JET-P(95)81

Effect of Temperature Gradient on the Characteristic of Langmuir Probe

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Preprint of a paper to be submitted for publication in
Plasma Physics and Controlled Fusion

December 1995

Abstract. The effect of a electron temperature gradient on the I-V_b (electron current-voltage) characteristic of the Langmuir probe is analysed. We find that $[\frac{d(\ln I)}{edV_b}]^{-1}$ does not simply represent the local electron temperature when the temperature gradient is taken into account. $[\frac{d(\ln I)}{edV_b}]^{-1}$ obtained from the probe measurements corresponds to an effective temperature, which is approximately the electron temperature a mean free path away from the probe, the electron energy for calculating the mean free path being that of electrons which are able to overcome the probe potential. A method of measuring the electron temperature profile using the probe is suggested.

I. INTRODUCTION

Langmuir probe is one of the oldest and simplest diagnostics used for measuring low temperature plasma parameters (Langmuir 1929). One can easily apply a voltage to a probe and measure the current to find the I-V_b (electron current and voltage) characteristic of the probe, from which the electron temperature, floating potential, and density are to be determined. However, the complexity in the interpretation of the I-V_b characteristic of the probe is not generally fully recognised (Stangeby 1989).

Recently it was shown that large modifications of plasma sheath can result when the electron temperature gradient is taken into account (Wesson 1995). This is due to the combined effect of the dominance of fast electrons and the rapid decrease in collision frequency with increasing particle velocity. The sheath is found to be dominated by the distant temperature when the mean free path of a thermal particle is greater than 1% of the length characterising the temperature change.

The purpose of the present work is to examine the effect of the electron temperature gradient on the I-V_b characteristic of the Langmuir probe.

When the probe potential is lower than plasma potential, electrons with low initial energy are repelled by the potential difference. Only the fast electrons with an initial kinetic energy larger than the potential energy are collected by the probe. If the electron temperature is higher away from the probe, high energy electrons in the remote plasma contribute to the probe current, and these electrons have a velocity distribution which is characterised by the electron temperature some distance from the probe. For a given electron velocity the characterising temperature will be that at the position of the last collision. Thus, this temperature is that at a distance of the particle's mean free path from the probe. Since the mean free path increases with the particle velocity, the characterising temperature is a function of the electron velocity.

Thus, the electrons collected by the probe have a velocity distribution quite from different from the local one and have a higher effective temperature.

In Section 2 our model and analysis are presented. The electron saturation current is calculated in Sec. 3. A more general expression for electron current is given in Sec.4. The electron temperature inferred from the I-V_b characteristic is analysed in Sec.5. A method of measuring electron temperature profile is suggested in Sec.6. The effect of potential gradient is given in Section 7, and the summary is presented in Sections 8.

2. Model and analysis

We consider the single langmuir probe, the direction in which the electron temperature varies being denoted by x.

The electron current density at the probe is

$$j = e \int_0^{-\infty} f(0,v)\, v\, dv \qquad (1)$$

where f(0,v) is the electron velocity distribution at the probe position, x=0. Since, in the absence of a collision, f is constant along a particle trajectory, we can approximate f(0,v) by

$$f(0,v) = f[x,v'(x)] \qquad (2)$$

up to the distance of the last collision, the velocity v'(x) being related to the velocity at the probe surface by the energy conservation equation

$$\frac{1}{2} m v^2 - e V_b = \frac{1}{2} m v'^2(x) - e \phi(x) \qquad (3)$$

where $\phi(x)$ is plasma potential at x, V_b is the probe voltage, e is electron charge, and m is electron mass.

