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Optimisation of Support Positions for Fusion In-vessel Components to Minimise Eddy Current Stresses

S Papastergiou, P Ageladarakis.

JET Joint Undertaking, Abingdon, Oxfordshire, OX14 3EA, UK.

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ABSTRACT

Understanding and minimising eddy current stresses is of major importance in the design of Fusion in-vessel components. These stresses determine often the life of components since the frequency of plasma disruptions may be a significant portion of the plasma pulses. Careful choice of the component support geometry can result in reducing stresses considerably. This is particularly true with large copper tiles installed inside the Vacuum Vessel as in the JET Neutral Injection (NI) Ducts. The low copper resistivity results in large eddy currents and particular care has to be exercised to minimise stresses.

The aim of this work is to define the optimum position of component support points to minimise the eddy current stresses. The components are idealised with linear or non-linear beam models depending on the eddy current force distribution. Then the position of the maximum value for the bending moment as a function of the support locations is estimated in order to obtain minimum moments and therefore stresses. ***It is shown that a factor of up to approximately 30 in stresses can be gained by careful choice of support positions. The near optimum location is independent of the component material, orientation, size and type of disruption***. Should there be geometrical difficulties in using the optimum locations, the effect of different support positions on stresses is quantified.

This work has been applied specifically to the design of the JET NI Duct tiles. It resulted in negligible stresses in the inconel support subframe and minimum stresses in the copper tiles. Although the detail analysis restricts itself to in-vessel tiles, the principles are general and the results can be applied to all in-vessel components which normally can be simulated with beam theory.

1. INTRODUCTION

Figure 1 shows the geometry of a typical tile. For relatively large components/tiles (like in the JET NI Duct protection) and in order to be able to take moments in all directions, we need at least a 3-point support system, as indicated. These support locations should incorporate disk springs to allow rotation at these points and minimise the stresses. We should show that by careful definition of the dimension x_1 (Fig. 1), we can minimise the stresses by a factor of at least 24-29, as compared to the worse case of a cantilever/fixed type of support. Such a fixed support can permit thermal expansion but does not allow rotation and resists moments. Figure 1 indicates the geometry of a typical in-vessel tile and different types of supports. The fixed supports result in large stresses while a free support reduces stresses. The stresses in the latter case vary considerably with the position of the support. The aim of this analysis is to define this relationship and to determine the optimum support position in order to obtain minimum stresses.

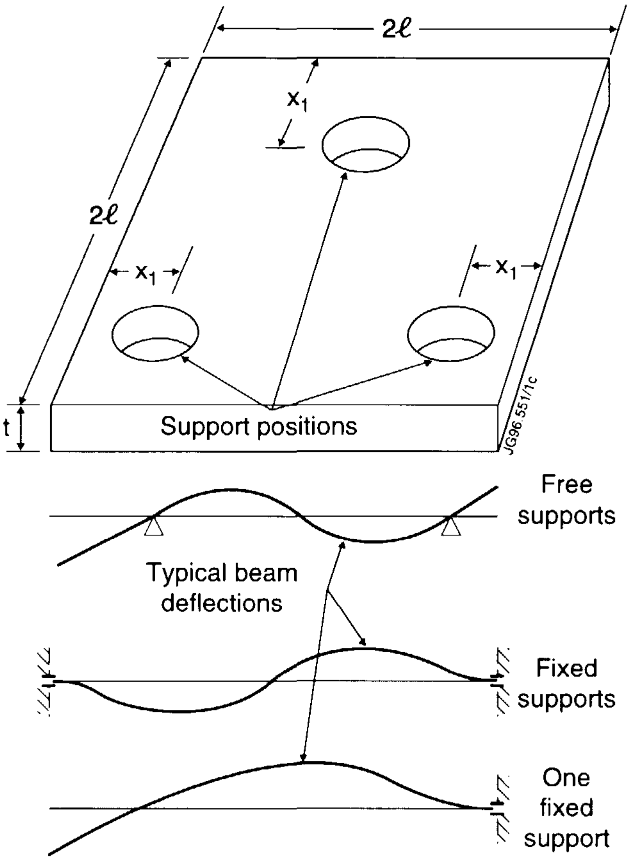

Figure 1. Geometry of a typical tile, different support types and characteristic beam deflections. All support types can permit thermal expansion. Only free supports allow rotation.

