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Snakes

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The injection of pellets into JET sometimes leaves a resonant localised structure commonly known as a snake. Snakes constitute a remarkable phenomenon, having both an intrinsic interest and a relevance to understanding transport. How are snakes formed? What maintains the magnetic island created by the snake? How does the confined density persist? And finally, why don't such structures arise spontaneously?

1. The experimental observations

When high speed deuterium pellets are injected into JET and reach the $q = 1$ surface a persistent modification of the equilibrium frequently occurs. This behaviour was first detected through its effect on the soft X-ray emission [1]. Typical signals measured by an array of soft X-ray cameras are reproduced in the time-space graph of figure 1. The snake-like appearance of these traces has given the phenomenon its name.

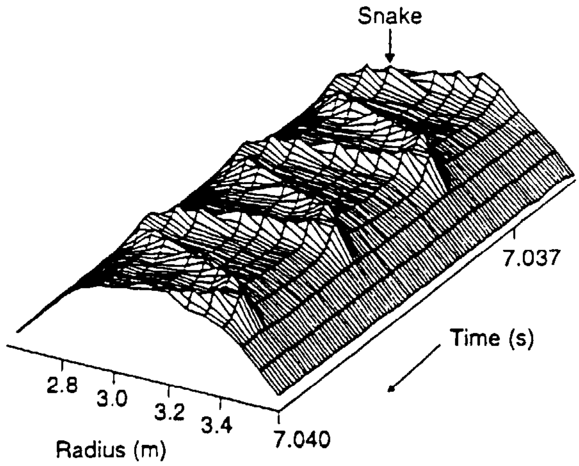

Figure 1. Typical snake as seen by the soft X-ray camera. The observed pattern is formed by a small region of localised soft X-ray emission rotating under the field of view [2]. .

A tomographic reconstruction of the soft X-ray signals, as illustrated in figure 2, shows the localised nature of the perturbation in a given poloidal cross-section. The region of enhanced emission appears to be concentrated at the $q = 1$ surface, and its $m = 1$, $n = 1$ structure enforces this view. The label $m = 1$ here does not refer to a Fourier component but just means that there is one peak in poloidal angle. The soft X-ray perturbations are typically localised to ~ 10% of the poloidal circumference of the $q = 1$ surface. The radial width is somewhat less than the poloidal width. The form of an $m = 1$, $n = 1$ perturbation on a toroidal surface is that of a tilted and displaced circle [3] as illustrated in figure 3.

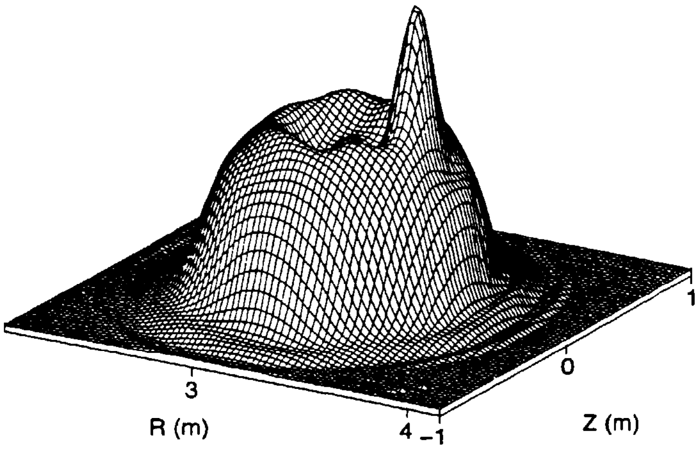

Figure 2. Tomographic reconstruction of the multi-channel soft X-ray signals showing the enhanced radiation from the snake.

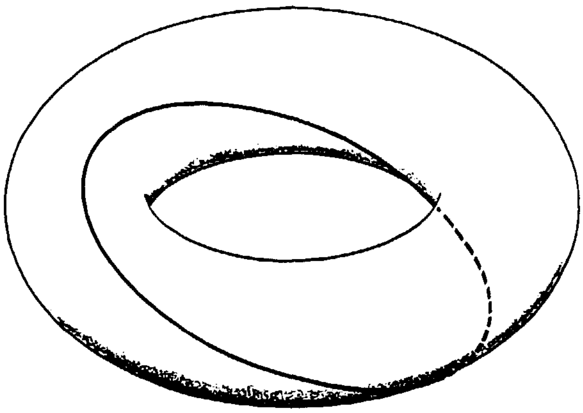

Figure 3. Showing that the snake has the geometry of a tilted and displaced circle.