In general we need to know T(x), n(x) and $\phi(x)$ for calculating the electron current density j. However the physics will be clearer if we first neglect the role of effect of n(x) and $\phi(x)$ outside the probe sheath. The more general case will be considered later. Thus putting $\phi(x)=V_p$ and substituting eqs.(2) and (3) into eq.(1),

$$j = e \int_{V_c}^{-\infty} f[x,v'(x)]\ v'\ dv' \tag{4}$$

where $V_c=2e(V_p-V_b)^{1/2}/m$, V_p denotes the plasma potential outside the probe sheath, and the electron velocity distribution $f[x,v'(x)]$ is assumed to be a locally non-drifting Maxwellian,

$$f[x,v'(x)] = n_0 \left(\frac{m}{2\pi T_{(x)}}\right)^{1/2} \exp\left(-\frac{mv'^2}{2T_{(x)}}\right) \tag{5}$$

Taking $x=\lambda(v')$, where λ is the mean free path, and noting that (see Wesson (1987))

$$\lambda(v')=4\lambda_0 y^2 \tag{6}$$

where $y=\frac{1}{2}mv'^2/T_0$ and λ_0 being the mean free path at $T=T_0$, we find the electron current

$$I= Aen_0 \left(\frac{T_0}{2m}\right)^{1/2} \int_{\frac{e(V_p-V_b)}{T_0}}^{\infty} \left(\frac{T_0}{T_{(y)}}\right)^{1/2} \exp\left(-\frac{T_0 y}{T_{(y)}}\right) dy \tag{7}$$

where A is the probe area, and $T(y)=T_{(x_{(y)})}$.

When $T(x)$ is constant, $T(x)=T_0$, and eq.(7) leads to the simple expression

$$I = I_s \exp\left(\frac{e(V_b - V_p)}{T_0}\right) \qquad V_b<V_p \tag{8}$$

where

$$I_s=Aen_0 \left(\frac{T_0}{2m}\right)^{1/2} \tag{9}$$

Eq.(8) is the conventional probe theory result.

When $T(x)>T_0$, it is evident from eq.(7) that I is larger than that obtained with $T(x)=T_0$. This is due to the contribution from the remote electrons which have a higher energy. These electrons can overcome the probe potential and be collected by the probe.

In general numerical calculation is needed to find the values of I for a given temperature profile. However, analytical expressions can still be obtained in some limiting cases, and those could be useful for experimental purposes.

3. Electron saturation current

When $V_b=V_p$, the electron current is at its saturation value. Defining L as the scale length of the temperature gradient, and the constant T_∞ as the electron temperature at x>>L, the electron saturation current is found from eq.(7) to be

$$I = I_S \qquad 4\lambda_0 << L \qquad (10)$$

$$I = I_S(\frac{T_\infty}{T_0})^{1/2} \qquad 4\lambda_0 >> L \qquad (11)$$

Comparing with the result obtained in the case of $T(x)=T_0$, eqs.(10) and (11) indicate that the electron saturation current is essentially not affected by the temperature gradient when $4\lambda_0/L << 1$ but is increased by $(\frac{T_\infty}{T_0})^{1/2}$ times when $4\lambda_0/L >> 1$.

The physics of eqs(10) and (11) is clear. When $4\lambda_0/L << 1$, the temperature of the electrons being considered is approximately T_0. Therefore, eq.(1) is the same as that obtained with $T(x)=T_0$. When $4\lambda_0/L >> 1$, the temperature of the remote electrons is increased to T_∞, so that the I of eq.(11) is also increased.

In general, I is given by

$$I = I_S(\frac{T_{eff0}}{T_0})^{1/2} \qquad (12)$$

in the case $V_b=V_p$, where T_{eff0} is defined by

$$(\frac{T_{eff0}}{T_0})^{1/2} = \int_0^\infty (\frac{T_0}{T_{(y)}})^{1/2} \exp(-\frac{T_0 y}{T_{(y)}})\, dy \qquad (13)$$

Obviously, $T_0 \le T_{eff0} \le T_\infty$. $T_{eff0} \approx T_0$ if $4\lambda_0/L<<1$, and $T_{eff0} \approx T_\infty$ if $4\lambda_0/L>>1$. So we see $4\lambda_0/L$ is the key parameter in determining the magnitude of the electron saturation current.

