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Equilibrium Analysis of JET Tokamak Discharges

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Abstract. Equilibrium analysis is one of the essential tools for determining the plasma shape and profiles, in particular the current density and safety factor, as well as global physical quantities. We report on recent developments of the equilibrium code EFIT, which is used for the routine analysis of JET Tokamak discharges. A new method to treat the iron transformer surrounding the JET device is presented. This is one of the prerequisites for the development of a real-time discharge analysis and feedback of physical quantities. A new type of constraint derived from the shape of the ECE temperature profile has been implemented which allows for an enhanced precision of the position of the magnetic axis and the safety factor.

I. Equilibrium analysis

The plasma equilibrium for an axisymmetric device is described by the Grad-Shafranov equation

$$-\frac{\partial^2\Psi}{\partial R^2}+\frac{1}{R}\frac{\partial\Psi}{\partial R}-\frac{\partial^2\Psi}{\partial Z^2}=\mu_0 R^2 P'(\Psi)+FF'(\Psi)+J_{ext}, \qquad (1)$$

where $\psi(R,Z)$ is the poloidal flux function, P the isotropic pressure, and $F=RB_z$. J_{ext} denotes the poloidal field coil currents and the magnetisation currents of an iron transformer. To reconstruct an equilibrium from measured data, an ansatz for the unknown profiles e.g. $P'(\Psi)=\sum_k c_k g_k(\Psi)$ with suitable functions g_k is selected. The Grad-Shafranov equation is then used as a constraint to determine the unknown coefficients c_k by minimising the functional

$$\chi^2=\sum_n \frac{1}{\sigma_n{}^2}(F_n^{calc}\{\Psi;C\}-F_n^{meas})^2+R(C) \qquad (2)$$

where F_n^{calc} is a functional which generates the value of the measured quantity from ψ and c_k. σ_n denotes a typical standard deviation of measurement, and R(C) controls higher derivatives in the current profile. The algorithm EFITJ[1] is used routinely to solve the reconstruction problem at JET. The code is based on the original EFIT code by Lao et al [2], modified to model the JET iron core. As input data for automatic analysis between discharges, we use magnetic data from 57 magnetic pick-up coils and 27 flux-loops located at the vacuum vessel of the torus. This intershot equilibrium reconstruction is in routine use for all JET discharges since the pumped divertor installation in 1994.

II. Iron transformer model

The presence of the iron transformer in the JET device complicates the equilibrium calculation, because the response of the iron to external magnetic fields is nonlinear and cannot be easily calculated. In the EFITJ code, the magnetisation currents of sections of the iron structure are added as free variables to the fitting procedure, and determined by the measurements[1]. This method gives good results for the equilibrium reconstruction, but it has a few drawbacks. The large number of parameters leads to long execution times. The algorithm is also unsuitable for predictive calculations. We have devised a model to pre-calculate the iron magnetisation with a permeability model. The field generated by the iron magnetisation **M** is given by

$$\mathbf{B}_{iron} = \nabla \times \mu_0 \left(-\oint \frac{\mathbf{M}' \times \mathbf{n}'}{|\mathbf{x} - \mathbf{x}'|} da' + \int \frac{\nabla \times \mathbf{M}'}{|\mathbf{x} - \mathbf{x}'|} d^3x' \right). \quad (3)$$

The total magnetic field is related to **M** via the permeability μ(B) through $\mathbf{B} = \mu\mu_0 / (\mu - 1)\mathbf{M}$, which inserted in (2) gives the nonlinear equation for **M**

$$\nabla \times \mu_0 \left(-\oint \frac{\mathbf{M}' \times \mathbf{n}'}{|\mathbf{x} - \mathbf{x}'|} da' + \int \frac{\nabla \times \mathbf{M}'}{|\mathbf{x} - \mathbf{x}'|} d^3x \right) + \frac{\mu}{\mu - 1} \mu_0 \mathbf{M} = \mathbf{B}_{plasma} + \mathbf{B}_{PFC}. \quad (4)$$

