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Effects of Density Asymmetries on Heavy-Impurity Transport in a Rotating Tokamak-Plasma

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ABSTRACT

The transport equations of heavy trace-impurities in a Tokamak plasma with strong toroidal rotation have been studied analytically in the collisional regime. It is found that the poloidal asymmetry of the impurity-density, which occurs because of the rotation, brings about a large enhancement of the diffusivity and indeed of the pinch velocity above the conventional Pfirsh-Schlüter values.

1. INTRODUCTION

The effects of plasma rotation on ion transport coefficients has been investigated in early theoretical works by Wong, Sigmar and others [1],[2],[3],[4]. In their works the centrifugal force was taken into account in the neoclassical flux producing an enhancement of the heat and particles transport. However, for historical reasons (the predicted extra flux was only a small correction on the neoclassical values for 1980's plasma parameters), and for lack of experimental measurements, the centrifugal force has not been included yet properly neither in the transport simulation codes nor in the equilibrium reconstruction codes which use the Grad-Shafranov equation. The renewed interest on the centrifugal force effects on plasma dynamics comes from the recent observations, done by using the JET soft X-ray tomography, of impurity-densities four times higher on the outer side of the plasma poloidal cross section than on the inner side during trace-impurity injection experiments [5]. This phenomena, in a less pronounced way, was already observed in the past at JET by Giannella [6]. In this paper we highlight the important role played by the asymmetries on the flux surfaces in determining the particles transport. Indeed, whereas in the Pfirsch-Schlüter regime without plasma rotation the particle density asymmetry over the magnetic surface is small (it is proportional to the second order in the inverse parallel diffusivity $D_{\parallel}^{-2}$), in the case of strong rotation this is no longer valid. Since poloidal asymmetries are in general associated with collisional transport in a torus, one also expects an enhancement of transport coefficients when a strong toroidal rotation is present. It is the goal of this work to evaluate the Pfirsch-Schlüter transport coefficients in the new regime of strong poloidal asymmetries. This program is carried out analytically, for a low-β plasma with circular flux surfaces, by employing a Fourier expansion of the impurity density in poloidal harmonics. A diffusion coefficient and pinch velocity ten times larger than the conventional Pfirsch-Schlüter ones are found for JET parameters.

2. THE TRANSPORT PROBLEM

The collisional cross field diffusion of impurity ions in a hydrogenic plasma is dominate by the collisions between proton and impurity; it is only when the impurity fluid moves at the same speed of the background ions that the weaker effect of collisions between electrons and impurities brings about a slower diffusion on a time scale comparable to the electron density evolution time. Since heavy impurities typically fall in the Pfirsch-Schlüter collisional regime (at least at the plasma edge where the ion and electron temperature is lower) it is interesting to solve the transport equations in this regime including the centrifugal force term and allowing strong poloidal asymmetries in the impurity density. Let m_I, Z be the impurity mass and charge while we indicate with m_i the mass of the hydrogen-like background ions. We assume trace impurities so that their time evolution does not effect the background plasma. A toroidal coordinate system (r,θ,Φ) is adopted with r denoting the toroidal surface minor radius, θ the poloidal angle Φ the toroidal angle; R_0 will be the flux surface major radius. The magnetic field is taken to be $\boldsymbol{B} = (0, B_{\theta 0}(r)/h, B_{\Phi 0}(r)/h)$, where $h = 1 + \varepsilon\cos\theta$ and e is the inverse aspect ratio r/R_0. The magnetic surfaces Ψ(r) are concentric toroidal surfaces. Following Braginskii [7], the force balance equations for background ions, impurities and electrons in the limit $m_I >> m_i$ and in a reference frame rotating at the toroidal angular frequency are :

$$
\begin{aligned}
-m_i n_i \Omega^2 R \mathrm{e}_R &= -\nabla p_i + n_i e\left(\boldsymbol{E} + \boldsymbol{V}_i \times \boldsymbol{B}\right) - \boldsymbol{F} \\
-m_I n_I \Omega^2 R \mathrm{e}_R &= -\nabla p_I + n_I Ze\left(\boldsymbol{E} + \boldsymbol{V}_I \times \boldsymbol{B}\right) + \boldsymbol{F} \\
0 &= -\nabla p_e - n_e e\left(\boldsymbol{E} + \boldsymbol{V}_e \times \boldsymbol{B}\right)
\end{aligned}
\tag{1}
$$

