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A Model of the Effect of Plasma Turbulence on Time Delay Measurements by Reflectometry

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ABSTRACT

A generic reflectometer is modelled as a device for launching and receiving radiation at a number of microwave frequencies, uniformly distributed over a specified bandwidth. The effect of density fluctuations is modelled as a phase error at each frequency. The phase distortion has a known correlation length in frequency space, allowing a semi-analytical prediction of the resultant error on the time delay. Specific techniques are compared to the generic model: Frequency sweep, differential phase, AM and sine FM modulation. There is no significant difference between them and the model in their response to the same type of phase error. An implication of the model is that there are situations where profile details are unrecoverable by certain types of reflectometer on a short time scale. These arise when the phase fluctuations are strong and the phase correlation length short, conditions found in the plasma edge.

1. INTRODUCTION

There are a number of measurement techniques available for reflectometry of fusion plasmas. In effect, all of them rely on measuring the group delay of the launched radiation as a function of frequency. This work introduces a generic model to help predict the limitations of reflectometry in plasmas, and to compare the attributes of different techniques. In particular it examines the effect of plasma density fluctuations on the group delay measurement.

The generic reflectometer is modelled by the launch and reception of electromagnetic radiation at N distinct frequencies, uniformly spaced by $\Delta\omega/2\pi$ and extending over a bandwidth B. It is assumed that the phase delay φ_i of each line ω_i is measured independently. In the absence of any noise or plasma fluctuation, the received set of electric field vectors S has a phase $\varphi_i = \omega_i \tau_0$. Without loss of generality τ_0 is set to zero. Each vector has superimposed a noise component N_p leading to a phase error of magnitude $N_p / S\sqrt{2}$. The new group delay τ is then found by a least squares fit of a line $\varphi_0 + \tau\omega$ to the set of points (ω_i, φ_i).

Plasma noise has two distinguishing features compared to noise associated for example with the input stage of a receiver. (1) The noise at each frequency is a fixed proportion of the launched power, provided the line width launched is wider than the typical fluctuation frequency in the plasma, ω_f. (2) The noise at different frequencies is correlated. Experimentally it is found that the frequency extent over which the noise is correlated, here termed correlation length, $\omega_l/2\pi$ varies from <50MHz to several GHz, depending on plasma conditions. The relationship between the phase fluctuation measured at each frequency and the density perturbation at the corresponding cut-off layer is not simple, particularly when $\tilde{\varphi} > 2\pi$. The separation in radius equivalent to ω_l is often smaller than 1-D calculations predict. It is thought (Mazzucato and Nazikian [1]) that 2-D interference effects account for this behaviour.

2. ERRORS INDUCED IN τ BY PLASMA NOISE

For a linear least squares fit, it is well known that the unknown parameters are given by (Matthews [2])

$$a = \begin{bmatrix} \varphi_0 \\ \tau\Delta\omega \end{bmatrix} = \mathrm{M}^{-1}\mathrm{C}\varphi \tag{1}$$

Here j is the set of N phase errors, M is a 2×2 matrix defined by $\mathrm{M}_{\mathrm{lj}} = \sum_{i=1}^{N} f_j(\omega_i) f_l(\omega_i)$, and C a 2×$N$ matrix, $\mathrm{C}_{\mathrm{ji}} = f_j(\omega_i)$, with f_1=1 and $f_2=\omega_i/\Delta\omega$. For uncorrelated errors in the phases

$$\langle \mathrm{a}_1 \mathrm{a}_\mathrm{j} \rangle = \mathrm{M}^{-1}\sigma_\varphi^2 \tag{2}$$

where σ_φ is the standard deviation of the phase. This leads to

$$\sigma_{lj}^2 = \mathrm{M}^{-1}\sigma_\varphi^2 = \begin{pmatrix} N & \frac{N(N+1)}{2} \\ \frac{N(N+1)}{2} & \frac{N(N+1)(2N+1)}{6} \end{pmatrix}^{-1} \sigma_\varphi^2 = \frac{12}{N(N-1)} \begin{pmatrix} \frac{2N+1}{6} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{N+1} \end{pmatrix} \sigma_\varphi^2 \tag{3}$$