4. Electron current for $V_b<V_p$

Experimentally the probes often work in the range $V_b<V_p$ to repel the electrons and reduce the probe current. From eq.(7) the electron current collected by the probe is

$$I = I_S \exp(\frac{e(V_b - V_p)}{T_0}) \qquad c<<1 \qquad (14)$$

$$I = I_S (\frac{T_\infty}{T_0})^{1/2} \exp(\frac{e(V_b - V_p)}{T_\infty}) \qquad c>>1 \qquad (15)$$

where

$$c = \frac{4\lambda_0}{L}(\frac{e(V_b - V_p)}{T_0})^2 \qquad (16)$$

Eq.(14) is the same as the conventional theory with $T(x)=T_0$. In the limit $c<<1$, the main contribution to the probe current is from the remote electrons that their mean free path is smaller than L, so the temperature of these remote electrons is approximately T_0 and the I given by eq.(14) is only dependent on T_0. On the opposite limit, $c>>1$, the mean free path of the remote electrons that contributing to the probe current is larger than L, and the temperature of the remote electrons is T_∞. In this case the I given by eq.(15) is dependent on T_∞ rather than T_0.

In general, I is given by

$$I = I_S (\frac{T_{eff}}{T_0})^{1/2} \exp(\frac{e(V_b - V_p)}{T_{eff}}) \qquad (17)$$

where T_{eff} is defined by

$$(\frac{T_{eff}}{T_0})^{1/2} \exp(\frac{e(V_b - V_p)}{T_{eff}}) = \int_{\frac{e(V_p - V_b)}{T_0}}^{\infty} (\frac{T_0}{T_{(y)}})^{1/2} \exp(-\frac{T_0 y}{T_{(y)}})\, dy \qquad (18)$$

Obviously, $T_0 \le T_{eff} \le T_\infty$. $T_{eff} \approx T_0$ if $c<<1$, and $T_{eff} \approx T_\infty$ if $c>>1$. So we see that c is the key parameter in determining the magnitude of the electron current for the case $V_b<V_p$.

5. Electron temperature inferred from I-V_b characteristic

From eq.(7) it is found

$$\frac{dI}{dV_b} = I_S (\frac{e}{T_0}) [(\frac{T_0}{T_{(x)}})^{1/2} \exp(\frac{e(V_b - V_p)}{T_{(x)}})]_{x=Lc} \qquad (19)$$

The ratio between I and $\frac{dI}{dV_b}$ gives the information about electron temperature.

Defining

$$T_b \equiv [\frac{d(\ln I)}{edV_b}]^{-1} \tag{20}$$

T_b is conventionally understood as the local electron temperature measured by the probe. When T(x) is constant , $T(x)=T_0$, it is found from eqs.(7) and (19) that $T_b= T_0$.

When electron temperature is increased with x, T_b contains the contributions both from the local electrons and from the remote electrons. Therefore, T_b have different meanings for different plasma parameters.

In the case $V_b=V_p$, it is found from eqs.(10), (11) and (19) that

$$T_b= T_0 \qquad 4\lambda_0/L<<1 \tag{21}$$

$$T_b= (T_\infty T_0)^{1/2} \qquad 4\lambda_0/L>>1 \tag{22}$$

The general expression for the case of $V_b=V_p$ is found from eqs.(12) and (19)

$$T_b= (T_{eff0}T_0)^{1/2} \tag{23}$$

where T_{eff0} is defined in eq.(13).

In the case $V_b<V_p$, it is found from eqs.(14), (15) and (19) that

$$T_b= T_0 \qquad c<<1 \tag{24}$$

$$T_b= T_\infty \qquad c>>1 \tag{25}$$

The general expression for the case of $V_b<V_p$ is found from eqs.(17) and (19)

$$T_b= \{(T_{eff}T_{(x)})^{1/2}\exp[(e(V_b - V_p)(\frac{1}{T_{(x)}}-\frac{1}{T_{eff}})]\}_{x=Lc} \tag{26}$$

where T_{eff} is defined by eq.(18), and c is defined by eq.(16).

