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ABSTRACT

In a toroidal plasma the distribution function of ions interacting resonantly with waves in the Ion Cyclotron Range of Frequencies (ICRF) can be described with a three dimensional orbit-averaged Fokker-Planck equation. This equation can be solved with a Monte Carlo method. Explicit expressions for the Monte Carlo operator describing wave particle interaction, within the framework of quasilinear theory, are given. Furthermore, properties of the operator are discussed.

1. INTRODUCTION

Resonant interaction between ions and waves in the Ion Cyclotron Range of Frequencies (ICRF) leads to a distortion of the velocity distribution function and a concomitant spatial transport of the resonating ions. In an axisymmetric toroidal device it is possible to reduce the Fokker-Planck equation, which describes wave particle interaction and collisions, to three dimensions by averaging it over unperturbed orbits, see e.g. Ref. [1]. The velocity distribution function of the resonating ions is then a function of three invariants of the unperturbed motion. A convenient way of solving the orbit-averaged Fokker-Planck equation is to use Monte Carlo technique. Previously, Monte Carlo operators for the orbit-averaged Fokker-Planck equation have been constructed [1].

The orbit-averaged quasilinear Monte Carlo diffusion operator for wave particle interaction presented in Ref. [1] is, however, not given in a fully explicit form. In particular, the part related to the expectation values in the Monte Carlo operator is given in terms of derivatives of the diffusion tensor with respect to the invariants. To carry out these derivatives numerically can be difficult. The purpose of this paper is to show that by using a special set of invariants the numerical evaluation of the expectation values can be considerably simplified.

The outline of the paper is as follows. In section 2 we write the orbit-averaged quasilinear diffusion operator for Fokker-Planck equations derived by Kaufman [2] in a general set of coordinates. We discuss some features of the quasilinear diffusion operator, particularly wave-induced spatial transport, in section 3. In section 4 we present the Monte Carlo operator in a form that is suitable for fast numerical evaluation, and in section 5 we illustrate the importance of proper evaluation of the operator using numerical computer simulations. Finally, section 6 summarizes our results.

2. THE ORBIT-AVERAGED QUASILINEAR DIFFUSION OPERATOR

As has been shown for example in Ref. [1], the orbit-averaged Fokker-Planck equation for ions interacting with waves can be written as

$$\frac{\partial f_0}{\partial t} = \left\langle C(f_0) + Q(f_0) \right\rangle, \tag{1}$$

where f_0 is the orbit-averaged distribution function, C is a collision operator and Q is a quasilinear operator for wave particle interaction. The orbit-average is defined as

$$\langle\cdots\rangle = \iiint(\cdots)d\theta_1 d\theta_2 d\theta_3, \tag{2}$$

where θ_1, θ_2 and θ_3 are the angles of an action-angle coordinate system (J, θ) [2], which exists in an axisymmetric magnetic configuration. In such a coordinate system the unperturbed Hamiltonian for a particle depends only on the actions J, and the angles evolve linearly in time:

$$\dot{\theta}_i = \frac{\partial H_0}{\partial J^i} = \Omega_i(J). \tag{3}$$

Here, $\Omega_1 = \partial H_0 / \partial J^1$ is the orbit-averaged cyclotron frequency, $\Omega_2 = \partial H_0 / \partial J^2 = 2\pi/\tau_b = \omega_b$ is the bounce frequency (τ_b is the time it takes for a particle to complete an orbit) and $\Omega_3 = \partial H_0 / \partial J^3 = \langle\dot{\varphi}\rangle$ is the orbit-averaged time derivative of the toroidal angle. The angles θ_1, θ_2 and θ_3 roughly describe the position of a particle in the Larmor rotation, the poloidal position along the guiding centre orbit and the toroidal position of the banana center, respectively. Furthermore, the actions J^1 and J^3 are given by $J^1 = m\mu/(Ze) = m^2 v_\perp^2/(2ZeB)$ and $J^3 = P_\varphi = Ze\psi_p/(2\pi) + mRv_{//}B_\varphi/B$, where m is the mass, Ze is the charge, B is the magnetic field strength, B_φ is the toroidal component of the magnetic field, , $v_{//} = \mathbf{v}\cdot\mathbf{B}/B$ and $\mathbf{v}$ is the particle velocity. Furthermore, R is the major radius coordinate, and ψ_p is the poloidal magnetic flux. The expression for J^2 is omitted since it will not be used later. For a detailed exposition of action-angle variables we refer the reader to Ref. [2].

