JET–P(99)20

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Suppression of Neoclassical Tearing Modes by the Magnetic Well in Shear-Optimised Tokamak Discharges

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Preprint of a Paper to be submitted for publication in Plasma Physics Reports

August 1999

ABSTRACT

It is suggested that the neoclassical tearing modes in the shear-optimised discharges in tokamaks can be suppressed by the "magnetic well" effect. The stabilising effect is shown to be significantly increased by shaped magnetic surfaces, due to the combined influence of ellipticity and triangularity.

The shear-optimised discharges [1,2] generate very promising regimes of enhanced plasma confinement in a tokamak power plant. It was predicted theoretically [3-9] and shown experimentally (see [10-12] and Refs. therein) that magnetic islands driven by the neoclassical bootstrap current may become an obstacle for achievement of high enough values of plasma pressure in tokamaks. Therefore a suppression of the neoclassical magnetic islands is one of the important issues for a future development of the shear-optimised scenarios. In the present work an idea is proposed that under specific conditions typical for the shear-optimised scenarios the neoclassical magnetic islands can be suppressed by the effect of "magnetic well" (see the definition in [13]), associated with the average curvature of the magnetic field lines. The stabilising effect of the magnetic well is also sometimes called in the literature "the stabilising effect of the Pfirsh-Shluter current induced by toroidicity and shaping of the poloidal cross-section" (see, e.g. [11]) or "the stabilising Glasser-Greene-Johnson effect due to the equilibrium pressure gradient and favourable curvature in the outer part of the island" (see, e.g. [12]).

Qualitatively the "magnetic well" effect for the neoclassical magnetic island can be explained in terms of the following equation of the island evolution:

$$\frac{\partial w}{\partial t} = \frac{c^2}{4\pi\sigma} C_1 \left(\Delta' + \frac{C_2}{w} \left(\Delta_{bs} - U_0 \right) \right). \tag{1}$$

Here w is the width of the magnetic island, c, σ are the speed of light and plasma conductivity, Δ' is the standard parameter of the tearing mode theory, which is assumed to be negative in our case ($\Delta' = -2m/r$ for high poloidal mode numbers, $m >> 1$), C_1, C_2 are constants,

$$\Delta_{bs} = \frac{\varepsilon^{1/2}\beta_{pe}}{S} K_{bs} \tag{2}$$

is the destabilising contribution of the bootstrap current, and

$$U_0 = \frac{\varepsilon^2 \beta_{pe}}{S^2} K_U \left(1 - \frac{1}{q^2} + g(e,\tau) \right) \tag{3}$$

is the "magnetic well". Here $\varepsilon = r/R_0$, r and R_0 are minor and major radii of the tokamak, $S = rq'/q$ is the magnetic shear, q the safety factor, $\beta_{pe} = 8\pi n_0 T_e / B_\vartheta^2$ the poloidal beta of electrons, B_ϑ is the poloidal magnetic field, $g(e,\tau)$ is a parameter which depends on the values of the plasma ellipticity e and triangularity τ (to be determined later) and K_{bs}, K_U are coefficients which depend on the plasma density n_0 and electron and ion temperatures, T_e, T_i, respectively.

It is obvious from (1)-(3) that the effect associated with the magnetic well U_0 is stabilising for the magnetic island. However, in discharges with $S \cong 1$, $g(e,\ \tau) \le 1$ and $K_U \cong K_{bs} \cong 1$ this stabilising effect is small in comparison with the bootstrap current drive:

$$\frac{U_0}{\Delta_{bs}} = \frac{\varepsilon^{3/2}}{S}\frac{K_U}{K_{bs}}\left(1-\frac{1}{q^2}+g(e,\tau)\right) \cong \varepsilon^{3/2} << 1, \qquad (4)$$

In the present paper we show that for the shear-optimised scenarios [1,2] typically satisfying the following conditions near the internal transport barrier:

$$S << 1\ ,$$
$$d\ln T_i/d\ln n >> 1, \qquad (5)$$
$$T_i > T_e,$$

the stabilising effect of the magnetic well is significantly enhanced and it can compete with the bootstrap-current effect. Furthermore, it is shown that this magnetic well effect can be increased by a proper choice of the plasma shape, which determines the value of $g(e,\tau)$.

In order to calculate all the necessary constants and coefficients K_U, K_{bs} and $g(e,\tau)$ we generalise the stationary magnetic island equation (17) from Ref.[8] to the case of a finite magnetic well. This equation relates the width w of the stationary magnetic island and the parallel current $J_{||}$ as follows:

$$\sum_{\sigma_\chi}\int_{-1}^{\infty} d\Omega \oint \frac{J_{||}\cos\xi}{\sqrt{\Omega+\cos\xi}}d\xi = \frac{c}{8\sqrt{2}}S\Delta'\frac{wB_0}{qR}. \qquad (6)$$

Here B_0 is the toroidal magnetic field, Ω is the surface function determined in [8], ξ is the angle variable of the island, $\sigma_\chi = sgn(\chi-\chi_s)$, χ is the poloidal magnetic flux and χ_s is the magnetic flux associated with the “centre” of the island. We follow an approach developed in [9] and represent the parallel current in the form:

$$J_{||} = J_{bs} + \tilde{J}_{||}, \qquad (7)$$

where J_{bs} is the bootstrap current and $\tilde{J}_{||}$ is the part of $J_{||}$ which depends on ξ and satisfies the condition $\left\langle \tilde{J}_{||}\right\rangle_\xi = 0$, where the averaging over the island magnetic surface $\langle\ldots\rangle_\xi$ is determined in [9]. The paper [9] will be cited as [I] and the formulas from this paper will be cited as (I. ...).