2. CALCULATION OF STRESSES

The eddy current forces increase with the size of the tile.

Figure 2 shows the eddy current forces acting on a tile. Depending on the tile geometry these forces may increase with the first, second or third power of the tile characteristic length.

The analysis in this report demonstrates the detailed calculations assuming both a linear and a non-linear (ie, second degree) force distribution. Higher order distributions result in very complicated mathematical expressions. However, an additional simple investigation of the third degree force distribution, gave practical design criteria valid for all cases.

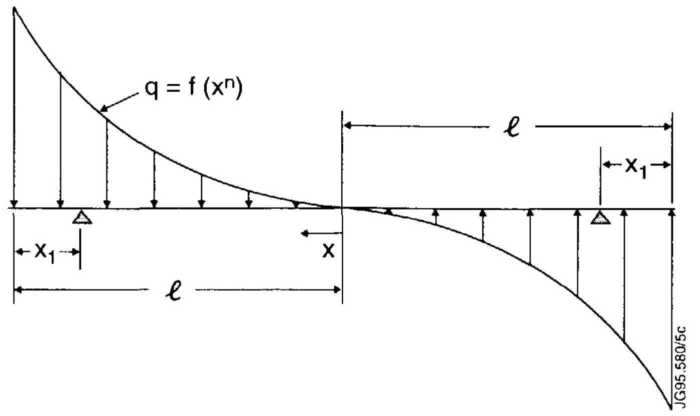

Figure 2. Typical eddy current force distribution.

Figure 3 indicates the idealised beam model used first in the analysis. In this, the tile forces and tile support positions are indicated. The aim of the work is to define x_1 in order to minimise the bending moments. Minimum bending moments result in minimum stresses. An important characteristic of the model is the "centre of gravity" (CG) of the applied forces.

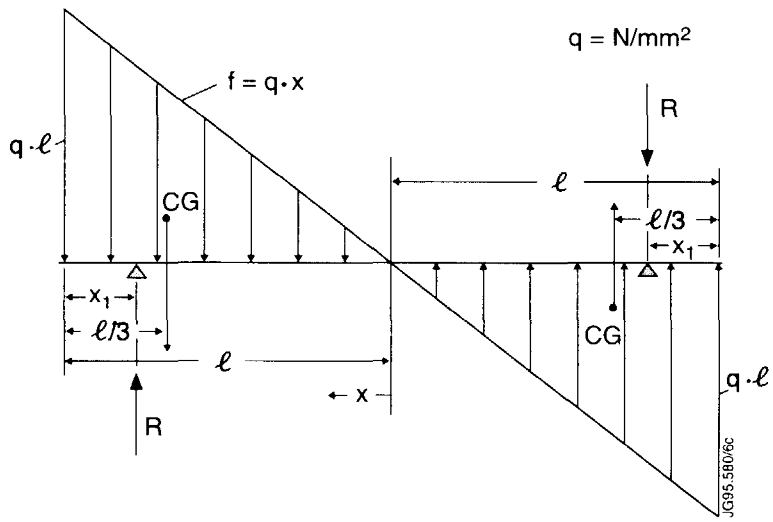

Figure 3. Linear eddy current force distribution.

Figures 4 and 5 illustrate diagrams of shear forces (Q) and bending moments (M_b) depending on the position of the supports (symmetrical around the tile centre) with respect to the CG, which is at $\ell/3$ from the end of the beam* .

* It is almost impossible to formulate relatively simple general equations for Q and M_b and then obtain the min M_b. It is thought that a significant feature of this analysis is to formulate equations for Q, M_b depending on the position of x_1 in relation to the CG. Then the equations $\frac{\partial M_b}{\partial x} = 0$ can be solved and obtain the min M_b.