There is an initial drop in temperature inside the snake but this decays away over tens of milliseconds leaving a density perturbation which can be of the same magnitude as the background density. This density perturbation sometimes persists as long as the observation time, up to 2 seconds. Furthermore the snake does not decay as would be expected. In fact in some cases it actually grows, as seen from figure 4. This points to the conclusion that, once formed, the snakes are essentially permanent.

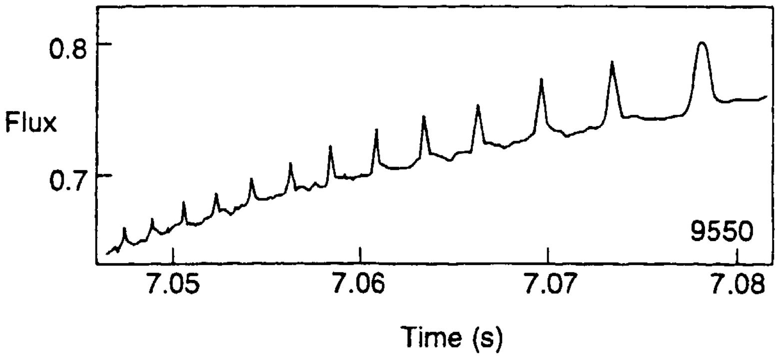

Figure 4. Showing the growth of the soft X-ray flux after the initial formation of the snake [2].

2. An overview

The basic interpretation of the observations is as follows. The pellet crosses the plasma radius in ~ 1 ms. As it does so it is evaporated, and the interaction with the plasma causes a cooling on each flux surface. However on rational surfaces the direct cooling is restricted to those field lines intersected by the pellet's trajectory. The lower thermal capacity of these regions means that they are more deeply cooled. This effect is strongest on the $q = 1$ surface where just a single field line transit leads to a rejoining of the field line. The cooling process is quite complicated involving thermal conduction, sound propagation and thermoelectric effects. However this does not need to be dealt with here.

The magnetic response to the cooling is also very complicated, partly because of the effect of the sheared field geometry on the cooling and partly because of the competition of thermal and electromagnetic time-scales. However, again the details of the transient behaviour are not necessary for an understanding of the snake itself.

The basic mechanism of the snake formation is that the localised cooling on the $q = 1$ surface increases the local resistivity, causing a drop in current density. As this occurs on a rational surface it leads to the formation of a magnetic island, and this magnetic island traps some of the particles released from the pellet, thus forming the snake.

We would now expect the snake to decay away. The initial fall in temperature which drove the island growth decays on a time-scale of tens of milliseconds. Furthermore, even if the island were maintained, the density perturbation would be expected to diffuse away. In many cases it does not. Since even the neoclassical loss time is much shorter than the existence time, the persistence of the snake cannot be understood in terms solely of good confinement.

Finally, if permanent perturbations of the snake type are natural to a tokamak plasma, what prevents the spontaneous growth of such structures?

It is surprising that little serious attention has been given to these problems, given that they introduce fundamental questions of plasma physics. More specifically we are concerned with the subject of confinement which is poorly understood and where insight should be sought wherever it can be found.

In what follows a set of suggested solutions to the above problems is proposed. While it is unlikely that they will all stand the test of time, at least they offer a framework for discussion.

3. Formation of the snake

The formation of the snake requires the formation of a magnetic island at the rational surface to prevent the spreading of the material of the pellet over the surface. This clearly requires that the island should grow sufficiently fast.

The island growth is driven by the local plasma cooling which is geometrically complicated. The immediate response involves the formation of a skin current around the cooled region and the diffusion of this current. However it is possible to obtain a simple description which avoids these complications, and this is outlined below.