The orbit-averaged quasilinear diffusion operator for wave particle interaction has been derived by Kaufmann and it reads [2]

$$\langle Q(f_0)\rangle = \frac{\partial}{\partial J^i}\left(\overline{D}^{ij}\frac{\partial f_0}{\partial J^j}\right). \tag{4}$$

Following the steps outlined in Ref. [1] and making the normal assumption that the stochastisation of the resonating ions is sufficient to prevent them from having super-adiabatic motion, the diffusion tensor $\overline{D}^{ij}$ can be written as

$$\overline{D}^{ij} = \sum_{n,N,\omega} D_0 n^i n^j, \tag{5}$$

where

$$D_0 = \frac{\pi}{\omega_b}\left|H_1(J, n, N, \omega)\right|^2, \tag{6}$$

$$, \qquad (7)$$

Here, $n^1 = n$ (cyclotron harmonic), $n^3 = N$ (the toroidal wave number of the wave), n^2 is given by the resonance condition $n^i \Omega_i = \omega$ [1], $\beta = k_\perp v_\perp / \omega_c$, $\omega_c = ZeB/m$, $\mathbf{v}_g$ is the guiding center velocity of a particle, $\mathbf{k}$ is the wave vector, J_n is the Bessel function of the order n, and E_+ and E_- are the left-hand and right-hand components of the electric field, respectively.

For a general set of invariants, $\mathbf{I} = \mathbf{I}(\mathbf{J})$, we can now write the quasilinear operator as

$$\langle Q(f_0) \rangle = \sum_{n, N, \omega} \frac{1}{\sqrt{g}} \frac{\partial}{\partial I^i} \left(\sqrt{g} D_0 n^k n^l \frac{\partial I^i}{\partial J^k} \frac{\partial I^j}{\partial J^l} \frac{\partial f_0}{\partial I^j} \right), \qquad (8)$$

where $\sqrt{g}$ is the Jacobian of the transformation. Later on we will find it convenient to write the orbit-averaged quasilinear operator in an the following alternative form (see appendix)

$$\langle Q(f_0) \rangle = \sum_{n, N, \omega} L_{n, N, \omega} \left(D_0 L_{n, N, \omega} f_0 \right)$$

$$\qquad (9)$$

$$L_{n, N, \omega} = n^k \frac{\partial I^i}{\partial J^k} \frac{\partial}{\partial I^i}.$$

For convenience, we now specialize to the invariants (E, Λ, P_φ), where $E = mv^2/2$, $\Lambda = \mu B_0 / E = v_\perp^2 B_0 /(v^2 B)$ and B_0 is the magnetic field at the magnetic axis. These invariants are frequently employed in the literature, but it should be emphasized that the analysis below can easily be adapted for other sets of invariants. In the adopted set of invariants $\langle Q(f_0) \rangle$ can be written as [1]

$$\langle Q(f_0) \rangle = \frac{1}{\sqrt{g}} \frac{\partial}{\partial I^i} \left(\sqrt{g} D^{ij} \frac{\partial f_0}{\partial I^j} \right), \qquad (10)$$

where the diffusion tensor elements $\left(D^{ij} = D^{ji} \right)$ are given by

$$D^{EE} = \sum_{n, N, \omega} \omega^2 D_0, \qquad (11)$$

$$D^{E\Lambda} = \sum_{n, N, \omega} \omega \frac{n \omega_{c0} - \Lambda \omega}{E} D_0, \qquad (12)$$