where $\boldsymbol{F}$ is the friction due to collisions between ions and impurities,

$$
\boldsymbol{F} \equiv -m_I \nu_{Ii} n_I \left(C_1 \boldsymbol{U}_{\parallel} + C_2 \boldsymbol{U}_{\perp}\right) - C_3 n_I \nabla_{\parallel} T_i - \frac{3}{2}\frac{\nu_{Ii} n_I}{\omega_{Ic}} \nabla_{\perp} T_i
$$

$\boldsymbol{U} = \boldsymbol{V}_I - \boldsymbol{V}_i$, the symbols ⊥ and II denote perpendicular and parallel direction to the magnetic field; C_1, C_2 and C_3 are numerical constants which take into account plasma anisotropy [8]. The term $mn\Omega^2 R\boldsymbol{e}_R$ is the centrifugal force, $R \equiv R_0 + r\cos\theta$ is the distance between the point on the toroidal surface and the axis and $\boldsymbol{e}_R$ is the unit vector in the direction of the line perpendicular to the torus major axis and passing through the point.

From equation (1) for the background ions one can derive, assuming quasi neutrality $\left(n_e \approx n_i\right)$, the parallel electric field produced by the displacement of the ions under the centrifugal force effect

$$
E_{\mathrm{II}\theta} = \frac{T_e}{T_e + T_i}\frac{m_i}{e}\Omega^2 R \sin\theta. \tag{2}
$$

The velocity $U_{\parallel\theta}$ can be calculated taking the parallel component of equation (1) for the impurity $\left(T_I = T_i\right)$

$$U_{\parallel\theta} = -\frac{1}{n_I} D_{\parallel} \frac{B_{\theta 0}^2}{B_0^2} \left(\frac{1}{r} \partial_\theta n_I + \frac{m^* n_I}{T_i} \Omega^2 R \sin\theta + \left(C_3 + 1\right) n_I \frac{1}{r} \frac{\partial_\theta T_i}{T_i} \right) \tag{3}$$

where $D_{\parallel} = T_i / C_1 m_I \nu_{Ii}$ is the parallel diffusion coefficient and $m^* \equiv \left(m_I - Z m_i T_e / (T_e + T_i)\right)$. The radial flux is calculated from equation (1) for the impurities taking the poloidal component and using the electric field (2) and the velocity (3)

$$n_I \boldsymbol{V}_{Ir} = -\frac{T_i B_{\Phi 0}}{Ze B_0^2} h \left(\frac{1}{r} \partial_\theta n_I + \frac{m^* n_I}{T_i} \Omega^2 R \sin\theta + \left(C_3 + 1\right) n_I \frac{1}{r} \frac{\partial_\theta T_i}{T_i} \right). \tag{4}$$

The particle density evolves transiently on the fast parallel-transport time-scale and subsequently on the slower time-scale of radial-transport. After the transient, the dominant term in the particle continuity equation is the divergence of the poloidal particle flow. Thus the poloidal component of the velocities satisfies the equation $\nabla \cdot n_j \boldsymbol{V}_j = 0$ (the index j denoting the species), that implies

$$n_j V_{j\theta} = g_j(r) / h \tag{5}$$

where g is, at this stage an arbitrary function of the minor radius, and toroidal symmetry is assumed. Equation (5) for the impurities can be written in terms of the one for the background ions

$$n_I V_{I\theta} = n_I V_{I\perp\theta} + n_I U_{\parallel\theta} + n_I V_{i\perp\theta} = g_I(r) / h \tag{6}$$

The function g for the background ions depends on the experimental conditions. The expression (6) can be written explicitly by assuming that the background plasma is not rotating poloidally (so that $g_i(r) = 0$) and substituting the velocity of equation (3) together with the perpendicular velocities of impurity and background ions calculated from equation (1)

$$\begin{aligned} &\frac{T_i B_{\Phi 0}}{Ze B_0^2} h \left(\partial_r n_I + \frac{m^* n_I}{T_i} \Omega^2 R \cos\theta - \frac{\partial_\theta p_i}{p_i} Z n_I \right) + \\ &- D_{\parallel} \frac{B_{\theta 0}^2}{B_0^2} \left(\frac{1}{r} \partial_\theta n_I + \frac{m^* n_I}{T_i} \Omega^2 R \sin\theta \right) = \frac{g_I(r)}{h} \end{aligned} \tag{7}$$