The helical flux function, ψ, is defined by

$$B_\theta(1-q) = \frac{\partial\psi}{\partial r} \qquad B_r = -\frac{1}{r}\frac{\partial\psi}{\partial\theta}$$

and the resistive diffusion is described by

$$\frac{\partial\psi}{\partial t} = \eta j \quad . \tag{1}$$

Taking the radial variation to be dominant, equation (1) can be integrated across the island width w to give the early growth

$$\frac{d\tilde{\psi}}{dt} = \langle\tilde{\eta}\rangle j_o \tag{2}$$

where $\tilde{\psi}$ is the time dependent part of ψ, $\langle\tilde{\eta}\rangle$ is the radially averaged resistivity perturbation within the islands, and j_0 is the unperturbed current density.

Equation (2) is transformed to an equation for the island growth using the geometric relation for the island

$$w = 4\left(\frac{q\tilde{\psi}}{q' B_\theta}\right)^{1/2}$$

to give

$$\frac{dw}{dt} = \frac{8\langle\tilde{\eta}\rangle j_o}{(q'/q)B_\theta}\frac{1}{w} \quad . \tag{3}$$

Taking $B_\theta \approx \mu_o j_o r/2$, the island width is given by

$$w = 4\left(\frac{2\langle\tilde{\eta}\rangle t}{(rq'/q)\mu_o}\right)^{1/2} \quad . \tag{4}$$

The time for spreading of the density over an angle θ at a distance w/2 from the rational surface is

$$\tau = \frac{R}{v_{Ti}}\frac{r\Delta\theta}{w}\frac{q^2}{rq'} \tag{5}$$

where R is the major radius and v_{Ti} is the ion thermal velocity. Substitution of $\tau = t$ from equation (5) into equation (4) gives a requirement for trapping of the density in the island

$$\frac{\langle\tilde{\eta}\rangle}{v_{Ti}} \geq \frac{\mu_o(rq'/q)^2}{32q\Delta\theta}\frac{w^3}{rR} \quad . \tag{6}$$

It is seen from inequality (6) that density trapping will occur at early times while w is small, and be halted at some critical island width. The quantities $\langle\tilde{\eta}\rangle$ and v_{Ti} are in reality time dependent but relation (6) allows a rough estimate of the requirement for density trapping. Using typical JET conditions and assuming a parabolic q-profile with $q_o = 0.75$, it is found that trapping up to an island width of 0.1m, with $r\Delta\theta \simeq w$, requires that the temperature be reduced to ~ 70 eV. If the shear were an order of magnitude lower, as indicated by the analysis by Gill [4], the required temperature would be ~700 eV. The actual temperature during the pellet ablation is not known but adiabatic cooling would produce an order of magnitude temperature fall within the island.

4. Sustainment of the magnetic island

Since the temperature perturbation within the snake decays away while the snake itself persists, another mechanism is required to explain the sustainment of the magnetic island. One possible mechanism is the accumulation of impurity ions within the snake.

In the frame of the plasma ions the large pressure gradient in the snake is balanced by the force of an electric field across its "minor radius". Impurity ions with a charge Ze will see this field and will be subject to a radial force which is Z time larger than that on the plasma ions. The force will be diminished by the effect of any relative velocity between the plasma ions and impurity ions. Since this depends on uncertain anomalous transport processes we introduce a factor ϕ to allow for this.

Thus, the force balance equations for the plasma ions and the impurity ions, with densities n_H and n_Z, are

$$T\nabla n_H = n_H e\underline{E}$$

and

$$T\nabla n_Z = \phi n_Z Ze\underline{E}$$

so that

$$\frac{n_Z}{n_{Zo}} = \left(\frac{n_H}{n_{Ho}}\right)^{\phi Z} .$$

where the subscript zero refers to values in the background plasma.

Taking, for example, a snake density $n_e = 1.5\ n_{eo}$ and an oxygen impurity with $\phi = 1$, we obtain $n_Z/n_{Zo} = 5$. If the background impurity level n_{Zo}/n_o were 1% this would produce a resistivity enhancement over the background plasma of a factor 1.8.