3

$$D^{EP_\varphi} = \sum_{n,N,\omega} \omega N D_0 \, , \tag{13}$$

$$\tag{14}$$

$$D^{\Lambda P_\varphi} = \sum_{n,N,\omega} \frac{n\omega_{c0} - \Lambda\omega}{E} N D_0 \, , \tag{15}$$

$$D^{P_\varphi P_\varphi} = \sum_{n,N,\omega} N^2 D_0 \, . \tag{16}$$

Alternatively, using Eq. (9) we obtain

$$\langle Q(f_0) \rangle = \sum_{n,N,\omega} L_{n,N,\omega}(D_0 L_{n,N,\omega} f_0) \tag{17}$$

where the operator L is given as

$$L_{n,N,\omega} = \omega \frac{\partial}{\partial E} + \frac{\omega_{c0} n - \Lambda\omega}{E} \frac{\partial}{\partial \Lambda} + N \frac{\partial}{\partial P_\varphi} \, . \tag{18}$$

Finally, let us turn to the diffusion coefficient D_0. We consider a case where the resonant interactions between a wave and a particle are well separated in space and de-correlated. The integral in Eq. (7) will then be made up of individual contributions from around the points where the phase Φ in Eq. (7) is stationary. These individual contributions can easily be calculated with the method of stationary phase and the result is,

$$D_0 = \frac{1}{4\omega^2} \sum_{\substack{stationary \\ points}} \frac{(Ze)^2}{|n\dot{\omega}_R|} v_{\perp R}^2 \left| E_+ J_{n-1}(\beta_R) + E_- J_{n+1}(\beta_R) \right|^2 \, , \tag{19ed0s}$$

where subscript R refers to a quantity evaluated at a stationary point, i.e. at a resonance.

3. PROPERTIES OF THE OPERATOR

The operator written in Eq. (18) is not so easy to identify with operators derived in earlier works. In order to relate it to the operator used by Stix in his seminal work on ICRF heating [3], let us now change invariants to the perpendicular velocity,   , the parallel velocity,   , and the poloidal flux, ψ_{p1}, at a given magnetic field $B = B_1$. This set of invariants can be related to our previous set as:

$$E = \frac{m}{2}\left(v_{\perp 1}^2 + v_{//1}^2\right), \qquad\qquad P_\varphi = mF(\psi_{p1})v_{//1}/B_1 + Ze\psi_{p1}/(2\pi), \tag{20}$$

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where $F(\psi_p) = RB_\varphi$. With this new set of invariants the operator $L_{n,N,\omega}$ in Eq. (18) can be expressed as

$$
\tag{21}
$$

For a particle which has a resonance at our chosen magnetic field, B_1, we have (in the small banana width limit) With these expressions one can easily see the close connection between velocity space part of the operator $L_{n,N,\omega}$ and the one used by Stix [3], which was originally derived by Kennel and Engelmann for a straight field line geometry [4]. In fact, the only difference is that a term containing a derivative with respect to the local poloidal flux appears in Eq. (21).

From the characteristics of the operator $L_{n,N,\omega}$ we can derive the following relationships between the change in the velocity and the poloidal flux (i.e. the spatial position) a particle receives as it passes a resonance point

$$
v_{\perp R}\Delta v_{\perp R} = \frac{1}{m}\frac{n\omega_{cR}}{\omega}\Delta E, \quad v_{//R}\Delta v_{//R} = \frac{1}{m}\frac{\omega - n\omega_{cR}}{\omega}\Delta E,
$$

$$
\Delta\psi_{pR} = \frac{RB_p\left(k_\varphi B_p - k_p B_\varphi\right) - B_R F(\psi_{pR})\dfrac{\boldsymbol{k}\cdot\mathbf{v}_D}{v_{//R}}}{B_R\omega m\left[\dfrac{\omega_{cR}}{2\pi} + v_{//R}\dfrac{\partial F(\psi_p)}{\partial\psi_p}\bigg|_{\psi_p=\psi_{pR}}\right]}\Delta E,
\tag{22}
$$

where $\mathbf{v}_D$ is the perpendicular component of the guiding center velocity and all quantities are evaluated at the magnetic field, B_R, where the particle orbit crosses the resonance. Note the complexity of the expression for the change in the poloidal flux at the resonance as compared to the expressions

$$
, \qquad \Delta P_\varphi = \frac{N}{\omega}\Delta E
\tag{23}
$$

for the change in Λ and toroidal angular momentum.