In equation (7) the poloidal gradient of the ion temperature has been neglected compered with the poloidal gradient of the impurity density, this assumption will be justified later when we will show that the impurity density depends strongly on the poloidal angle. Once the function g is expressed in terms of measured quantities, equation (7) allows for the poloidal density derivative to be written in terms of the radial one and so to compute the flux (4). The poloidal equilibrium of the impurity has been calculated by Wesson [9] and the radial equilibrium calculated from equation (7) has been published by the authors in [10]. The next step is to expand the impurity density in Fourier components

$$n_I(r,\theta) = \sum_m n_{Icm}(r)\cos m\theta + n_{Ism}(r)\sin m\theta$$

and express the function g in terms of the Fourier coefficients of the density by taking the average of equation (7) on the magnetic surface

$$\overline{f}_\Psi = \frac{1}{2\pi}\int_0^{2\pi} f(\Psi,\theta)h(\theta)d\theta$$

and using the periodicity $n_I(\theta=0) = n_I(\theta=2\pi)$. The function g is found to depend only on the coefficients of the first and second harmonics of the density. Substituting g in equation (7) and then using equation (7) in equation (4) the expression for the radial flux in terms of the radial profiles is obtained.

The average of the radial flux on the magnetic surface to the order ε^2 becomes

$$\langle n_I V_{Ir}\rangle_\Psi = \left(\frac{T_i}{ZeB_{\theta 0}}\right)^2 \frac{1}{D_\parallel}\left[\begin{array}{l} \varepsilon\left(-\partial_r n_{Ic1} + \dfrac{\partial_r p_i}{p_i} Zn_{Ic1} + \dfrac{m^*\Omega^2 R_0}{T_i} n_{Ic1}\right) + \\ \varepsilon^2\left(-2\partial_r n_{I0} + 2\dfrac{\partial_r p_i}{p_i} Zn_{I0} + 3\dfrac{m^*\Omega^2 R_0}{T_i} n_{I0}\right) + \\ \dfrac{3}{4}\varepsilon^2\left(-\partial_r n_{Ic2} + \dfrac{\partial_r p_i}{p_i} Zn_{Ic2} + \dfrac{5}{3}\dfrac{m^*\Omega^2 R_0}{T_i} n_{Ic2}\right) \end{array}\right] + \\ -\left(\frac{T_i}{ZeB_{\Phi 0}}\right)\left(1-\frac{\varepsilon^2}{2}\right)\frac{m^*\Omega^2 R_0}{T_i}\left(n_{Is1} + \frac{\varepsilon}{2} n_{Is2}\right) \qquad (8)$$

For $\Omega \to 0$ the flux (8) is the Pfirsch-Schlüter result, in that limit $n_{IC1,2} \sim O(D_\parallel^{-2})$. From the sinus and cosines components of equation (7) we find (to the order ε^2)

$$n_{Ic1} = \frac{m^* \Omega^2 R_0^2}{T_i} \varepsilon n_{I0}$$

$$n_{Is1} = -\frac{1}{D_{\parallel}} \frac{T_i B_{\Phi 0}}{ZeB_{\theta 0}^2} \left[\begin{array}{l} \varepsilon R_0 \left(-\partial_r n_{Ic1} + \frac{\partial_r p_i}{p_i} Z n_{Ic1} + \frac{m^* \Omega^2 R_0}{T_i} n_{Ic1} \right) + \\ \varepsilon^2 R_0 \left(-2\partial_r n_{I0} + 2\frac{\partial_r p_i}{p_i} Z n_{I0} + 3\frac{m^* \Omega^2 R_0}{T_i} n_{I0} \right) \end{array} \right] \qquad (9)$$

