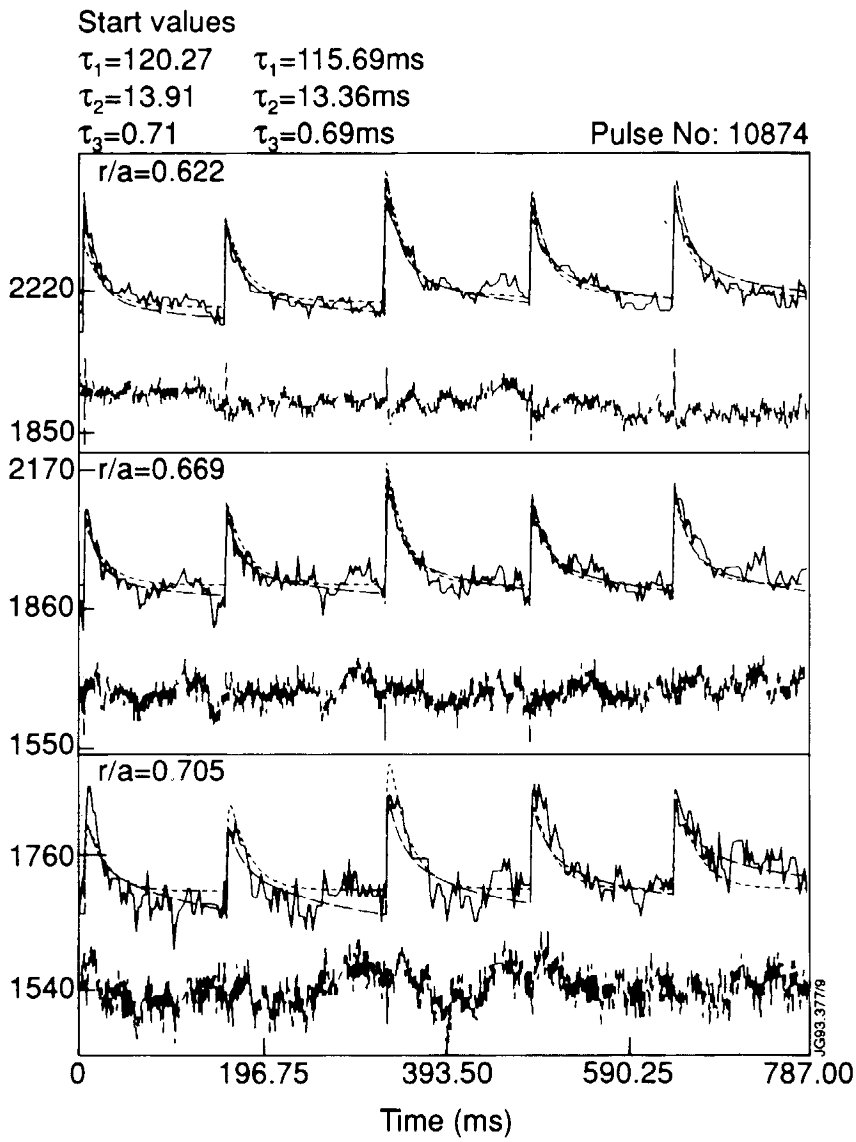



Figure 7: The fit of a series of sawteeth for different channels at a time. The dashed line shows the result of the dynamic response identification and the full line the result of the identification of the exponential functions.