Assuming for a plasma T_0=25ev, n=5×10^{19}m^{-3}, the effective Z is Z_{eff}=2, L=1m, and $[e(V_p-V_b)/T_0]$ =6, it is found that $\lambda_0/L\approx4.6\times10^{-2}$, $c\approx6.6$. Then eq.(25) leads to

$$T_b \approx T_\infty$$

However, if T_0=5ev, $[e(V_p-V_b)/T_0]$ =4, and the other parameters are the same as above, it is found $\lambda_0/L\approx0.18\times10^{-2}$, $c\approx$ 0.1. Then eq.(24) indicates

$T_b \approx T_0$

So we see that the meaning of T_b depends on the plasma parameters the probe measured. On the other hand, eqs.(24) and (25) suggest that it is possible to use the probe to measure both the local and the remote electron temperature. When the probe voltage is swept in the region C<<1, $T_b \approx T_0$ is obtained. When the probe voltage is swept in the region C>>1, $T_b \approx T_\infty$ is obtained.

When C>>1 or C<<1, the detailed profile of the electron temperature is not important and we only need to know the value of T_0 and T_∞.

When C~1, the electron temperature profile is needed for calculating T_b. An example of the calculation of T_b as a function of $e(V_p-V_b)/T_0$ is shown in Fig.1 for the temperature profile

$$T = T_0 + (T_\infty - T_0)\tanh(x/L)$$

and $T_\infty=2T_0$, where curves T1 and T2 correspond to $4\lambda_0/L=0.1$ and 0.01 respectively.

6. Mesurement of temperature profile using probe

Eqs.(26) and (18) can be used to calculate the probe electron current for a given electron temperature profile and the value of (V_b-V_p). However, it is a inverse problem experimentally. That is, to obtain the electron temperature from the measured values of the current and voltage of the probe.

It can be found from eq.(19) that

$$T_p \equiv \frac{e\dfrac{dI}{dV_b}}{\dfrac{d^2I}{dV_b^2}} = T_{(x)}\{1+\frac{Lc}{2T_0}[1+\frac{e(V_b-V_p)}{T_{(x)}}]\frac{dT}{dx}\}^{-1}_{x=Lc} \qquad (27)$$

Eq.(27) gives a simple formula for obtaining the electron temperature profile. For a group of measured values of T_p obtained by swept the probe voltage, it is possible to find a particular temperature profile that satisfies eq.(27)

Since the temperature gradient is smaller for higher temperature plasma and the remote plasma has a higher temperature, it is reasonable to assume that the temperature gradient is small at x=Lc and the second term on the right hand side of eq.(27) can be neglected, then eq.(27) is simplified as

$$T_p = T_{(x)}|_{x=Lc} \tag{28}$$

When $T(x)=T_0$ is assumed, it is easily to find that $T_p=T_0$. So that T_p is a constant in this case and does not depend on the values of (V_b-V_p). When $T(x)>T_0$, T_p is varied with (V_b-V_p). T_p is changed from approximately T_0 to T_∞ when (V_b-V_p) changes from zero to large negative values.

7. Effect of potential gradient

For simplicity we assume plasma density is constant. Electron force balance requires

$$n\nabla T = ne\nabla\phi \tag{29}$$

Above equation gives

$$e(\phi(x)-V_p) = T_{(x)} - T_0 \tag{30}$$

Using eq.(3), it is found

$$\frac{1}{2}m\,v^2 - e\,V_b = \frac{1}{2}m\,v'^2(x) - e\,V_p - T_{(x)} + T_0 \tag{31}$$

Defining c1 as

$$c_1 = \frac{4\lambda_0}{L}\left[\frac{e(V_p - V_b)}{T_0} + \frac{T_{(x)}}{T_0} - 1\right]^2_{x=c_1 L} \tag{32}$$

With a derivation similar to previous section and the assumption $8\lambda_0 \frac{dT}{dx} y/T_0 << 1$ for $y > Lc_1/4\lambda_0$, it is found that both I and $\frac{dI}{dV_b}$ obtained in Sec. 4 and 5 are reduced by a factor $\exp(T_0/T_{eff}-1)$. However, the expressions for T_b is not changed. The valid regions of the solution found in Sec. 4 and 5 are modified by replacing c with c_1.

