JET–P(99)22

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Preprint of a Paper to be submitted for publication in European Physical Journal D

August 1999

ABSTRACT

On the basis of tearing mode theory a simple but physically explicit model of the evolution of toroidally coupled rotating magnetic islands has been developed. The basic mechanism identified by the model in the island evolution is the locking in phase of rotating islands that leads to rapid destabilisation of an initially stable mode**.** Destabilisation of marginally stable (2,1) and (3,1) modes is analysed in several scenarios. It is shown that mode coupling is an effective way of destabilising a m=3 island in a low-β plasma. The numerical examples presented show the individual roles of coupling, inertia and a resistive wall. The model was applied for the analysis of MHD observations of an ASDEX discharge.

1. INTRODUCTION

Magnetic islands have been observed in many tokamak discharges showing a strong influence on both global stability and transport and leading to confinement degradation. The understanding of how rotating islands evolve and interact with the plasma and surrounding vessel is of great importance in the avoidance of locked modes and disruptions, which are both key issues for ITER.

The conditions of stability of modes of different helicity coupled by toroidal, shaping and finite pressure effects was developed analytically in first order approximation by Connor et al. [1,2]. Stability analysis in toroidal geometry can also be obtained with a variety of linear full-magnetohydrodynamic (MHD) codes [3]. These very complete models focus on the conditions of stability of the interacting modes, but have not yet been used to display the full temporal evolution. Computational non-linear evolution of modes in toroidal geometry has been addressed by Carreras et al [4]. Although in principle both linear and non-linear codes can be adapted to study the numerical evolution of coupled modes, they would not provide simple analytic terms for inclusion in the present studies of feedback stabilisation developed in cylindrical geometry [5-7].

In order to study the evolution of amplitude and rotation of coupled tearing modes, an alternative simpler approach is considered here. The evolution of two coupled modes is treated by analogy with the problem of a single mode driven by external electro-dynamic fields . This similarity has been noted in Ref. [5].

A full consideration of tokamak geometry on (m,n) mode coupling implies inclusion of finite pressure and flux surface shaping effects, in addition to toroidal ones. However an essential feature of the toroidal geometry, surviving even in the β=0 limit and circular cross section torus, is the appearance of coupling with side-band harmonics modes. Thus the electro-dynamic coupling can, quite generally, be represented for any given mode as the effect of suitable "external" current distribution, provided one can relate the phase and magnitude of the "effective" currents to the amplitude and phase of the side-band modes coupled to the mode under consideration. Unlike the problem of feedback stabilisation by an external current, in the study of coupling of a (m,n) mode with a (m±1,n) mode, a current sheet of helicity (m±1,n) is located inside the plasma.

In this paper, we consider contributions to the stability parameter Δ' coming from the (m,n) eddy currents excited in the resistive wall and from the internal time varying current sheets located on the surfaces where the side-band harmonics are resonant. A full-time derivative of the island angular momentum is taken into account, along the lines of Ref. [8]. We consider resistive modes with poloidal mode numbers m>1. (A similar approach applied for the study of the coupling of the m=1 and m=2 harmonics can be found in [9].). Having determined the complex stability parameter for the (m,n) mode, suitable for a numerical solution, we will then adopt a tenuous-plasma approximation in order to obtain simpler analytic expressions for the equations describing the time evolution of island width and rotation.

The model was applied to the study of destabilisation of small islands. Simulations were also carried out for the analysis of MHD observations of an ASDEX tokamak discharge.

2. FORMULATION OF THE PROBLEM

In the case of toroidal geometry, the magnetic perturbations δB related to magnetic islands with different poloidal and toroidal mode numbers (m,n) have not only a single (m,n) component, but also side band harmonics. Each mode evolves driven by its own free energy and by the effect of the electro-dynamic (magnetic) fields generated by other modes, considered as external sources. Therefore the prototype problem is the investigation of the interaction of a magnetic island with a time varying current sheet source with specified helicity (m,n), amplitude and phase, located within the plasma. The model is fully specified when the localised "source currents" are associated uniquely to the amplitude and phase of the interacting modes. The tearing mode equation for each mode (m,n) with inhomogeneous source terms is then solved in order to obtain the complex stability parameter Δ' which is subsequently used in a non-linear Rutherford type equation for island growth [10], associated with the momentum balance equation governing island rotation.