The first of equations (9) shows that the cosines component of the density is of the same order of the zero component (in terms of the expansion in parallel diffusion coefficient powers), while the impurity temperature is not affected by the centrifugal force and has a much weaker dependence on the poloidal angle; this argument is used to neglect the contribution of the temperature poloidal-gradient in equations (4) and (7). The second harmonic coefficients of the impurity density are found being of the order $n_{IC2} \sim O(D_{\parallel}^{-1}), n_{Is2} \sim O(D_{\parallel}^{-2})$. Finally substituting equations (9) into equation (8) and using

$$n_{I0}(r) = \frac{\langle n_I \rangle_{\Psi}}{\left(1 + \varepsilon^2 \frac{m^* \Omega^2 R_0}{T_i} \right)}$$

derived from the definition of average on the magnetic surface together with the first of equations (9), the flux to the order ε^2 becomes

$$\begin{aligned} \langle n_I V_{Ir} \rangle_{\Psi} &= -D_{\Omega} \left(\partial_r \langle n_I \rangle_{\Psi} - \frac{\partial_r p_i}{p_i} Z \langle n_I \rangle_{\Psi} \right) + P \langle n_I \rangle_{\Psi} \\ &= -D_{\Omega} \partial_r \langle n_I \rangle_{\Psi} + V_p \langle n_I \rangle_{\Psi} \end{aligned} \qquad (10)$$

where

$$D_{\Omega} = D_{PS} f_{\Omega} = D_{PS} \left(1 + \frac{m^* \Omega^2 R_0^2}{2T_i} \right)^2$$

$D_{PS} = 2q^2 D_{\perp} = 2\varepsilon^2 (T_i / ZeB_{\theta 0})^2 D_{\parallel}^{-1}$ is Pfirsch-Schlüter diffusion coefficient and

$$P = D_\Omega (1 - f_\Omega^{-1/2}) \left(\frac{2 + \varepsilon^{-1} + f_\Omega^{1/2}}{R_0} - 2 \frac{\partial}{\partial_r} \ln \Omega + 2 \frac{\partial}{\partial_r} \ln T_i + \frac{m_i}{m^*} \frac{Z T_i^2}{\left(T_e + T_i\right)^2} \frac{\partial}{\partial_r} \left(\frac{T_e}{T_i} \right) \right)$$

3. COMPARISON WITH JET DATA

Typical values for the diffusion coefficient of heavy impurities found at JET by laser ablation experiments in hot ion H-mode plasmas are $1-2m^2s^{-1}$ at the plasma edge (normalised radius r = 0.8-0.9) much smaller values are found toward the plasma centre, pinch velocities of the order of $50ms^{-1}$ are also found at the edge [11]. Hot ion H-mode plasmas are characterised by high neutral beam injected power (typically 20MW in recent pulses) and therefore they present a strong toroidal rotation $\Omega \approx 10^5 rads^{-1}$. The diffusion coefficient D_Ω and pinch velocity V_p have been evaluated for the JET pulse #38684, in which the impurity in-out asymmetry is very strong. The nickel was injected in a hot ion H-mode plasma, the ion temperature was 7keV at r=0.5 plasma normalised radius while the electron temperature was 4keV; Zeff was estimated to be 2.0 and the ion density $7 \times 10^{19} m^{-3}$. The plasma was heated by 17MW neutral beam and its rotational frequency was $\Omega = 1.2 \times 10^5 rads^{-1}$. The diffusion coefficient D_Ω and the pinch velocity obtained using these plasma parameters are showed in Fig. 1 and Fig. 2. The values obtained by modelling the time behaviour of nickel line radiation together with the soft X-ray emissivity for the present shot are of the same order of magnitude [12]. In JET pulse #38684 the asymmetry induced enhancement factor for heavy impurity is of the order of $f_\Omega \approx 20$ and the enhanced transport is expected to take place at the plasma edge where heavy impurities are in the Pfirsh-Schlüter regime of collisionality.