As can be seen from Eqs (22) and (23), the change in the poloidal flux at the resonance depends not only on the wave frequency but also on all three components of the wave vector (k_φ, k_p, $\boldsymbol{k}\cdot\nabla\psi_p/|\nabla\psi_p|$), whereas the changes in the invariants Λ and P_φ only directly depend on the wave frequency and the toroidal mode number ($N = k_\varphi R$). Thus, in spite of the apparent importance of k_p and $\boldsymbol{k}\cdot\nabla\psi_p/|\nabla\psi_p|$ for the change in ψ_{pR} (and thus for the change in the

spatial position where the orbit crosses $B = B_R$), for a given ΔE it is only the toroidal mode number that is important for the spatial transport of resonating ions. The reason for this somewhat surprising fact, which at first sight seems to be a contradiction, is that as k_p and $\mathbf{k} \cdot \nabla \psi_p / |\nabla \psi_p|$ change, both the location of the interaction and the relative changes in $v_{\perp R}$, $v_{//R}$ and ψ_{pR} are modified. The combined effect of these modifications is such that the relative changes of the invariants Λ and P_φ are independent of k_p and $\mathbf{k} \cdot \nabla \psi_p / |\nabla \psi_p|$. It is only through the diffusion coefficient, D_0, that k_p and $\mathbf{k} \cdot \nabla \psi_p / |\nabla \psi_p|$ play a role.

The change in the spatial position at the resonance when $k_\varphi B_p - k_p B_\varphi \neq 0$ is the same effect as has been discussed in the context of interaction between short wavelength waves, e.g. Lower Hybrid waves, and fast ions [5]. As discussed above, this effect does not play a role for the spatial transport of ions interacting with ICRF waves.

One should note that for wave propagation in a torus the launched toroidal mode number spectrum is invariant, while the parallel wave number $k_\parallel$ can upshift significantly especially in the case of weakly damped waves. The reason for the upshift in $k_\parallel$ is that modes with higher poloidal mode numbers m than emitted by the antenna are excited through mode coupling, and $k_{//} = \dfrac{m}{r} \dfrac{B_p}{B} + \dfrac{N}{R} \dfrac{B_\varphi}{B}$ where r is the minor radius. As our results show, such an upshift in $k_\parallel$ does not directly affect the radial transport of the resonating ions since the toroidal mode number is conserved.

Finally, we see that for a symmetric toroidal mode number spectrum the wave-induced transport of trapped particles is purely diffusive, whereas for an asymmetric spectrum there also is a convective component.

4. MONTE CARLO OPERATOR

In the Monte Carlo approach to solve the orbit-averaged Fokker-Planck equation, a large number of "particles" are followed in invariant space (a particle in this context does not represent a real single particle but an ensemble of particles with the same invariants that are distributed along an orbit). The invariants of a "particle" at a time step, t_n, are changed to those at the next time step, $t_{n+1} = t_n + \Delta t$, according to

$$I^i(t_{n+1}) = I^i(t_n) + \Delta I^i. \tag{24}$$

The components of the Monte Carlo operator, ΔI^i, are stochastic variables, whose expectation values and covariances are given by

$$E\left[\Delta I^i\right] = \frac{d\mu^i}{dt} \Delta t \tag{25}$$

$$C\left[\Delta I^i, \Delta I^j\right] = \frac{d\sigma^{ij}}{dt}\Delta t. \qquad (26)$$