The current $\tilde{J}_{||}$ due to the curvature of the magnetic field satisfies the equation

$$\left(\nabla_{||}\right)_{\vartheta}\tilde{J}_{||}+\left(\nabla\cdot\mathbf{j}_{\perp}\right)_{\vartheta}=0, \tag{8}$$

where

$$\mathbf{j}_{\perp}=\frac{c}{\mathrm{B}^2}[\mathrm{B}\times\nabla p], \tag{9}$$

B and p are the total magnetic field and plasma pressure (the sums of equilibrium and perturbed parts). The operation $(...)_{\vartheta}$ denotes the averaging over poloidal angle ϑ at fixed ξ.

Taking into account (7) we represent (6) in the form (cf. (I.5.3)):

$$\frac{\Delta'}{4}+\frac{\Delta_{bs}+\Delta_U}{w}=0, \tag{10}$$

where Δ_{bs} is the bootstrap current contribution and Δ_U is the part due to the effect of the magnetic field curvature. The value Δ_{bs} was calculated in [I] and according to (I.5.51) is equal to

$$\Delta_{bs}=2c_{bs}K_{bs}\ . \tag{11}$$

Here K_{bs} and c_{bs} are given by

$$K_{bs}=-1.23\varepsilon^{1/2}\beta_{pe}\frac{r}{S}\left[\left(1+\tau_i\right)\frac{n_0'}{n_0}+0.40\frac{T_e'}{T_e}-0.17\tau_i\frac{T_i'}{T_i}\right], \tag{12}$$

$$c_{bs}=-\frac{1}{4}\int_1^{\infty}d\Omega\frac{k}{E(k)}\left\langle\left(\Omega+cos\,\xi\right)^{1/2}\right\rangle\oint\frac{cos\,\xi d\xi}{\left(\Omega+cos\,\xi\right)^{1/2}}=0.79, \tag{13}$$

where $k=\left[2/(\Omega+1)\right]^{1/2}$, $E(k)$ is a complete elliptical integral of the second kind, $\tau_i=T_i/T_e$.

According to (6), (7), (10)

$$\Delta_U=-\frac{2\sqrt{2}qR_0}{cS}\sum_{\sigma_\chi}\int_{-1}^{\infty}d\Omega\oint\frac{\tilde{J}_{||}\,cos\,\xi}{\sqrt{\Omega+cos\,\xi}}d\xi. \tag{14}$$

Calculation of Δ_U is given in the Appendix and the result has the following form:

$$\Delta_U=-2c_{bs}U_0\equiv-1.58U_0\ , \tag{15}$$

where

$$U_0=-\frac{4\pi p_0'w'q^2R^2}{S^2B_0^4}\ . \tag{16}$$

Substituting (11) and (15) into (10), we obtain

$$\frac{\Delta'}{4}+\frac{1.58}{w}\left(K_{bs}-U_0\right)=0\ . \tag{17}$$

It follows from (17) that for $\Delta' < 0$ the stationary magnetic island cannot exist if

$$U_0 \geq K_{bs} \, . \tag{18}$$

This inequality represents the criterion of the island suppression by the magnetic well.

According to Eq.(2.79) of [13] the vacuum value of w' in a noncircular tokamak can be represented as

$$w' = -\frac{2B_\vartheta^2}{r}\left[1 - q^2\left(1 + 6\frac{e\tau}{\varepsilon}\right)\right], \tag{19}$$

where e is the ellipticity, τ the triangularity of magnetic surfaces. These values are determined in such a way that the equation for a magnetic surface of average radius r is represented as

$$\rho^2(1 + e\cos 2\vartheta + 2\tau\cos 3\vartheta) = r^2, \tag{20}$$

where $\rho,\ \vartheta$ are the polar co-ordinates related to the centre of the surface. Using (16), (19), we find

$$U_0 = -\frac{\varepsilon^2\beta_{pe}r}{S^2}\left[(1+\tau_i)\frac{n_0'}{n_0} + \frac{T_e'}{T_e} + \tau_i\frac{T_i'}{T_i}\right]\left(1 - \frac{1}{q^2} + 6\frac{e\tau}{\varepsilon}\right). \tag{21}$$

Criterion (18) can be represented in the form

$$f(r) \geq 1, \tag{22}$$

where

$$f(r) \equiv U_0 / K_{bs} \, . \tag{23}$$

Substituting (12) and (21) into (22), we obtain the criterion of the neoclassical island suppression as follows:

$$f(r) = 0.81\frac{\varepsilon^{3/2}}{S}\frac{1+\tau_i+\eta_e+\tau_i\eta_i}{1+\tau_i+0.40\eta_e-0.17\tau_i\eta_i}\left(1-\frac{1}{q^2}+6\frac{e\tau}{\varepsilon}\right) \geq 1, \tag{24}$$

where $\eta_\alpha = \partial \ln T_\alpha / \partial \ln n_0$, $\alpha = e, i$.

