# Penetration of the Triangularity Shaping in High- $\beta$ Tokamaks and Stability of the Internal Kink Mode 

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#### Abstract

Stability of the internal $n=1$ kink mode in tokamak depends strongly on the ellipticity and triangularity of the plasma cross-section at $q=1$ surface. Penetration of the plasma shaping from the edge to the centrally localised $q=1$ surface has been investigated numerically using the equilibrium code HELENA [1] for $q$ - profiles close to the Bussac equilibrium and for a more realistic scenario with a flatter $q$ - profile. The internal $n=1$ kink mode growth rates are computed by the Ideal MHD MISHKA-1 code [2] for the equilibria considered. Plasma shaping optimal for the kink stability is discussed.


## INTRODUCTION

One of the plasma instabilities which is posing concern on the performance of Tokamaks as fusion reactors is the internal $n=1$ kink mode which appears when the resonant magnetic surface $q=1$ is present in the plasma core [3, 4]. This instability causes sudden (few milliseconds) loss of plasma temperature and particle confinement and the consequent flattening of the relevant profiles. Although the details of the process triggered by the internal kink instability is still debated and not properly understood [5-7], it was shown by Bussac et al. [3] that the linear theory of the instability can be described quite satisfactory in the ideal MHD approximation. Main result of [3] was the finding that in toroidal geometry the internal $n=1$ kink mode can become unstable only if the plasma pressure gradient overcomes a threshold value (for large aspect ratio circular tokamak with a parabolic current profile and a small $q=1$ radius the mode is stable when $\beta_{p}<\sqrt{13 / 144} \approx 0.3$ [3]).

Later theoretical studies [4, 8-11] have shown that the shape of plasma cross-section is playing an important role in the stability of internal $n=1$ mode. It was particularly shown that the internal kink mode is destabilized by ellipticity $[4,8,11]$, while the instability threshold can be stretched if the plasma is made more triangular [8-10]. In a recent paper Mikhailovskii [12] has analytically shown that in a high- $\beta$ tokamak with a finite plasma current gradient an additional part of the central triangularity is induced by the wall ellipticity and the plasma pressure as follows:

$$
\begin{gather*}
\tau_{0}=k_{30} \tau_{*}+k_{3 e} e_{*} \beta_{p} / R+k_{3 \beta} \beta_{p}^{3} a_{*}^{2} / R^{3}  \tag{1}\\
e_{0}=e_{*}+a_{*}^{2} \beta_{p}^{2} / 2 R^{2} \tag{2}
\end{gather*}
$$

where $k_{30}, k_{3 e}$ and $k_{3 \beta}$ are numerical coefficients determined by the current profile, $\beta_{p}$ characterizes the ratio of plasma pressure to poloidal magnetic field pressure, $a_{*}$ and $R$ are minor and major radii of the torus, the ellipticity $e$ and triangularity $\tau$ were characterized in [12] through the equation of magnetic surfaces in the form

$$
\begin{equation*}
\rho_{0}^{2}\left(1+e \cos 2 \omega_{0}+2 \tau \rho_{0} \cos 3 \omega_{0}\right)=a^{2} \tag{3}
\end{equation*}
$$

and $\tau_{0}, \tau_{*}$ and $e_{0}, e_{*}$ are the triangularities and ellipticities at the plasma centre and at the plasma edge correspondingly. It is seen from (1) that the triangularity at the plasma centre depends on the ellipticity at the edge. A problem can be considered then to maximise the triangularity penetration into the plasma centre and minimise therefore the kink instability region.

In the present paper we consider two typical $q$-profiles, one of which is close to the Bussac equilibrium and second is a bit flatter $q$-profile. For these two current profiles we extend numerically Mikhailovskii's study scanning over a large range of ellipticity and triangularity values and compute kink growth rates for the equilibria obtained. To carry out this work we made use of the equilibrium code HELENA [1] and ideal MHD stability code MISHKA1 [2]. The input to MISHKA-1 is a plasma equilibrium in terms of the flux function. The ideal MHD equations are then linearised to study small perturbations and the solution is Fourier analysed in terms of the natural cyclic variables $\theta$ (poloidal angle with associated wave number $m$ ) and $\phi$ (toroidal angle with associated wave number $n$ ). The equations are also Fourier transformed in time (associated frequency $\omega$ and growth rate $\gamma$ ), with the usual convention that the real part of the frequency gives the actual frequency of the mode while the positive imaginary part gives the growth-rate of the instability (the negative imaginary part gives the damping rate). The result is a set of differential equations for the radial dependent part of the perturbed fields, the solution of which depends on the parameters $m, n, \gamma$. For a fixed $m, n$ the equations reduce to a single eigenvalue problem which is solved to give the growth rate of the $m, n$ mode. Details of the implementation can be found in [2]. By extrapolating the growth rates we then obtain the marginal stability limit for the mode of interest.