An estimate of the resulting island size can be obtained by balancing the drive term, given in equation (3), with the usual stabilising term $(\eta_o/\mu_o)\,\Delta'$ with $\Delta' \approx 2k_\theta$ where k_θ is the effective poloidal wave-number. The resulting island width is

$$k_\theta w = 8\frac{\langle\tilde{\eta}\rangle}{\eta_o}\left(\frac{q}{rq'}\right) .$$

The observed value of $k_\theta w$ is of order one and consequently, for $(rq'/q) \sim 1/2$, a resistivity enhancement of ~ 10% would be adequate to sustain the magnetic island. The enhancement factor of 1.8 obtained in the numerical example above is seen therefore to be more than adequate, and this perhaps indicates that the impurity density gradient factor ϕ is less than one.

5. Sustainment of the density

At first sight the persistence of the density perturbation in the snake in the presence of the very large local pressure gradient seems to be explained by good confinement. However the life of the snake exceeds even the neoclassical confinement time, and it appears that if the discharge were permanent, the snake would be also.

The neoclassical confinement time is given by

$$\tau_{neo} = \left(\frac{w_s}{w_b}\right)^2 \tau_{coll} \frac{1}{f}$$

where w_s and w_b are the snake and electron banana widths, τ_{coll} is the electron collision time and f is the fraction of trapped particles. For a typical case, $w_s = 0.15$m, $w_b = 0.6$mm, $\tau_{coll} = 2\ \mu$s and $f = 1/5$, giving a confinement time, $\tau_{neo} = 0.6$s, to be compared with life times of ~ 2s and decay times which appear to be effectively infinite.

It is seen therefore that an explanation of the persistence of the density perturbation is called for. This has to be in terms of a density source.

A possible source is the particle pinch. It is believed that such a pinch operates in tokamaks and it is often modelled by the form

$$v_p = -\alpha r \quad .$$

This constitutes a source $S = -\nabla \cdot n\underline{v}_p$, that is

$$S = 2\alpha n \quad .$$

Since this source is operative in the normal plasma it is possible to estimate the diffusion coefficient required to explain the snake by a simple comparison with the whole plasma. Thus in both cases the source balances the diffusion and equating the diffusion for the two cases

$$\frac{D_s}{(w/2)^2} n_s = \frac{D_a}{a^2} \hat{n}$$

where D_s and D_a are the diffusion coefficients characterising the snake and the bulk plasma, n_s and $\hat{n}$ are the enhancement of the density in the snake and the peak density in the plasma, and w and a are the width of the snake and the radius of the plasma. Thus

$$\frac{D_s}{D_a} = \frac{\hat{n}}{n_s}\left(\frac{w}{2a}\right)^2 \quad .$$

Typically $w/2a = 0.1$ and $\hat{n}/n_s = 2$, so that we require

$$\frac{D_s}{D_a} \sim \frac{1}{50} \quad . \qquad (7)$$

Since D_a is typically greater than the neoclassical diffusion coefficient by a factor $\sim (m_i/m_e)^{1/2}$, relation (7) indicates a diffusion coefficient of the order of the neoclassical value.

An alternative possibility is that the snake is subject to its own pinch effect, operating across its minor radius. If such a pinch were dominant, the required diffusion coefficient in the snake would be correspondingly increased.

6. Absence of spontaneous† snakes

Since the snake, once formed, seems to be a natural and permanent feature of the equilibrium, the question arises as to why such structures are not seen as part of normal plasmas.

To understand the general behaviour we examine the time trajectories in the space (n_s,w) of the snake density perturbation and island width. The governing equations take the form

$$\frac{dw}{dt} = \eta_o\left(\beta\frac{n_s^\gamma}{w} - C\right) \tag{8}$$

$$\frac{dn_s}{dt} = S - \lambda\frac{n_s}{w^2} \quad . \tag{9}$$

The first term on the right hand side of equation (8) describes the drive resulting from the impurity concentration, and C represents the stabilising effect of line bending. In equation (9) S represents the source, possibly due to the pinch, and $\lambda n_s/w^2$ is the loss of density due to diffusion out of the snake.