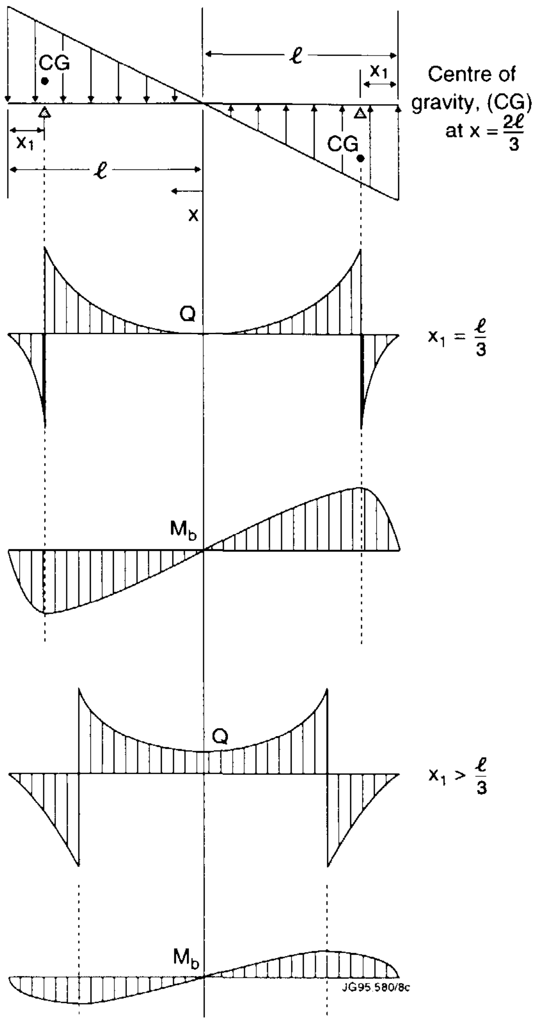

Figure 4. Diagrams of shear forces (Q) and bending moments (M_b) for different support positions.

The gain (stress) factor G, used throughout in this analysis, is defined as the ratio of the applied eddy current moment (M_e) to M_b.

It is clear that if $x_1 \geq \ell/3$, the maximum bending moment is in the support position . This is because always $\frac{\partial M_b}{\partial x} = 0$, when Q = 0; and in order to get $(M_b)_{max}$, Q should equal zero [1]. (However, if $x_1 < \ell/3$, then Q = 0 in two positions, ie: at the support position x_1 and at position x_o (see Fig 5)).

It can be shown that the bending moment at the support is given by the equation:

$$M_b(x_1) = \frac{q\ell^3}{6} - \frac{q\ell^2 x_1}{2} - \frac{q(\ell - x_1)^3}{6} \qquad (1)$$

q is in [N/mm^2]