## 1. THE EQUILIBRIUM

We began the study by solving numerically the Grad-Shafranov equation for a circular toroidal plasma with $a_{*} / R=0.1$ imposing a current and pressure profile close to the analytical approximate equilibrium chosen by Bussac et al. in the original work [3], i.e. we assumed the profiles shown in Figure 1:

$$
\begin{gather*}
p=p(0)(1-\bar{\psi}),  \tag{4}\\
\left.\langle j\rangle\right|_{\bar{\psi} \rightarrow 0}=j(0)(1-\bar{\psi}), \tag{5}
\end{gather*}
$$

where $\langle j\rangle$ is the current density averaged over the flux surface, and $\bar{\psi}$ is the normalised poloidal flux; note that $\bar{\psi}=(r / a)^{2}$ close to the magnetic axis.

In order to proceed toward a more realistic case we also consider the flatter $q$ - profile shown in figure 2 and pressure profile (4), keeping $q_{0}=0.85$ and $\beta_{p}=0.95$. The numerical analysis has been carried out assuming an inverse aspect ratio of 0.1 .


Fig.1: Relevant profiles for the Bussac-like equilibrium.
By keeping constant the value of $q$ in the plasma centre $\left(q_{0}=0.85\right)$ we varied $\beta_{p}$ at the edge from 1 to 0.35 in order to estimate in the further stability analysis the critical value $\beta_{B u s s a c}^{\text {crit }}$ above which the internal kink mode is unstable; $\beta_{\text {Bussac }}$ is defined as

$$
\begin{equation*}
\beta_{\text {Bussac }}=\left[2 \mu_{0} / B_{m}^{2}\left(r_{0}\right)\right] \int_{0}^{r_{0}}\left(r / r_{0}\right)^{2}(-d p / d r) d r, \tag{6}
\end{equation*}
$$

and $r_{0}$ is the radial position of the $q=1$ surface. We therefore generated a number of equilibria with different pressure values but with the same central $q$ - values and same radii of the $q=1$ magnetic surface. After that we proceeded to finite ellipticity and triangularity cases keeping the values $q_{0}$ and $\beta_{p}$ constant.

With reference to Figure 2, the definition of ellipticity and triangularity used throughout this work is


Fig.2: Relevant profiles for the flat-q equilibrium

$$
\begin{equation*}
e=\frac{y_{b}-y_{a}}{x_{d}-x_{a}} ; \quad t=\frac{x_{b}-x_{a}}{x_{d}-x_{a}} \tag{7}
\end{equation*}
$$

where $a$ is the position of the magnetic axis, $b$ is the lowest point of the last closed magnetic surface and $d$ is the outer most point of the last closed magnetic surface.

We then studied the penetration of edge triangularity and ellipticity for both $q$ - profiles. The results are plotted in Figures 4-6.


Fig.3: Plasma poloidal cross section showing the position of the magnetic axis (a), of the outermost point on the last closed magnetic surface (d) and of the lowest point on the same surface (b).


Fig.4: Ellipticity penetration in the two cases of Bussac and flat-q equilibrium


Fig.6: Triangularity penetration for the flat-q equilibrium varying the edge ellipticity between 1.0 and 1.5 .


Fig.5: Ellipticity penetration in the flat-q case varying the edge triangularity. Lines are coinciding for different values of triangularity.

The flat $q$ - profile helps the penetration of ellipticity at zero triangularity, which is consistent with the findings in [8]. There is no influence of triangularity on ellipticity penetration, in agreement with (2). By increasing the ellipticity we found a decrease of $1 \%$ in the triangularity at $q=1$.

We proceed now investigating how ellipticity and triangularity affects the value of $\gamma^{2}$. By keeping $q_{0}=0.85$ and $\beta_{p}=0.95$ constant we calculated the growth rate for various values of ellipticity and triangularity.

## 2. THE KINK STABILITY

We first reproduce the Bussac circular cross-section stability limit for the Ideal MHD kink mode. We found that for high values of the edge $\beta_{p}$ the square of the linear growth rate depends linearly on $\beta_{\text {bussac }}^{2}$ (in agreement with the Bussac findings), a linear extrapolation to $\gamma^{2}$ equal zero gives a result close to that reported by Bussac et al in their paper ( $\beta_{\text {bussac }}^{c}=\sqrt{13} / 12$ ). After that we proceeded to finite ellipticity and triangularity cases keeping the values $q_{0}$ and $\beta_{p}$ constant.

The code MISHKA- 1 has been run accounting for the interaction of 13 poloidal harmonics around $m=1$. In Fig. 7 we report the values of $\gamma$ as a function of $\beta_{\text {bussac }}^{2}$, obtained by varying $\beta_{p}$ at the edge (the values taken are $\beta_{p}=1.0,0.95,0.9,0.85$ ), the data represent four ellipticity cases ( $e=1.0,1.1,1.2,1.3$ )

The dependence of $\gamma^{2}$ on ellipticity and triangularity is shown in Fig. 8, the scan has been carried out for the equilibrium of Fig. 1 keeping fixed $q_{0}=0.85, \beta_{p}=0.95$


Fig.7: Growth rates versus beta Bussac squared for various values of edge ellipticity.