This equation is solved by the integral equation method [e.g. 5]. The region containing iron is subdivided into finite elements, and the magnetisation is assumed to be constant within the elements. A similar method was suggested in [4]. The resulting algebraic equation is solved with a Newton-Raphson method. This gives the iron contribution of the magnetic field in equation (1). As a further simplification to solve equation (4), the 8 JET transformer limbs are modelled with an axisymmetric equivalent geometry [5]. Details are given in figure 1. The method described leads to the first full domain equilibrium reconstruction code for iron core Tokamaks. A comparison with the existing database of JET equilibria however shows negligible differences, which proves that the old model was good enough for its purpose. The number of fitting variables, on the other hand, is reduced to those for the plasma current profiles, giving a reduction from typically 67 parameters to 5. The singular value decomposition as the fitting algorithm now consumes negligible CPU time, so that one obstacle for real-time application is removed. On a Pentium 150MHz PC, the algorithm for the iron model itself takes about 400 ms CPU time per time-slice, the rest of only partially optimised EFIT code about 1 sec, with the aim of cutting it down to 50 ms on a suitable platform. The code is applicable to a wide range of Tokamaks, and allows for equilibrium reconstruction and prediction. It is suitable for future

Tokamak devices, such as ITER, which require continuous processing of diagnostic data for monitoring and control

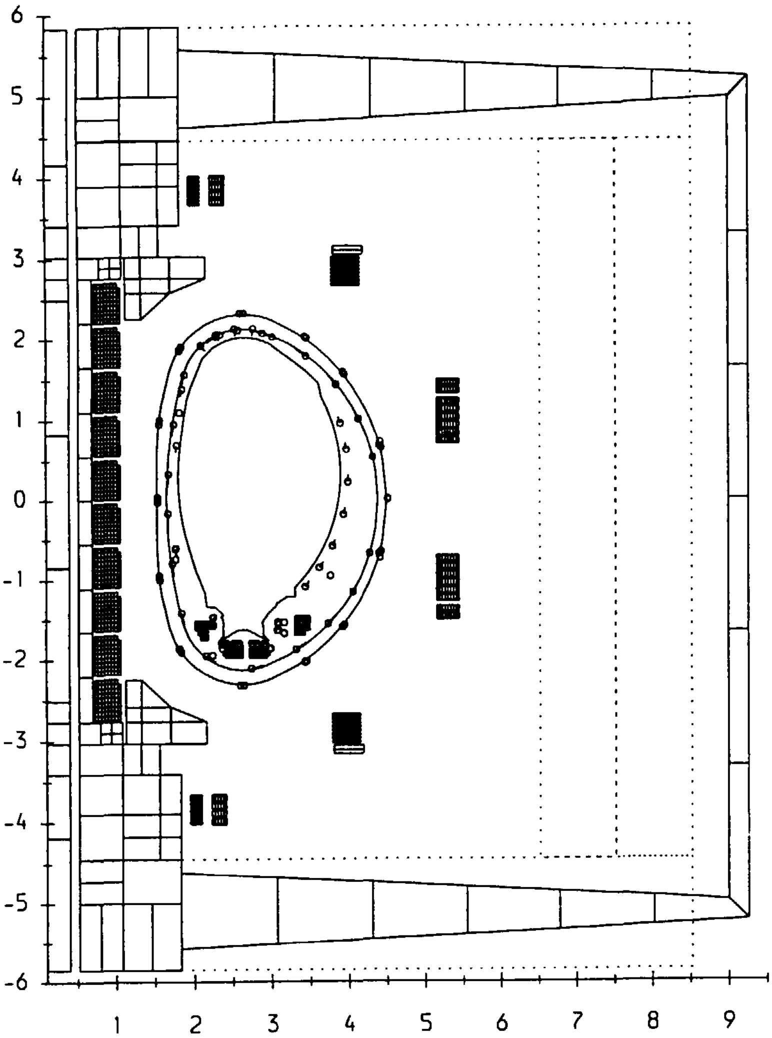

Figure 1.

Cross section of the JET device. It shows the vacuum vessel with magnetic diagnostics marked with circles, the poloidal field coils and the finite elements of the axisymmetric iron model. The dashed lines mark the real shape of the transformer limbs.