We adopt a notation appropriate for the large aspect ratio Reduced MHD description [11]. A helical magnetic field perturbation eventually evolving into a "magnetic island" is expressed in terms of a helical flux function

$$\delta\vec{B} = \mathrm{Re}(\vec{\nabla} x(\Psi_{mn}\vec{b}_{mn})) \tag{1}$$

where $\vec{b}_{mn} = \frac{nr}{mR_0}\hat{\theta} + \hat{z}$ and $\Psi_{mn} = \psi_{mn}(r).e^{i\left(m\theta - \frac{n}{R_0}z + \phi\right)}$. The width of the island

$$W = \sqrt{\frac{16R_0q^2(r_s)}{r_sBq'(r_s)}\psi_{mn}(r_s)} \tag{2}$$

is related to the amplitude of ψ at the rational flux surface q(r) (defined as $q(r) = \frac{rB}{R_0 B_\theta}$, where R_0 is the tokamak major radius and r the radius of the magnetic surface). The phase of the mode ϕ is such that $\frac{d\phi}{dt} = \omega$, ω being the mode frequency.

The linear tearing mode equation for ψ_{mn} is written in our notation as

$$-\frac{B_\theta}{\mu_0 rB}\nabla_*^2\psi_{mn}.(m - nq) + \frac{m}{r}\lambda_0'\psi_{mn} = 0 \qquad (3)$$

where $\lambda_0' = \frac{d}{dr}\left(\frac{J_{0//}}{B}\right)$, and $J_{0//}$ is the equilibrium current density directed along the magnetic field . The stability parameter obtained from the solution of (3) in the vicinity of the singular point $r=r_s$ is defined as

$$\Delta'\psi_{mn}(r_s) = \psi'_{mn}(r_{s+}) - \psi'_{mn}(r_{s-}) \qquad (4)\ .$$

The evolution of the width and rotation frequency of a magnetic island are a consequence of Faraday-Ohm's law and the current continuity equation. The key role in the growth and rotation of an island of mode numbers (m,n) is held by the stability parameter Δ' which describes the presence of a current sheet at the resonant surface where q(r)=m/n.

The real part of Δ' governs the island growth. In its non-linear stage the time evolution of the island width is well described by Rutherford's expression, as given in Ref. [10]

$$\frac{dW}{dt} = 1.16\frac{Z_{eff}\eta}{\mu_0}\mathrm{Re}(\Delta') \qquad (5),$$

where Z_{eff} is the plasma effective charge and η is the Spitzer resistivity.

The imaginary part of Δ' gives the electromagnetic torque on the island defined as

$$T_\phi = \frac{8\pi^2 R_0}{4\mu_0}.n.r_s\ \mathrm{Im}\left(\psi_{mn}^2(r_s).\Delta'\right) \qquad (6).$$

The toroidal momentum of inertia of an island (I_ϕ) with mode numbers (m,n) is, in terms of the island width (W), given by

$$I_\phi = n_e(r_s)Am_p.8\pi r_s R_0^3.W \qquad (7),$$

where n_e is the electron density, m_p the proton mass and A the atomic mass of the plasma. The magnetic island, moving in the toroidal direction with a angular velocity equal to $\frac{\omega}{n}$, will be accelerated or decelerated by this torque. In this work we neglect viscosity.

In the absence of any torque, inertia will slow down a growing island in order to conserve angular momentum. Including these two sources the frequency of the mode will evolve in time according to

$$\frac{d\omega}{dt} = \frac{1}{I_\phi}\left[nT_\phi - \omega \frac{dI_\phi}{dt} \right] \qquad (8).$$

The simplest picture is obtained by solving the tearing mode equation (3) for the mode (m,n) localised at r_s, with the addition of a source term within the plasma with a current sheet $I_v = I \exp\left[i(m\theta - \frac{n}{R_0} z + \phi_c) \right]$ placed at $r=r_c$ and a resistive wall at r=d (see Fig. 1). The solution of (3) is obtained by the method of variation of parameters and it is matched across the different regions shown in the Figure 1.