Considering typical plasma parameters in the region of the internal transport barrier in the JET shear-optimised discharges, $q \approx 2$, $S << 1$, $T_i \approx 2T_e$, $\eta_i \approx 3$, $\eta_e \approx 1$, one obtains an estimate

$$f(r) \approx 3.4\left(0.75 + 6\frac{e\tau}{\varepsilon}\right)\frac{\varepsilon^{3/2}}{S} \geq 1. \tag{25}$$

It is seen therefore that in the case of the magnetic shear small enough and the magnetic surfaces shaped enough, $6e\tau/\varepsilon > 1$, the criterion of the neoclassical island suppression (24) is satisfied if $e\tau\varepsilon^{1/2}/S \geq 0.05$.

APPENDIX

It follows from [I] that

$$\left(\nabla_{||}\right)_{\vartheta} = k_{||}\frac{\partial}{\partial \xi} , \tag{A.1}$$

where

$$k_{||} = -\frac{mSw\sigma_{\chi}}{\sqrt{2}qR_0 r}\left(\Omega + cos\,\xi\right)^{1/2}. \tag{A.2}$$

From (9) we find that in terms of the variables $\xi,\ \vartheta$

$$\nabla \cdot \mathrm{j} = \frac{cw'}{rB_0^3}\frac{\partial p}{\partial \vartheta} , \tag{A.3}$$

where

$$w' \equiv \frac{d}{dr}\left\langle \mathrm{B}_0^2 + 8\pi p_0 \right\rangle, \tag{A.4}$$

p_0 is the equilibrium plasma pressure, $\langle ... \rangle$ is the averaging over the equilibrium magnetic surfaces.

It follows from [I] that the total plasma pressure is a function of the island magnetic surface, i.e. $p = p(\Omega)$. Then in terms of $\xi,\ \vartheta,\ \Omega$

$$\left[\left(\frac{\partial p}{\partial \vartheta}\right)_r\right]_{\vartheta} = m\frac{\partial p}{\partial \Omega} sin\,\xi\ . \tag{A.5}$$

Then we obtain from (A.3)

$$\left(\nabla \cdot \mathrm{j}_{\perp}\right)_{\vartheta} = \frac{cw'm}{rB_0^3}\frac{\partial p}{\partial \Omega} sin\,\xi\ . \tag{A.6}$$

Using (A.1), (A.2) and (A.6) we represent (8) in the form

$$\frac{\partial \tilde{J}_{||}}{\partial \xi} = \frac{\sqrt{2}cw'\sigma_{\chi}}{wSB_0^3}\frac{sin\,\xi}{\left(\Omega + cos\,\xi\right)^{1/2}}\frac{\partial p}{\partial \Omega}\ . \tag{A.7}$$

Hence

$$\tilde{J}_{||} = -\frac{2\sqrt{2}cw'\sigma_{\chi}}{wSB_0^3}\left[\left(\Omega + cos\,\xi\right)^{1/2} - \left\langle \left(\Omega + cos\,\xi\right)^{1/2} \right\rangle\right]\frac{\partial p}{\partial \Omega}. \tag{A.8}$$

Then (9) takes the form

$$\Delta_U = -\frac{8R_0^2 q^2 w'}{S^2 w B_0^4}\sum_{\sigma_\chi}\sigma_\chi \int_{-1}^{\infty} d\Omega \frac{\partial p}{\partial \Omega}\left\langle (\Omega + cos\,\xi)^{1/2}\right\rangle \oint \frac{cos\,\xi d\xi}{(\Omega + cos\,\xi)^{1/2}}. \tag{A.9}$$

One can obtain from [I] that

$$p = p_0(\chi_s) + \frac{1}{R_0 B_\vartheta}\frac{dp_0}{dr} h(\Omega), \tag{A.10}$$

where $h(\Omega)$ is the function introduced in [I]. Then

$$\frac{\partial p}{\partial \Omega} = \frac{1}{R_0 B_\vartheta}\frac{dp_0}{dr}\frac{dh}{d\Omega}\ . \tag{A.11}$$

According to (I.2.20), (I.5.50)

$$\frac{dh}{d\Omega} = \frac{\pi}{8}\frac{k}{E(k)}\sigma_\chi w R_0 B_\vartheta . \tag{A.12}$$

It follows from (A.9), (A.11), (A.12) and (13) that Δ_U can be represented in the form (15), (16).

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