8. Summary

In the present work the effect of a electron temperature gradient on the I-V_b characteristic of the Langmuir probe is analysed. We find that $T_b \equiv [\frac{d(\ln I)}{edV_b}]^{-1}$ does not simply represent the local electron temperature when the temperature gradient is taken into account. T_b obtained from the probe measurements corresponds to an effective temperature, which is approximately the electron temperature a mean free path away from the probe, the electron energy for calculating the mean free path being that of electrons which are able to overcome the probe potential. The meaning of T_b depends on the magnitude of the parameter $c=\frac{4\lambda_0}{L}(\frac{e(V_b - V_p)}{T_0})^2$. In the limit c<<1, $T_b=T_0$. On the opposite limit c>>1, $T_b=T_\infty$. A method of measuring electron temperature profile is suggested.

Acknowledgment

This work is partly supported by the Commission of European Communities under the contract CT1*CT94-0609.

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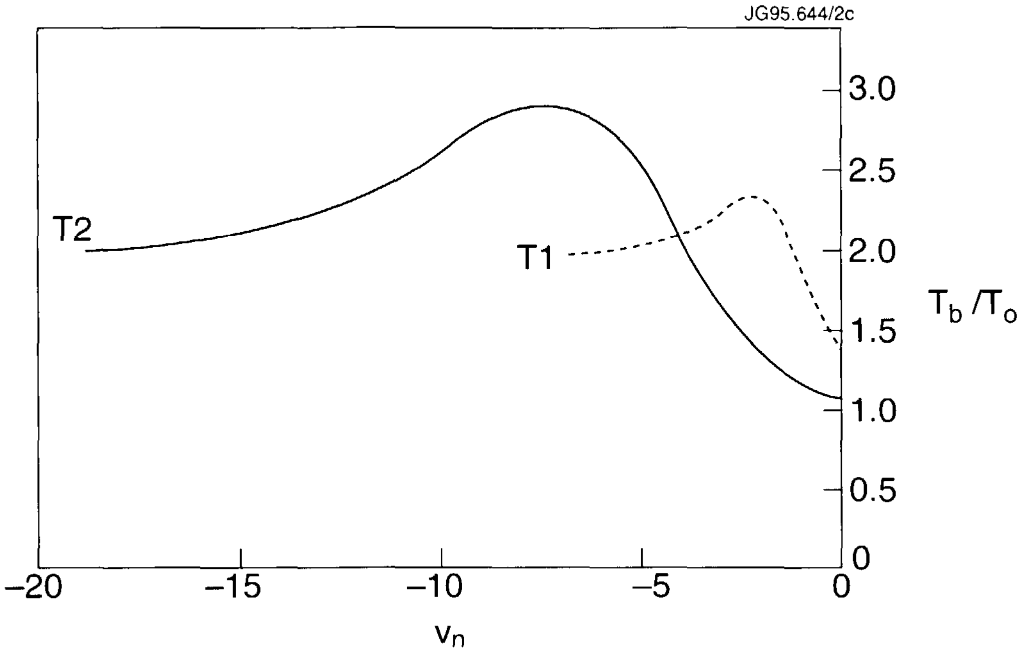

Fig.1: T_b versus $V_n=e(V_b-V_p)/T_0$ for $T_\infty=2T_0$. Curves T1 and T2 correspond to $4\lambda_i/L=0.1$ and 0.01 respectively.