III. Improvement of reconstruction with ECE constraint

The equilibrium reconstruction from external magnetic data is a mathematically ill-posed problem of Hadamard type, since flux and derivative (magnetic field) are given as boundary conditions at approximately the same location. Therefore, the reconstructed profiles can become quite inaccurate. One solution is to add additional internal data for the fitting procedure. The EFIT code is presently capable of using Faraday rotation data, pressure profile, q-profile (e.g. from soft X-ray), and motional Stark data. We present a new type of constraint derived from the shape of the temperature profile from the electron cyclotron emission (ECE) measurement, which is potentially available in real time. The measurement gives the electron temperature profile $T_e(B_{total})$ as a function of the total magnetic field, since it is proportional to the electron cyclotron frequency. We assume that the temperature peak is close to the magnetic axis, such that both radial locations coincide. We then impose as constraint

$$B_z(R_p) = 0 \qquad \text{at } B_{\text{total}}(R_p) = B_{\text{ECE}} \tag{5}$$

where R_p is the unknown radial position of the peak and B_{ECE} the corresponding field at the peak. The constraint is linearised and introduced into the fitting procedure. We analysed discharge 40554 with this new condition. Figure 2 shows a comparison of two equilibrium reconstructions, with magnetic data only, and with the additional ECE constraint. There is a significant shift of the magnetic axis with the ECE data applied.

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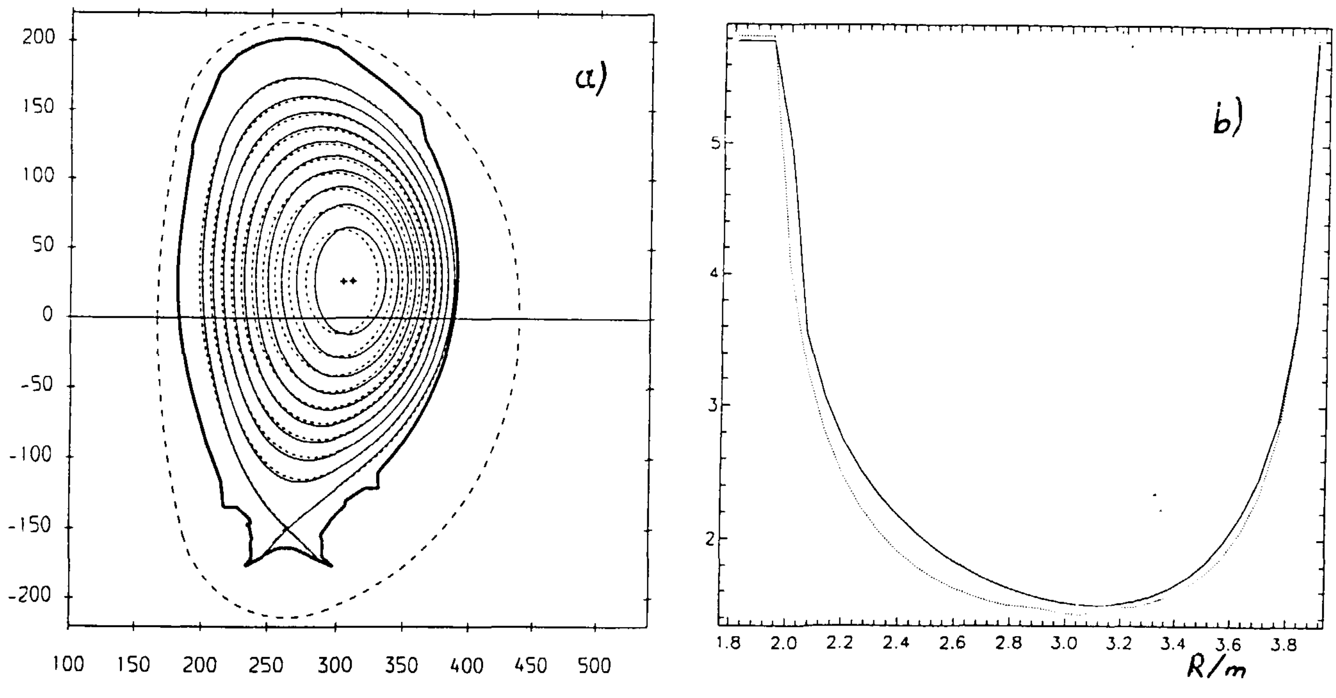

Figure 2. Flux contours(a), and safety factor(b) of discharge 40554, t=46.1 sec. Solid lines are for the reconstruction with the ECE constraint, dashed with only magnetic data.