The time derivatives of the expectation values, μ^i, and covariances, σ^{ij}, are obtained from the orbit-averaged Fokker-Planck equation by following the time evolution of a distribution function, $f_0 = g^{-1/2}\delta(\boldsymbol{I} - \boldsymbol{I}_0)$ representing a single particle at $t = t_n$,

$$\frac{d\mu^i}{dt} = \dot{\mu}^i = \int I^i \frac{\partial f_0}{\partial t}\sqrt{g}d^3 I = \frac{1}{\sqrt{g}}\frac{\partial}{\partial I^j}\left(\sqrt{g}D^{ij}\right) = \frac{1}{\sqrt{g}}\frac{\partial}{\partial I^j}\left(\sqrt{g}D_0 n^k n^l \frac{\partial I^i}{\partial J^k}\frac{\partial I^j}{\partial J^l}\right), \qquad (27)$$

$$\frac{d\sigma^{ij}}{dt} = \dot{\sigma}^{ij} = \int (I^i - \mu^i)(I^j - \mu^j)\frac{\partial f_0}{\partial t}\sqrt{g}d^3 I = 2D^{ij}. \qquad (28)$$

The Monte Carlo operator can now be written as a sum of two components,

$$\Delta I^i = \frac{d\mu^i}{dt}\Delta t + A^{ik}\xi_k\sqrt{\Delta t}, \qquad (29)$$

where ξ_k are uncorrelated stochastic variables with zero expectation value and unit variance; and the matrix A^{ik} must fulfill the relation: $\sum_k A^{ik}A^{jk} = d\sigma^{ij}/dt$ (note that this is not a tensor summation, i.e. k is a contra-variant index in both of the tensors). There is no unique solution for A^{ik}, two methods for obtaining a solution are discussed in Refs. [6, 1]. Following Ref. [6] one can obtain the following explicit expression:

$$\Delta E = \dot{\mu}^E \Delta t + \xi_E \sqrt{\dot{\sigma}^{EE}\Delta t} \qquad (30)$$

$$\Delta\Lambda = \dot{\mu}^\Lambda \Delta t + \left[\xi_E \dot{\sigma}^{E\Lambda}/\sqrt{\dot{\sigma}^{EE}} + \xi_\Lambda \sqrt{\dot{\sigma}^{\Lambda\Lambda} - \left(\dot{\sigma}^{E\Lambda}\right)^2/\dot{\sigma}^{EE}}\right]\sqrt{\Delta t} \qquad (31)$$

$$\Delta P_\varphi = \dot{\mu}^{P_\varphi}\Delta t + \left[\xi_E \dot{\sigma}^{EP_\varphi}/\sqrt{\dot{\sigma}^{EE}} + \xi_\Lambda S^{\Lambda P_\varphi} + \xi_{P_\varphi}\sqrt{\dot{\sigma}^{P_\varphi P_\varphi} - \left(\dot{\sigma}^{EP_\varphi}\right)^2/\dot{\sigma}^{EE} - \left(S^{\Lambda P_\varphi}\right)^2}\right]\sqrt{\Delta t}. \qquad (32)$$

where $S^{\Lambda P_\varphi} = \left(\dot{\sigma}^{P_\varphi\Lambda} - \dot{\sigma}^{E\Lambda}\dot{\sigma}^{EP_\varphi}/\dot{\sigma}^{EE}\right)/\sqrt{\dot{\sigma}^{\Lambda\Lambda} - \left(\dot{\sigma}^{E\Lambda}\right)^2/\dot{\sigma}^{EE}}$ is zero in the case of interaction with a single frequency.

In order to keep the computation time down, it is essential to evaluate $d\mu^i/dt$ and $d\sigma^{ij}/dt$ efficiently. While the time derivative of the variances are just two times the diffusion tensor elements, the time derivative of the expectation values are much more complicated to evaluate, involving derivatives with respect to the invariants. Since the diffusion tensor elements can be precalculated and stored in a table, allowing for fast subsequent calculation by

interpolation, one could in principle easily evaluate the expectation values by numerical differentiation using the elements in the table. However, this method only works when a particle interacts resonantly with the same toroidal mode numbers and frequencies in all of the grid points used for the interpolation. In practice one will frequently encounter particles for which this condition is not fulfilled or particles which are outside the grid (e.g. with energies above the maximum energy of the grid). For such particles it is important to evaluate the Monte Carlo operator directly as efficiently as possible.