It follows that the error in τ is given by

$$\sigma_\tau = \frac{\sigma_{22}}{\Delta\omega} = \frac{\sigma_\varphi}{\Delta\omega}\sqrt{\frac{12}{N(N^2-1)}} \tag{4}$$

The resultant $N^{-1.5}$ scaling of (4) for large N is the same as predicted by the Woodward formula for direct pulse delay measurements. In that case (Minkoff [3]),

$$\sigma_\tau = \frac{1}{\alpha B}\sqrt{\frac{N_0}{E_T}} \tag{5}$$

Here α is a form factor depending on the pulse shape, E_T is the total energy on the pulse, and N_0B is the total power in the noise. Substituting $2\pi E_T = N\Delta\omega S^2$, $2\pi N_0 = N\Delta\omega N_p^2/BT$, where T is the total pulse length we get $\sigma_\tau = \dfrac{N_p}{\alpha BS\sqrt{BT}}$. For a top hat spectrum, as assumed by our model, BT=N and $\alpha = \pi/\sqrt{3}$ so that

$$\sigma_\tau = \frac{2\sqrt{3N_p}}{\Delta\omega S\sqrt{N^3}} = \sqrt{2}\sqrt{\frac{12}{N^3}}\frac{\sigma_\varphi}{\Delta\omega} \tag{6}$$

This result is the same as (4) to within a factor of $\sqrt{2}$, for large N. The difference arises from the contribution both of amplitude and phase noise to the pulse detector scheme used in [3].

When there is correlation between the noise contribution for each frequency, equation (2) becomes [2]

$$\langle a_l a_j \rangle = M^{-1} C \rho M^{-1} C \sigma_\varphi^2 \tag{7}$$

Here ρ is the next *NxN* correlation matrix between the errors. The equivalent of equation (3) now is

$$\frac{\sigma_{lj}^2}{\sigma_\varphi^2} = M^{-1} \sum_{ip} \begin{pmatrix} \rho_{ip} & p\rho_{ip} \\ i\rho_{ip} & ip\rho_{ip} \end{pmatrix} M^{-1} \tag{8}$$

To make further progress, it is necessary to postulate a form for the correlation matrix. A heuristic Gaussian representation is sometimes used [1], so that

$$\rho_{ip} = e^{-\left(\frac{i-p}{\omega_l/\Delta\omega}\right)^2} \tag{9}$$

Evaluating (8) using (9), with $\Delta\omega$ held fixed gives the results shown in fig.1 as contour plots of $\sigma_\tau \Delta\omega$. For any fixed ratio of $2\pi B/\Delta\omega$, the worst case error in the time delay occurs when $\omega_l \approx \pi B$. In 1-D, this corresponds to coherent structures of scale length comparable to the maximum separation of the cut-off layers. Figure 2 shows the case of ω_l fixed. The error is independent of the number of spectral peaks once $\Delta\omega < \omega_l$ for fixed B, but decreases with B. In practice, B will be limited by dispersion in the plasma, since the position of the cut-off is a function of frequency.

3. EQUIVALENCE WITH REFLECTOMETRY SCHEMES

The technique most closely related to this model is the frequency sweep. The frequency is varied between set limits, and the phase is measured at each frequency. In this situation the problem can be reduced to a least squares fit. For a slow sweep, phase errors between adjacent points become completely decorrelated, which means that the error for fixed B and $\Delta\omega$ increases, unless $\omega_l \le 0.35\sqrt{(2\pi B)^3/\Delta\omega}$, an approximate relation derived from fig.1.

Straight pulse detection (as in [3]) is very similar to the frequency sweep. However the equivalent $\Delta\omega$ can be very small. In most cases it will be limited by the fluctuation frequency ω_f, of order 1MHz. Differential phase, in which the phase difference for two adjacent frequencies is measured directly, can trivially be shown exactly equivalent to the model in its response to phase fluctuations, with N=2, so that $\sigma_\tau = \frac{\varphi_u - \varphi_l}{\Delta\omega}$ (u and l are the subscripts for the upper and lower frequency).