For $x_1 = \frac{\ell}{2} \Rightarrow M_b = -\frac{5}{48} q\ell^3$

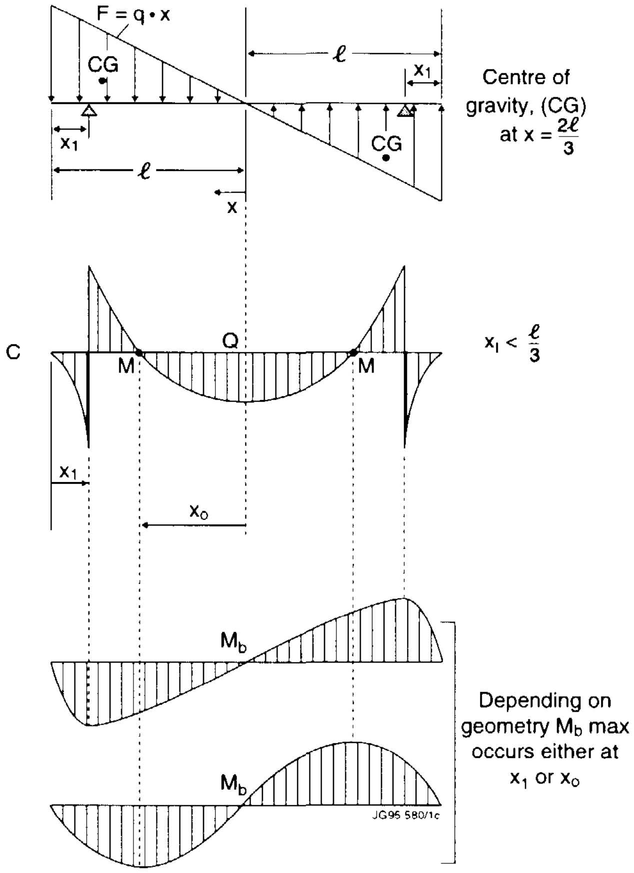

Figure 5. Diagrams of shear forces (Q) and bending moments (M_b), (for $x_l < \ell/3$)

The total eddy current force, acting at the CG in each half of the plate is:

$$F_e = \int_0^\ell F(x)dx = \frac{q\ell^2}{2} \qquad (2)$$

Thus the eddy current moment (M_e) developed on the plate is given by forces equal to F_e acting at the CG positions (separation = 4/3 ℓ)

$$M_e = \frac{2}{3}\, q\ell^3 \qquad (3)$$

[Note that if the tile was cantilevered with one fixed support then $M_e = M_b\,(x_1)$].

The gain factor, G, is always defined as:

$$G = \frac{M_e}{M_b},$$

(G=1 for a cantilever tile with one fixed support; G=2 with two fixed supports)

So, if we support the tile at $\ell/2$, then G = 6.4 for $x_1 = \frac{\ell}{3}$, $M_b = -\frac{4}{81} q\ell^3$ and G = 13.5

If $x_1 < \ell/3$ then we should check the bending moment at x_1 and at x_0 (Fig 5).

For $x < (\ell - x_1)$:

$$M_b(x) = qx\left[\frac{\ell^2}{2} - \frac{\ell^3}{3(\ell - x_1)} - \frac{x^2}{6}\right] \text{ and} \tag{4}$$

$$Q(x) = \frac{q\ell^2}{2} - \frac{q\ell^3}{3(\ell - x_1)} - q\frac{x^2}{2}, \tag{5}$$

The maximum M_b is obtained as follows:

when $Q = 0 \Rightarrow \frac{\partial M_b}{\partial x} = 0$;

which leads to:

$$x_o = \ell\sqrt{1 - \frac{2\ell}{3(\ell - x_1)}} \tag{6}$$

Thus, for

$x_1 = \ell/4 \quad \Rightarrow x_0 = 0.33\,\ell \qquad \Rightarrow G = 23.8$

$x_1 = \ell/5 \quad \Rightarrow x_0 = 0.408\,\ell \qquad \Rightarrow G = 29.3$(optimum position)

$x_1 = \ell/6 \quad \Rightarrow x_0 = 0.447\,\ell \qquad \Rightarrow G = 22.3$

$x_1 = \ell/9 \quad \Rightarrow x_0 = 0.5\,\ell \qquad \Rightarrow G = 16.2$

$x_1 = \ell/20 \quad \Rightarrow x_0 = 0.546\,\ell \qquad \Rightarrow G = 12.3$

The different values of G are calculated as follows. For each position of x_1, we obtain first the maximum $[M_b(x_1,), M_b(x_0)]$ and then compare it with the M_e. Figure 6 illustrates the values of $M_b(x_1,), M_b(x_o)$. It is clear that the minimum M_b is obtained at position $x_1 = \ell/5$. In this case, the factor G is 29 which is the maximum value and the optimum support position. The effect of different support positions on G is shown , ie: G = 23 at $x_1 = \ell/4$