Using the result from the appendix we can write the expectation values as

$$\frac{d\mu^i}{dt} = \sum_{n,N,\omega} n^k \frac{\partial I^j}{\partial J^k} \frac{\partial}{\partial I^j}\left(D_0 n^l \frac{\partial I^i}{\partial J^l}\right) = \sum_{n,N,\omega} L_{n,N,\omega}\left(D_0 n^l \frac{\partial I^i}{\partial J^l}\right). \tag{33}$$

From this form we can see that in order to evaluate the expectation values directly in a numerically efficient way, it is essential to find a set of invariants in which $L_{n,N,\omega}$ takes a suitable form. Such sets of invariants exist. For example, with the set of invariants given by

$$I_\perp = E\frac{\Lambda\omega}{n\omega_{c0}}, \quad I_{//} = E\left(1 - \frac{\Lambda\omega}{n\omega_{c0}}\right), \quad I_\varphi = P_\varphi - \frac{N}{\omega}E \tag{34}$$

the operator $L_{n,N,\omega}$ takes the simple form:

$$L_{n,N,\omega} = \omega\frac{\partial}{\partial I_\perp} \tag{35}$$

and we have

$$\frac{d\mu^i}{dt} = \sum_{n,N,\omega} \omega\left[D_0 \frac{\partial}{\partial I_\perp}\left(n^l \frac{\partial I^i}{\partial J^l}\right) + n^l \frac{\partial I^i}{\partial J^l}\frac{\partial D_0}{\partial I_\perp}\right]. \tag{36}$$

Thus, we can evaluate the expectation values for the invariants $\boldsymbol{I} = (E, \Lambda, P_\varphi)$ as

$$\frac{d\mu^E}{dt} = \sum_{n,N,\omega} \omega^2 \frac{\partial D_0}{\partial I_\perp} \tag{37}$$

$$\frac{d\mu^\Lambda}{dt} = \sum_{n,N,\omega} \frac{n\omega_{c0} - \Lambda\omega}{E}\omega\left[\frac{\partial D_0}{\partial I_\perp} - \frac{2}{E}D_0\right] \tag{38}$$

$$\frac{d\mu^{P_\varphi}}{dt} = \sum_{n,N,\omega} N\omega\frac{\partial D_0}{\partial I_\perp}, \tag{39}$$

which is relatively easy since we only need to calculate $\partial D_0/\partial I_\perp$ numerically. In practice this means that D_0 must be evaluated in two points around the central point, i.e. at

$$E = E_0 \pm \Delta I_\perp \tag{40}$$

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$$\Lambda = \Lambda_0 + \frac{n\omega_{c0}}{\omega}\left[\frac{I_\perp \pm \Delta I_\perp}{I_{//} + I_\perp \pm \Delta I_\perp} - \frac{I_\perp}{I_{//} + I_\perp}\right] \tag{41}$$

$$P_\varphi = P_{\varphi 0} \pm \frac{N}{\omega}\Delta I_\perp . \tag{42}$$

5. NUMERICAL ILLUSTRATION

In order to illustrate the importance of proper evaluation of the Monte Carlo ICRF operator for modeling of ICRF heating, we use the FIDO code [7]. This code solves Eq. (1) using the Monte Carlo operators described in Section 4, with the expectation values of the invariants (E, Λ, P_φ) calculated according to Eqs (37)–(39). The term $\partial D_0/\partial I_\perp$ in Eqs (37)–(39) is obtained by evaluating the diffusion coefficient D_0 in two points given by Eqs (40)–(42) around the central point. For ions with resonances well separated along the orbit we assume the interactions to be decorrelated. The integral in Eq. (7) is then made up of individual contributions from around the points where the phase Φ in Eq. (7) is stationary and the resulting D_0 is given by

$$D_0 = \frac{1}{4\omega^2}\sum_{\substack{stationary \\ points}}\frac{(Ze)^2}{|n\dot{\omega}_{cR}|}v_{\perp R}^2\left|E_+ J_{n-1}(\beta_R) + E_- J_{n+1}(\beta_R)\right|^2 , \tag{43}$$

where subscript R refers to a quantity evaluated at a stationary point, i.e. at a resonance. For ions which have resonances close to each other along the orbit (i.e. particles which are near tangency resonance where $d\omega_{cR}/dt = \dot{\omega}_{cR} \to 0$) the contributions to Eq. (43) are modified and involve Airy functions [8]. In FIDO D_0 for particles having near tangency resonance is calculated using Eq. (43) imposing a lower limit to $|n\dot{\omega}_{cR}|$ so that the contribution to D_0 does not exceed the maximum contribution obtained from the correct expression involving Airy functions.