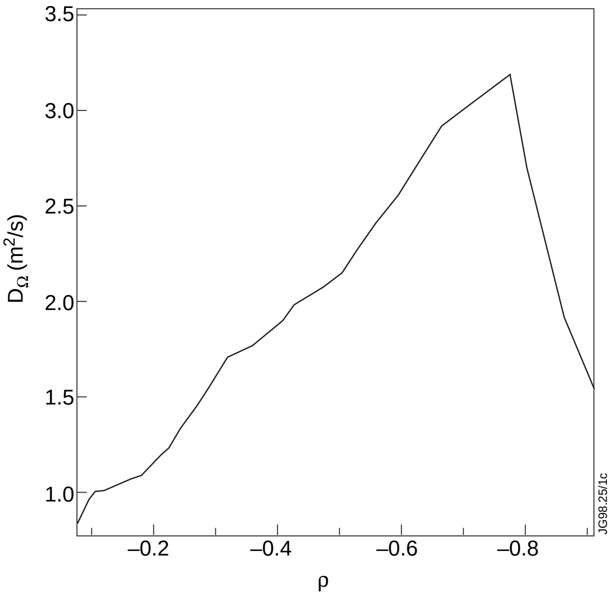

Fig.1: Diffusion coefficient from equation (10) calculated taking the plasma parameters relative to the JET Pulse 38684, time 52.92 s from the start of the discharge.

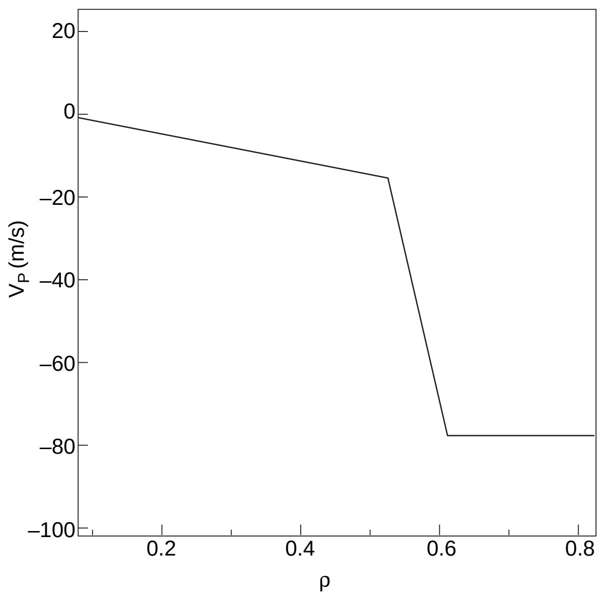

Fig.2: Pinch velocity from equation (10) calculated taking the plasma parameters relative to the JET Pulse 38684, time 52.92 s from the start of the discharge.

4. CONCLUSIONS

We have investigated the role of density asymmetries on the impurity neoclassical flux in the circular symmetric cross section geometry. A two scale ordering of the particles flux in power of the inverse parallel diffusion coefficient $(D_{\|})^{-1}$ and in the inverse aspect ratio ε has been adopted. In the absence of toroidal rotation the poloidal asymmetry is negligibly small due to the large parallel conductivity (the cosines components of the density have coefficients $n_{Ic1,2}(r)$ of order $O(D_{\|}^{-2})$ and therefore they are negligible compared to $n_0(r)$ which is of order $O(D_{\|}^{0})$). For fast plasma rotations the heavy impurity Mach number is greater than one and the asymmetry in the particles density becomes of the same order of the averaged density (the fast rotation brings about a $n_{Ic1}(r)$ of the same order of $n_0(r)$ i.e. $O(D_{\|}^{0})$ and allows strong asymmetries on the magnetic surface to survive); as a consequence of the asymmetry the diffusion coefficient is enhanced to values of the order of $1m^2s^{-1}$ and the pinch velocity becomes very large since the density poloidal asymmetry dominates over the temperature; this leads to an inward pinch of at least $10ms^{-1}$ for heavy impurities. We consider this result a noteworthy conceptual advance. Indeed finding an explanation for an enhancement of the impurity pinch over the standard neoclassical levels of the magnitude required to explain the experiment (typically a factor 10) had previously eluded the theoretical effort - unlike the enhancement of the diffusivity which could be otherwise obtained including a modest amount of electrostatic turbulence. The order of magnitude of the obtained diffusion coefficient and pinch velocity is in the right range needed to simulate the JET data. A detailed numerical comparison of the result against the simulation requires the flux (10) to be written in terms of a general magnetic field and to be implemented in the simulation codes, this is left for future developments.

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