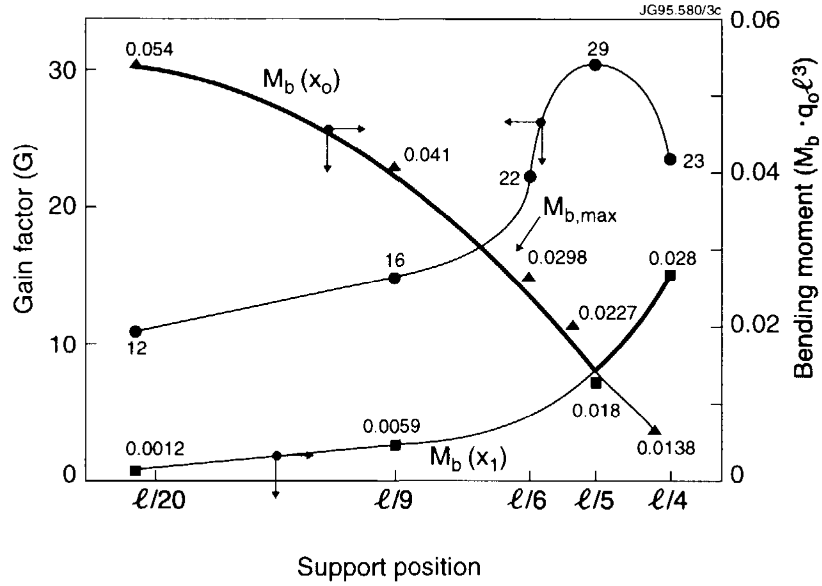

Figure 6. Comparison between the bending moments at the support position x_l and point x_O (where Q=0) in the case of linear force distribution. The gain factort G is also displayed in the diagram.

3. EFFECT OF NON-LINEAR DISTRIBUTION OF EDDY CURRENT FORCES

Figure 7 gives a typical eddy current force distribution as a function of the square of the characteristic tile length ℓ.

The CG of the distribution is at $\ell/4$ from the tile edge. The diagrams of Q and M_b are similar to the ones given at Figures 4, 5; depending on x_l and its relation to $\ell/4$ (ie: not $\ell/3$ as in the linear case).

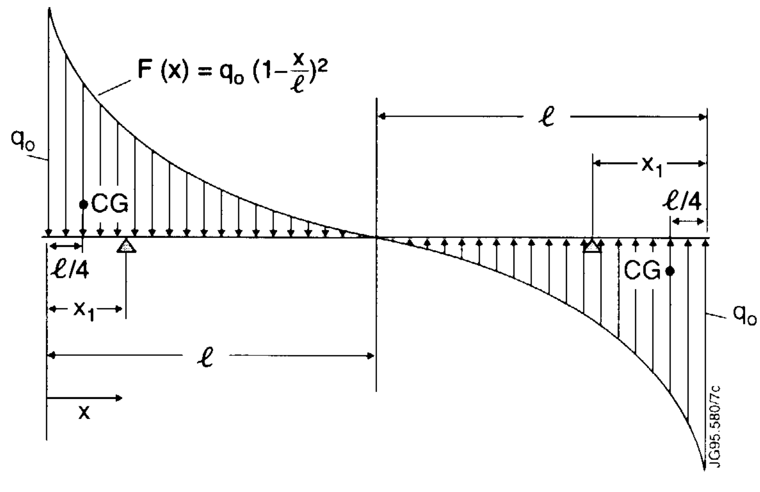

Figure 7. Eddy current force distribution as a function of the square of the characteristic tile length.

The general equations for the shear forces (Q) and bending moments (M_b) for $x \geq x_1$ are given by the following expressions:

$$Q(x) = \frac{q_o \ell^2}{4(\ell - x_1)} - q_o \left(\frac{x^3}{3\ell^2} - \frac{x^2}{\ell} + x \right) \tag{7}$$

$$M_b(x) = \frac{q_o \ell^2 (x - x_1)}{4(\ell - x_1)} - q_o \left(\frac{x^4}{12\ell^2} - \frac{x^3}{3\ell} + \frac{x^2}{2} \right) \tag{8}$$

q_o is in [N/mm] and represents the force at the tile edge; therefore q_o is equivalent to $q\,\ell$.