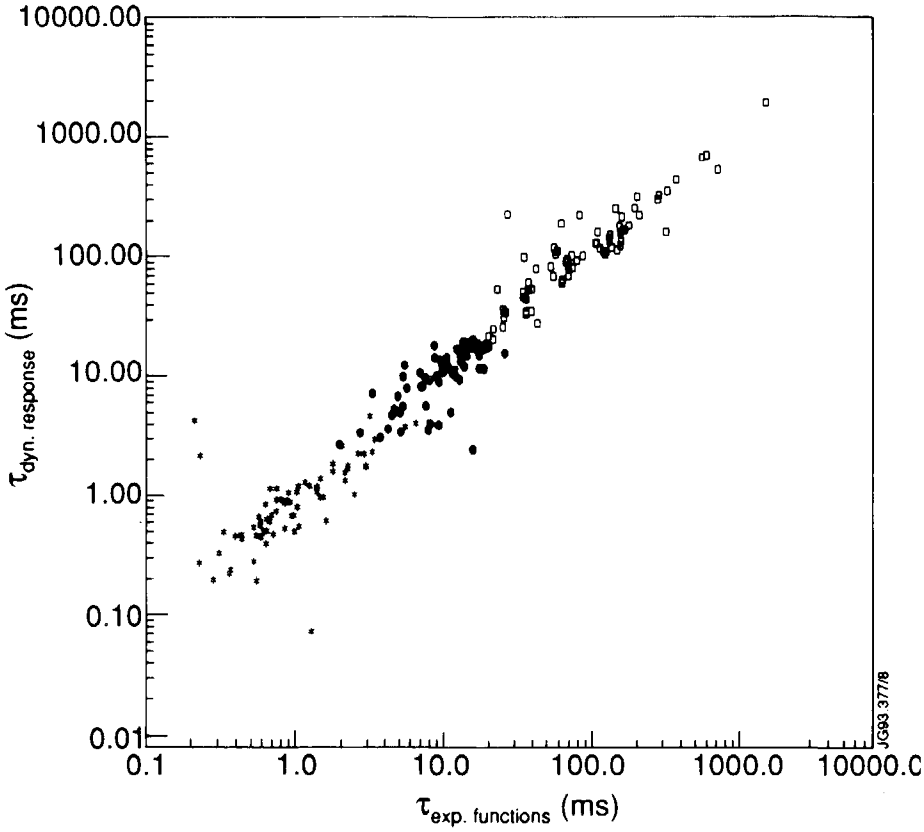



Figure 8: τ values obtained from the dynamic response versus τ values obtained from the identification of the exponential functions.

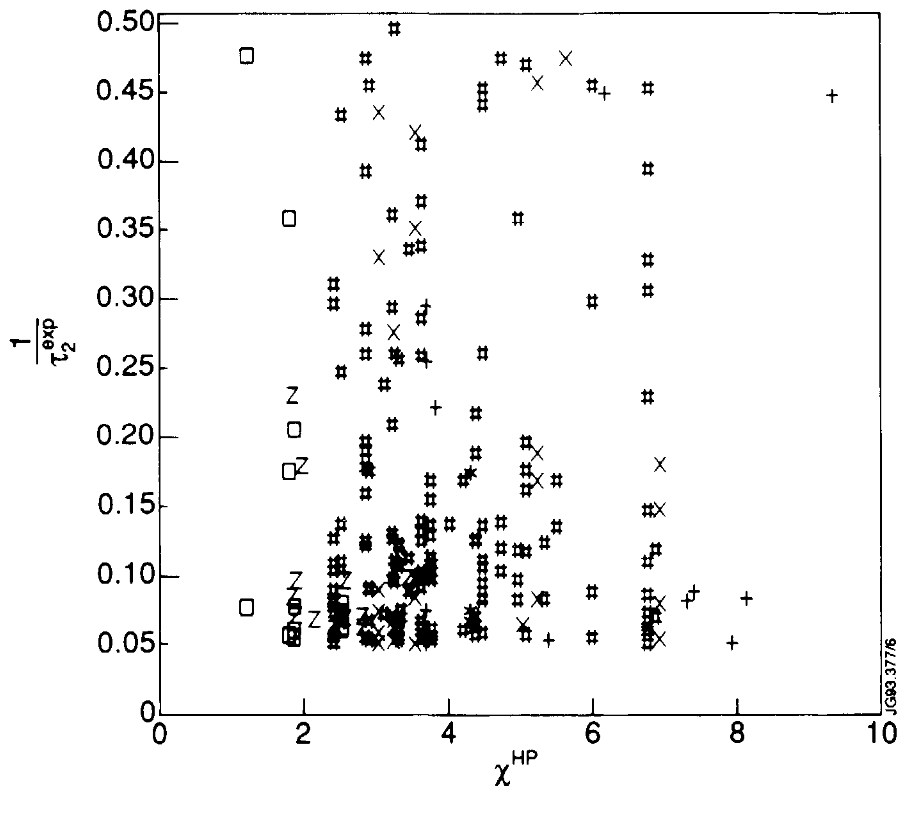



Figure 9: Plot of $1/\tau_2$ versus $\chi^{hp}$ for raw data (the lack of any correlation is obvious). Similar results are obtained for the fastest and slowest time constants ($\tau_3$ and $\tau_1$ respectively).