In the simulations we use the steady-state parameters of an ICRF-only deuterium plasma on JET with fundamental minority heating of hydrogen. A magnetic field of 3.5 T and a plasma current of 3.3 MA are used, and the fundamental hydrogen resonance is located on the high field side about 10 cm from the magnetic axis. Furthermore, the central electron density is 4.1×10^{19} m^{-3}, and the central electron and ion temperatures are 3.5 keV and 2 keV, respectively. The hydrogen concentration is 1 %.

The wave parameters and the power deposition profiles used in FIDO are taken from the ICRF code PION [9]. In particular, we have $k_\perp = 35$ m^{-1}, $E_-/E_+ = 4$, and the total hydrogen absorption is 3.5 MW. We have performed calculations for a symmetric toroidal mode number spectrum $(N = 28, -28)$ which is representative of the phasing normally used in JET. In addition we have studied effects of asymmetric toroidal mode number spectra with a single toroidal mode number $(N = 28$ and $N = -28)$, using the same parameters. For $N = 28$ the wave-induced spatial transport of resonating particles is inwards and for $N = -28$ it is outwards.

As can be seen in Fig. 1, the ICRF-induced spatial transport of resonating ions plays an important role for the fast-ion pressure profile. In particular, the pressure profile in the case where the ICRF-induced spatial transport of resonating ions is outwards ($N = -28$) is broader than in the other two cases. Also, the differences in the calculated fast-ion pressure profiles between the inward drift case ($N = 28$) and the symmetric phasing ($N = 28, -28$) are smaller than compared to the outward drift case. The main reason is that for the

Fig. 1 Pressure profiles of the resonating ions for different toroidal mode number spectra.

inward drift case and for the symmetric phasing, most of the fast ions have wide non-standard orbits which pass near the plasma center, whereas in the outward drift case standard banana orbits which do not pass near the plasma center dominate. These general observations are consistent with experimental evidence for the existence of ICRF induced drifts [10].

To assess the sensitivity of the fast-ion pressure profile to possible errors in the calculation of the Monte Carlo ICRF operator, we have performed three further simulations with the FIDO code for each toroidal mode number spectrum studied. When calculating $\partial D_0 / \partial I_\perp$ in these simulations we have still used Eqs (40)–(42), but we have neglected in turn the variation in E in Eq. (40), in Λ in Eq. (41) and in P_φ in Eq. (42). This corresponds to neglecting the term proportional to the derivative of E, Λ and P_φ, respectively, in the operator L given in Eq. (18).

Fig. 2 Pressure profiles of the resonating ions for $N=-28$ calculated with the correct expectation values and, for assessing the sensitivity, when the E, Λ and P_φ in turn have been kept constant in Eqs.(40) - (42).