In AM reflectometry, the source is amplitude modulated at a frequency ω_m. The time delay is deduced from the phase delay of the modulation envelope. For small phase fluctuations, AM reflectometry is equivalent to the model with N=2 and $\Delta\omega=2\omega_m$. This is because the error in the phase of the modulating frequency is given by $\tan(\sigma_t\omega_m) = \frac{\sin(\varphi_u-\varphi_c)-\sin(\varphi_l-\varphi_c)}{\cos(\varphi_u-\varphi_c)+\cos(\varphi_l-\varphi_c)}$, so that $\sigma_\tau \approx \frac{\varphi_u-\varphi_l}{2\omega_m}$. (Here c is the subscript identifying the carrier.)

Sinusoidal FM modulation (sine FM) is another possibility. For this technique, as yet not tested on a plasma, the carrier is modulated at frequency ω_m, while being swept relatively slowly to produce a density profile. Following Terman [5], the electric field can be written

$$E = E_0 \sin\left((\omega + \omega_{max}\cos(\omega_m t))t\right) = E_0 \sin\left(\omega t + M\sin(\omega_m t)\right) \tag{10}$$

Here M is the modulation index ω_{max}/ω_m. For $M<1$, FM reflectometry is equivalent to AM reflectometry in its response to phase fluctuations (two important sidebands in the launched spectrum).

For large M it is not obvious how to find an analytical representation of the phase error of the modulation envelope. Numerical simulation shows that sine FM is approximately equivalent to the model if one takes $N=2*M+1$ and $\Delta\omega=\omega_m$ (Fig.3).

4. LARGE PHASE FLUCTUATIONS

When the fluctuation level is such that the phase difference between adjacent peaks becomes comparable to π, most reflectometry techniques encounter difficulties. For the simple frequency sweep, phase tracking in frequency fails. For FM and AM modulation and differential phase, the error in the phase of ω_m exceeds π. This leads to failure of phase tracking in time. For pulse compression there are large components above the Nyquist delay, which determines the range of the instrument. The resultant aliasing renders averaging meaningless. Straight pulse detection suffers only from peak broadening, so that the average position of the peak is in principle recoverable by averaging. These effects reconcile reflectometry methods that have $\Delta\omega \propto B$, such as differential phase (Hanson [5]) with the model. Errors in the profile for those systems appear to decrease with B. This is because of their sensitivity to phase errors between frequency peaks. For fluctuations with short correlation length, obtaining a reasonable estimate of the time delay can take excessive amounts of time, unless $B>>\omega_l$: Taking $\Delta\omega = \omega_\ell = 50$MHz and $\sigma_\phi = \pi/4$ gives $\sigma_\tau = 2.1$ns for a differential phase system (left axis of fig.2). An error of 0.05ns (7.5mm in vacuum for a mirror) then requires ~$1.8*10^3$ independent measurements, which for a 10μs correlation time for the fluctuations corresponds to T=18ms. Although T could be reduced to ~50μs by increasing the bandwidth from 50MHz to 2GHz, a typical optimum value for O-mode systems (Laviron et al [16]), phase tracking would be compromised. Increasing N to 40 (by going to an FM system) from this point eliminates phase tracking problems and gives $\sigma_\tau = 0.05\text{ns}$. Another consideration is the dependence on the correlation function. One can construct a function for which there is always an advantage in *decreasing* the bandwidth. For the family $\rho_{ip} = e^{-\left(\frac{i-p}{\omega_l/\Delta\omega}\right)^m}$, m integer, $\sigma_\tau \to 0$ when $B >> \omega_l$ *and* when $B << \omega_l$ but only for m>2. However as $\Delta\omega \to 0$ it is a requirement for a homogeneous fluctuation field that (1-ρ_{ip}) must tend to zero as $\Delta\omega^2$. (Iwama et al, [7]). This is satisfied for m=2 (Gaussian). It follows that for a sufficiently small B all correlation functions of homogeneous fields behave like a Gaussian so there will certainly not be any advantage in decreasing B further. Of course, phase errors are not necessarily homogeneous so there may be some gain in reducing the bandwidth in certain regions (for example to avoid averaging over one or more magnetic islands). This could become a second parameter in an adaptive reflectometer system, that not only optimises the reflectometer parameters for the density gradient, (Doyle et al [8]) but also for the fluctuation level characteristics.