For $x_1 \geq \ell/4$ the maximum of M_b occurs at $x = x_1$

Thus, for

$$x_1 = \frac{\ell}{4} \Rightarrow M_b = -0.026 \cdot q_o \ell^2$$

$$x_1 = \frac{\ell}{3} \Rightarrow M_b = -0.044 \cdot q_o \ell^2$$

$$x_1 = \frac{\ell}{2} \Rightarrow M_b = -0.088 \cdot q_o \ell^2$$

The total eddy current force, acting at the CG in each half of the plate is:

$$F_e = \frac{q_o \ell}{3} \tag{9}$$

and thus, the eddy current moment M_e, for this case is:

$$M_e = 0.5\, q_o\, \ell^2 \tag{10}$$

Therefore the gain factor G is

at

$$x_1 = \frac{\ell}{4} \Rightarrow G = 19$$

$$x_1 = \frac{\ell}{3} \Rightarrow G = 11.3$$

$$x_1 = \frac{\ell}{2} \Rightarrow G = 5.6$$

For $x_1 < \ell/4$ the maximum M_b occurs either at x_1 or at x_o (see Fig 5). It can be shown that

$$\frac{\partial M_b}{\partial x} = 0; \qquad when\ Q = 0 \Rightarrow$$

$$x_o^3 - (3\ell)x_o^2 + (3\ell^2)x_o - \frac{3\ell^4}{4(\ell - x_1)} = 0 \qquad (11)$$

The real root of this equation is given by [2]:

$$x_o = \ell\left[1 + \frac{\sqrt[3]{(x_1-\ell)^2\,(4x_1-\ell)}}{2^{2/3}\,(\ell - x_1)}\right] \qquad (12)$$

If, $x_1^i = \frac{\ell}{i}$ where $i \geq 5$, then it can be shown that the above equation results in

$$x_o^i = \ell\left[1 + \frac{\sqrt[3]{(\ell - i)^2\,(4 - i)}}{2^{2/3}\,(i - 1)}\right] \qquad (13)$$

Hence, the bending moment at the support position x_1^i is given by:

$$M_b\left(x = x_1^i\right) = q_o\,\ell 2\left[\frac{1}{12i^4} - \frac{1}{3i^3} + \frac{1}{2i^2}\right] \qquad (14)$$

and at position x_o^i

$$M_b\left(x = x_o^i\right) = q_o\,\ell 2\left(\frac{\left(x_o^i - \frac{\ell}{i}\right)}{4\left(\ell - \frac{\ell}{i}\right)}\right) -$$

$$- q_o\left[\frac{\left(x_o^i\right)^4}{12\ell^2} - \frac{\left(x_o^i\right)^3}{3\ell} + \frac{\left(x_o^i\right)^2}{2}\right] \qquad (15)$$

Thus for

$x_1 = \ell/5 \quad \Rightarrow i = 5 \quad \Rightarrow x_o = 0.603\,\ell \quad \Rightarrow G = 29$

$x_1 = \ell/6 \quad \Rightarrow i = 6 \quad \Rightarrow x_o = 0.536\,\ell \quad \Rightarrow G = 41$

(optimum position)

$x_1 = \ell/7 \quad \Rightarrow i = 7 \quad \Rightarrow x_o = 0.499\,\ell \quad \Rightarrow G = 33$

$x_1 = \ell/8 \quad \Rightarrow i = 8 \quad \Rightarrow x_o = 0.477\,\ell \quad \Rightarrow G = 28$

$x_1 = \ell/9 \quad \Rightarrow i = 9 \quad \Rightarrow x_o = 0.461\,\ell \quad \Rightarrow G = 23$

Figure 8 gives the values of the above bending moments together with the values of the gain factor G. It is shown that the best position is at $\sim \ell/6$, where G=41. Again for the calculation of G we use the max of $[M_b(x = x_o^i),\ M_b(x = x_l^i)]$.