Fig.8: Growth rates versus ellipticity for various edge triangularities. Bussac like equilibrium with edge beta poloidal 0.95 and $q$ on the magnetic axis 0.85 .

Together with the stabilising effect of triangularity what we learned from Fig. 8 is that for a triangular plasma there is an optimum value of ellipticity, for which the growth rate is minimised and which turns out to be $e=1.2$.

The effect of triangularity on $\beta_{\text {bussac }}^{c}$ has been studied then by fixing the ellipticity to the optimum value. The results are reported in Fig.9. It is seen that the triangularity plays a strongly stabilizing role.

The linear scaling showed in Fig. 7 is lost when $\beta_{p}$ at the edge approaches small values as it can be seen in figure 6 where $\beta_{p}$ is brought down to 0.35 , Fig. 9 shows the dependence of $\gamma$ on $\beta_{\text {bussac }}^{2}$ for $t=0.0$ and $t=0.1$

The same study has been repeated for the flat-q equilibrium showed in Fig.2, by keeping constant $q_{0}=0.85, \beta_{p}=0.95$ and varying the edge ellipticity and triangularity.


Fig.9: Growth rates versus beta Bussac squared for various values of edge triangularity. Bussac like equilibrium.

The non monotonic behaviour observed in the Bussac case is not resolved because of the very small growth rates obtained at $t=0.1$ and $e$ less than 1.3 (values of $\gamma^{2}$ the order of $10^{-}$ ${ }^{11}$ are below the resolution of MISHKA-1). At $t=0.15$ any ellipticity value give extremely small growth rates below the resolution of the code.

The critical $\beta$ is also affected by triangularity and we find the behaviour reported in Figure 11.

We conclude therefore that the flat-q profile helps the penetration of ellipticity at zero triangularity, which is consistent with the detected larger growth rates. The strong stabilising effect observed when triangularity


Fig.10: Growth rates versus ellipticity for various edge triangularities. Flat-q equilibrium with edge beta poloidal 0.95 and $q$ on the magnetic axis 0.97 .


Fig.11: Growth rates versus beta Bussac squared for various values of edge triangularity. Flat-q equilibrium. is increased cannot be due to a less penetrative ellipticity as it is shown in Fig. 5 where the penetration of edge ellipticity is plotted in the two cases of triangularity 0.0 and 0.05 .

The point that there is no influence of triangularity on ellipticity penetration to justify the difference observed in the mode growth rate when going from 0 triangularity to 0.05 is seen from Fig.5. The effect must be entirely due to the penetration of edge triangularity to the $q=1$ surface, the latter is plotted in Fig. 6 for ellipticity values that range between 1.0 and 1.5. By
increasing the ellipticity there is a decrease of $1 \%$ in the triangularity at $q=1$, this variation is responsible for the larger growth rates.

That a variation of $1 \%$ in triangularity can indeed cause a very strong effect on the growth rate can be seen in Figs 12 and 13.


Fig.12: Growth rate versus triangularity for the flat-q equilibrium and edge ellipticity 1.5. The constant central $q$ is 0.97 and the poloidal beta 0.95 .

## CONCLUSION

We have studied the effect of plasma shaping on the internal kink mode by calculating numerically the mode linear growth rate for two Tokamak relevant plasma equilibria: a Bussac like equilibrium and a flat-q equilibrium (reported in Fig. 1 and Fig. 2 respectively). The edge ellipticity has been varied between 1.0 and 1.5 (the increment step adopted is 0.05 ), the edge triangularity between 0.0 and 0.15 (the step being 0.05 ) and the poloidal $\beta$ at the edge has been scanned between 1.0 and 0.35 (scan step 0.05 ) the latter scan has been undertaken for the purpose of finding the critical $\beta$ below which the mode is stable. The value of $q$ on the magnetic axis has been kept constant when varying the edge ellipticity and triangularity, this made also the radial position of the $q=1$ surface constant and located at $r=0.55$ (where r is the plasma normalised minor radius), the values of $q_{0}$ taken are 0.85 and 0.97 for the Bussac like and for the flat-q equilibrium respectively.

In the Bussac like equilibrium we find that increasing the edge ellipticity the mode is destabilised at low edge triangularity. By increasing the triangularity at the edge the mode is stabilised and an interesting feature is that the dependence of the mode linear growth rate on ellipticity becomes non-monotonic as shown in Fig.8. The critical $\beta$ is increased by increasing the edge triangularity of $5 \%-10 \%$ as shown in Fig.9.

The flat-q equilibrium is more favourable for plasma confinement since allows a deeper penetration of the edge shape at the $q=1$ resonant surface, as it can be seen from Figs 4-6. The effect is that this equilibrium is very sensible to small variation of edge ellipticity and triangularity. As in the previous case ellipticity has a destabilising effect increasing the mode linear growth rate (as shown in Figure 10) but at the same time the addition of 5\% edge triangularity brings about a very large stabilising effect and when triangularity reaches $10 \%$ the mode appears to be linearly stable. The strong stabilising effect that small variation of triangularity have on the growth rate can be clearly seen from Figs 12 and 13.

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