CONCLUSIONS

Subject to constraints imposed by the plasma profile, in the presence of homogeneous phase fluctuations produced by fluctuations in the plasma density, it is a desirable feature of any reflectometer to maximise the bandwidth covered for the production of each time delay point. For bandwidths exceeding the correlation length of the noise, there is further advantage in increasing the density of coverage. Reflectometry techniques such as differential phase cannot comply with the consequent requirement of simultaneous good phase tracking and maximum signal to noise in the time delay. Only techniques with variable spectral density coverage, such all types of FM and pulse compression have the requisite flexibility to adapt to the characteristics of the noise.

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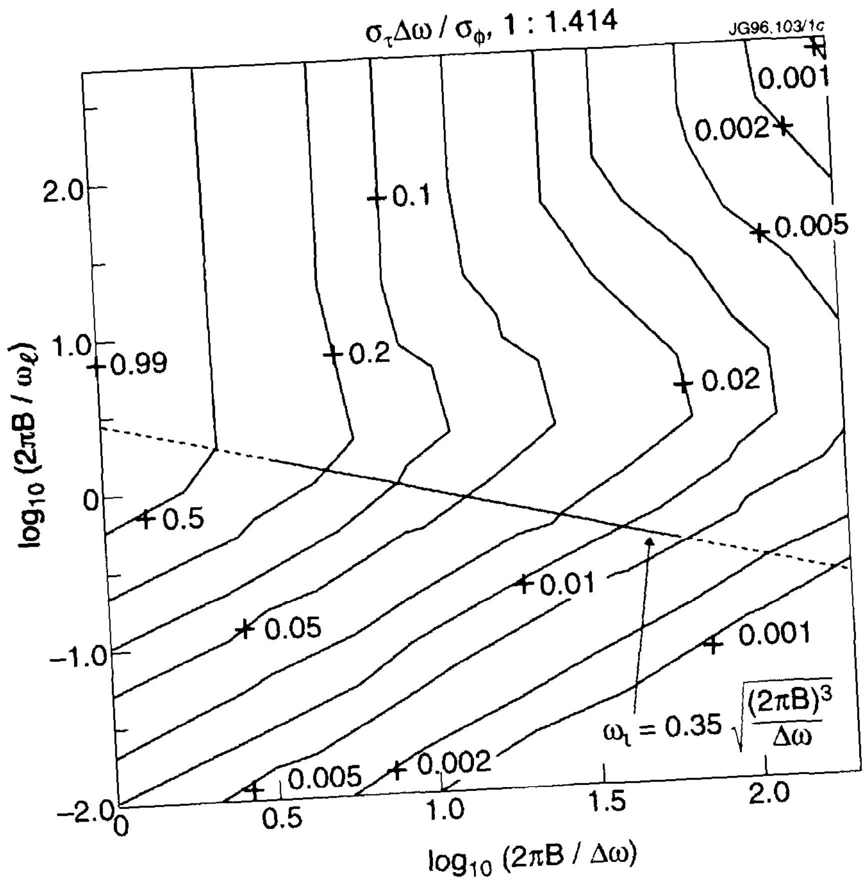

Fig.1: Contour plot of σ_τ for fixed $\Delta\omega$.