Figure 5 illustrates the behaviour schematically. The two lines represent the zeros of dw/dt and dn_s/dt and the dots represent equilibria. The arrows show the direction of the time trajectories and it is seen that small density and island size perturbations decay into the origin. This explains the absence of spontaneous snakes. The diagram has no stable point and it is clear that some further non-linear effects must be included. Examples would be the removal of the thin island assumption and non-linear transport within the island. There is no value at present in including such uncertain effects, but it is reasonable to assume that there will indeed be a stable saturated state since this is what experiment shows. Including such a point in the (n_s,w) diagram leads to a trajectory flow diagram of the type illustrated in figure 6. The dashed line shows the critical boundary to the left of which perturbations decay to zero and to the right of which the state moves to the stable snake equilibrium.

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† *It should be noted that in the paper by Gill et al. [2], the adjective spontaneous is used with a different meaning from that used here. There it means "not induced by a pellet", the snakes being produced by the sawtooth instability. Here spontaneous means "arising without an immediate cause".*

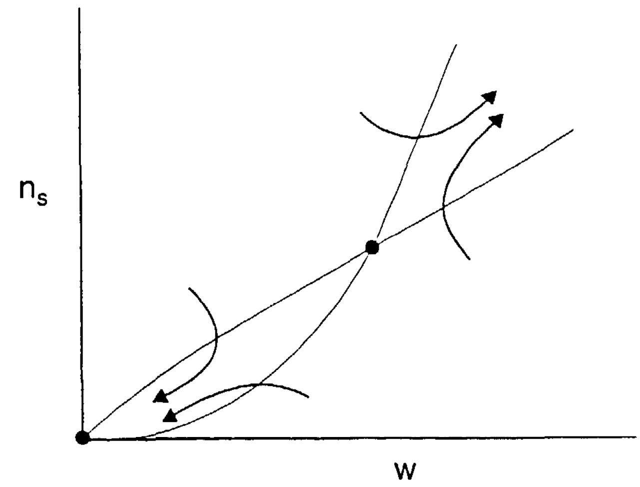

Figure 5. Time trajectories in the space of snake density and island width, showing the decay of possible snakes with small n_S and w.

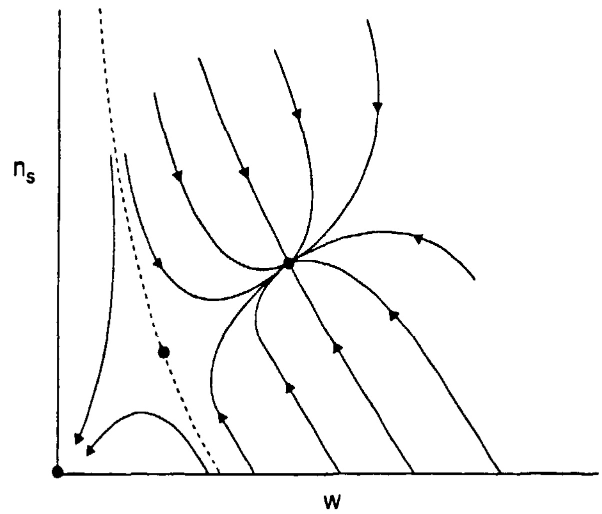

Figure 6. Showing the behaviour of snakes on the two sides of the critical line in (n_S,w) space. Above the critical line the state moves to a stable snake-like equilibrium.

7. Summary

The localised and persistent density concentration following pellet injection has been analysed. The formation is attributed to the growth of a magnetic island at the q = 1 surface, driven by the localised cooling. This island traps some of the ionised pellet material, but the long life of the density perturbation cannot be explained by this initial trapping. It would be expected that on the observed timescale the density perturbation and the magnetic island would decay away. The suggested explanation of their persistence is in terms of two processes which support each other. On the one hand the density perturbation increases the resistivity through impurity concentration, and this maintains the magnetic island. And on the other hand the magnetic island affects the local confinement in such a way as to increase the density. A possible source for the background density perturbation is the divergence of the pinch velocity.

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References

[1] Weller, A. et al. 1987, Phys. Rev. Letts. **59** 2303.

[2] Gill, R.D. et al. 1992, Nuc. Fus. **32** 723.

[3] Such circles are called the Circles of Villarceau after the astronomer and mathematician Yvon Villarceau (1813-1883).

[4] Gill, R.D, et al., 1989, Nuc. Fus. **29** 821.