The results from these simulations with $N = -28$ are displayed in Fig. 2, together with the result from the simulation where $\partial D_0 / \partial I_\perp$ has been calculated correctly using Eqs (40)–(42). While the fast-ion pressure profile calculated by neglecting the variation in P_φ does not differ much from the pressure profile calculated properly, neglecting the variation either in E or Λ has a major effect on the pressure profile. The reason for these differences is the relatively strong dependence of D_0 on E and Λ, and its relatively weak dependence on P_φ. Somewhat smaller differences are found in the case of inward drift ($N = 28$) and the symmetric phasing ($N = 28, -28$).
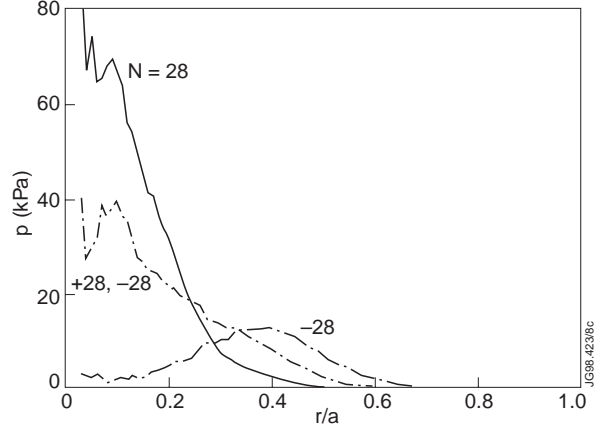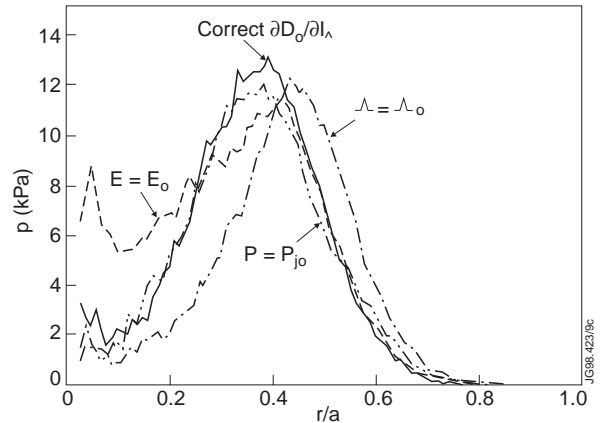
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6. SUMMARY

A 3D Monte Carlo operator for the orbit-averaged Fokker-Planck equation describing wave particle interaction in a toroidal geometry has been derived from a quasilinear differential operator. Based on earlier work [1], the operator has been developed here to make it more suitable for numerical implementation. Furthermore, properties of the orbit-averaged quasilinear differential operator have been discussed. By rewriting the differential operator in a special set of invariants, the close relationship between this operator and the more familiar quasilinear operator for a straight field line geometry could be demonstrated. Examination of the characteristics of the quasilinear differential operator shows that, although all three components of the wave vector influence the local change in the radial position an ion receives as it passes a resonance, the relative changes in the particle invariants depend only on the toroidal component of the wave vector. Consequently, it is only the toroidal component of the wave vector that is of direct importance for wave-induced spatial transport, the other component can only have an indirect effect through the diffusion coefficient.

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APPENDIX

From Eq. (8) we obtain

$$\langle Q(f_0)\rangle = \sum_{n,N,\omega}\left[n^k\,\frac{\partial I^i}{\partial J^k}\,\frac{\partial}{\partial I^i}\left(D_0 n^l\,\frac{\partial I^j}{\partial J^l}\,\frac{\partial f_0}{\partial I^j}\right) + D_0 n^l\,\frac{\partial I^j}{\partial J^l}\,\frac{\partial f_0}{\partial I^j}\,\frac{n^k}{\sqrt{g}}\,\frac{\partial}{\partial I^i}\left(\sqrt{g}\,\frac{\partial I^i}{\partial J^k}\right)\right]. \quad (A1)$$

The last term in the above expression is zero, which can be shown as follows. Let $A = (1,0,0)$; hence $\nabla_J \cdot A = 0$. Transformation from J^i space to I^i space yields

$$\nabla_I \cdot A = \frac{\partial A^i}{\partial I^i} = \frac{1}{\sqrt{g}}\,\frac{\partial}{\partial I^i}\left(\sqrt{g}\,\frac{\partial I^i}{\partial J^j}\,A^j\right) = \frac{1}{\sqrt{g}}\,\frac{\partial}{\partial I^i}\left(\sqrt{g}\,\frac{\partial I^i}{\partial J^1}\right) = 0. \quad (A2)$$

Following the same procedure for $A = (0,1,0)$ and $A = (0,0,1)$, we obtain

$$\frac{1}{\sqrt{g}}\,\frac{\partial}{\partial I^i}\left(\sqrt{g}\,\frac{\partial I^i}{\partial J^j}\right) = 0. \quad (A3)$$

Hence, $\langle Q(f_0)\rangle$ in Eq. (8) can alternatively be written as in Eq. (9).