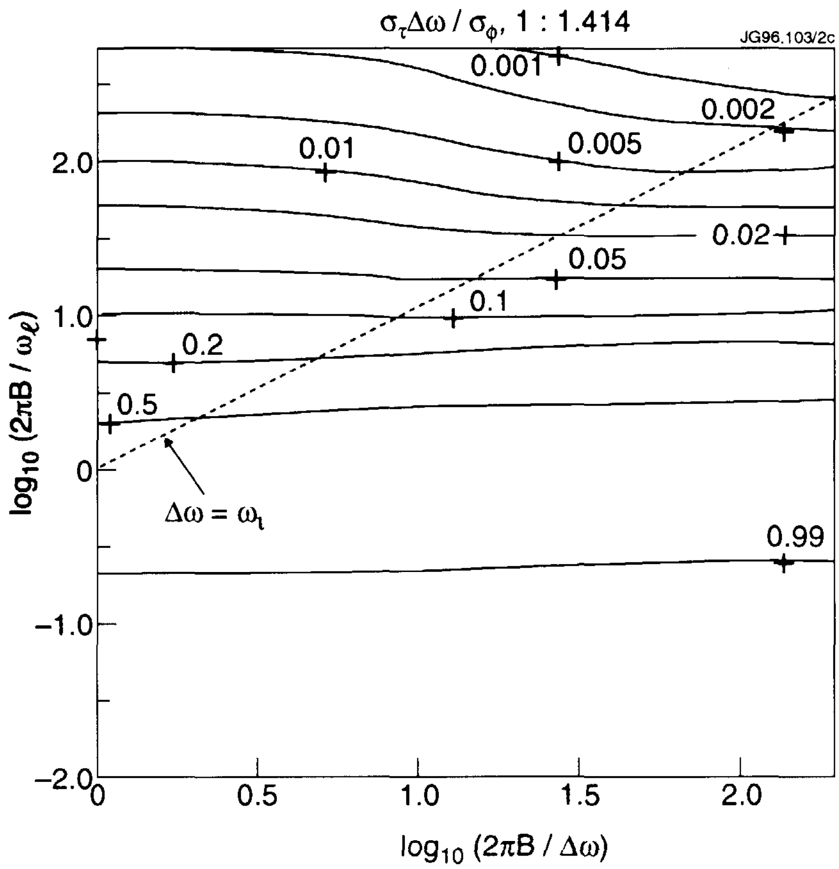

Fig.2: Contour plot of σ_τ for fixed ω_l.

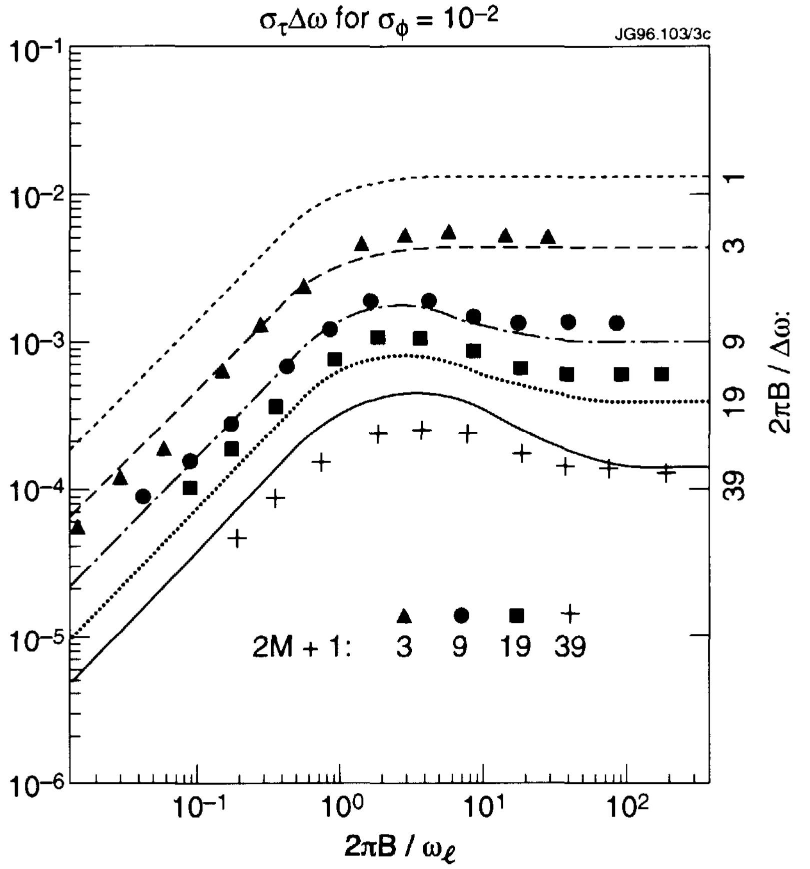

Fig.3: Comparison of model (lines) with numerical simulations of FM modulation (points, with standard deviation ±5%).