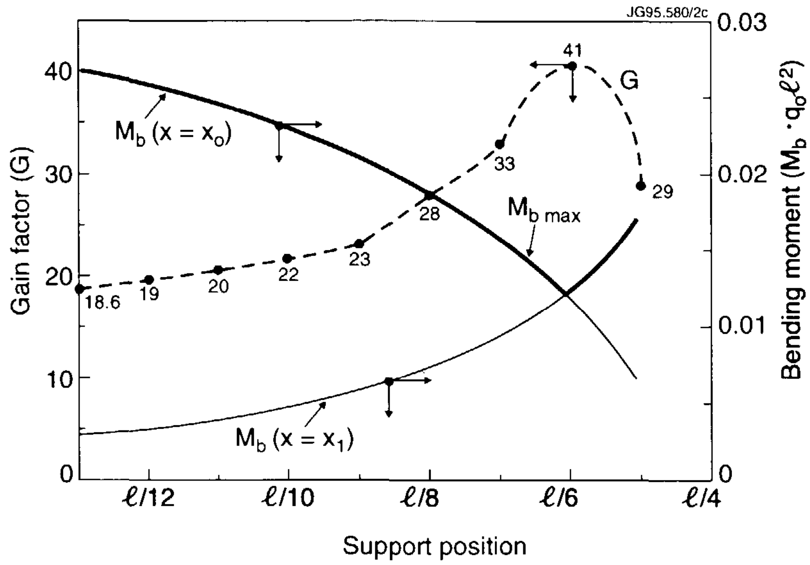

Figure 8. Comparison between the bending moments at the support position x_l and point x_0 (where Q=0), when the force distribution is a function of the square of the tile characteristic length, and $x_l < \ell/3$. The gain factor, G is also displayed for different x_l values.

In the case that the eddy current distribution is a non-linear function of the third degree of the characteristic length '*l*', the results are summarised below:

CG is at $\ell/5$ from the tile edge

$$M_e = \frac{2}{5} q \ell^2 \qquad (16); \qquad\qquad F_e = \frac{q_o \ell}{4} \qquad (17)$$

$x_l = \ell/2$ $\Rightarrow G = 5.2$

$x_l = \ell/3$ $\Rightarrow G = 10$

$x_l = \ell/4$ $\Rightarrow G = 16.4$

$\mathbf{x_l = \ell/5}$ $\Rightarrow$ **G = 24 (optimum position)**

It should be noted that the above analysis does not depend on the time duration of the disruption and its relationship with the natural period of the component. Very fast disruptions can be dealt with impact type of analysis, while long disruptions can be analysed with static calculations (3). However, in all cases, the position of the component support as determined by the above analysis can reduce drastically the eddy current stresses.

4. CONCLUSIONS.

Eddy current stresses on in-vessel components depend on the acting forces and on component support types and positions. The effect of support types and positions on stresses has been quantified.

The near optimum support position of an in-vessel component/tile to minimise eddy current stresses, is at $\ell/5$ from the edge; where ℓ is half of the component length. Such a support should permit rotation. The stress gain factor is approximately 24 to 29 as compared to a cantilever fixed support. This value is irrespective of the component material, size, orientation, eddy current force distribution and type of disruption.

Depending on the exact type of eddy current force distribution one could obtain even better support arrangements. However, positioning the supports at $\ell/5$ from the tile edge, offers a high gain over a wide range of conditions.

NOMENCLATURE

ℓ	=	tile/component characteristic length
CG	=	"centre of gravity" of applied force
F(x)	=	eddy current force at a distance "x"
F_e	=	total eddy current force
G	=	gain factor
M_b	=	bending moment
M_e	=	eddy current moment
Q	=	shear force
q	=	normalised eddy current force distribution (N/mm^2)
R	=	reaction force
t	=	tile thickness
x	=	distance
x_1	=	distance of support position
x_o	=	distance of point where the shear force becomes zero

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