

Alfvén Eigenmode Induced Energetic Particle Transport in JET

L C Appel¹, H L Berk², D Borba, B Breizman², T C Hender¹,
G T A Huysmans, W Kerner, M Pekker², S D Pinches³,
S E Sharapov⁴.

JET Joint Undertaking, Abingdon, Oxfordshire, OX14 3EA, UK.

¹ UKAEA/Euratom Fusion Association, Culham, Abingdon, Oxfordshire, UK.

² IFS, University of Texas at Austin, Texas 78712, USA.

³ University of Nottingham, Nottingham, UK.

⁴ Visiting from RRC Kurchatov Institute, Moscow, Russia.

"This document is intended for publication in the open literature. It is made available on the understanding that it may not be further circulated and extracts may not be published prior to publication of the original, without the consent of the Publications Officer, JET Joint Undertaking, Abingdon, Oxon, OX14 3EA, UK".

"Enquiries about Copyright and reproduction should be addressed to the Publications Officer, JET Joint Undertaking, Abingdon, Oxon, OX14 3EA".

ABSTRACT

A Hamiltonian guiding centre particle following code has been developed to study the fast particle motion in the presence of arbitrary time-dependent electromagnetic perturbations. In conjunction with an MHD stability code, this code was used to analyse TAE/KTAE-induced α -orbit diffusion and α -losses in JET plasmas. Resonant α -orbits are studied below and above the stochasticity thresholds, in the presence of single or several TAE and KTAE modes. Monte Carlo randomised ensembles of α -particles in the presence of finite-amplitude TAE/KTAE modes are followed and estimates for the stochastic diffusion coefficients are obtained. Generalisation of the method towards the self-consistent wave-particle evolution is described.

1. INTRODUCTION

A key issue for a burning plasma in a tokamak-reactor is the transport of fusion-born α -particles, since this will determine the plasma heating profile, the magnitude of the α -losses and the distribution of the Helium ash. It has long been recognised that the α -particle transport may be strongly affected by α -driven, kinetic plasma instabilities and, in particular, by the Toroidal Alfvén Eigenmode (TAE) instability [1, 2]. Experiments with neutral beam injection [3, 4] and ion cyclotron heating (ICRH) [5, 6] have confirmed that such Alfvén instabilities can indeed be driven by energetic particles and can lead to anomalous transport and losses of energetic particles.

The analysis of TAE - induced transport of α -particles should take into account the fact that the gradient-B and curvature drifts of highly energetic α -particles are significantly larger than their $\bar{\mathbf{E}} \times \bar{\mathbf{B}}$ drift and consequently, their motion has to be analysed in terms of an orbit approach. The anomalous α -particle transport can thereby be studied in an extended Monte Carlo simulation showing stochastic diffusion in the parameter space given by the constants of motion. A numerical algorithm has been developed for the TAE-induced α -particle losses in Ref. [2] based on a guiding centre Hamiltonian formalism for the description of α -orbit motion in the presence of resonant Alfvén perturbations with $\omega_A \ll \omega_{B\alpha}$, where $\omega_{B\alpha}$ is the gyrofrequency of α -particles. It was shown in [2], that the constants of motion (P_ϕ , E , μ ; $\sigma \equiv v_{||}/|v_{||}|$) are the most appropriate ones for studying the α -orbit stochastic maps in the guiding centre approximation and that the transport process corresponds to a variation of the toroidal angular momentum P_ϕ and the particle energy E with the magnetic moment μ being constant. Thus in the presence of a fixed amplitude TAE, α -loss mechanisms have been studied. It is however emphasised that such an approach with the TAE-amplitude held constant is not self-consistent since the TAE instability itself depends on the redistribution of the fast particles.

Recently a new approach has been developed for predicting saturation of TAE-modes in Refs. [9 - 12], where it is shown under which condition the unstable TAE-modes can lead to stochastic diffusion due to overlapping of α -orbit resonances. If the TAE saturates above the

overlapping threshold, then TAE amplitude bursts appear due to global phase space mixing, resulting in significant anomalous transport of α -particles.

The subject of the present work is the analysis of the fast particle transport due to Alfvén instabilities in the JET tokamak. The orbit motion of highly energetic particles is studied by a guiding centre particle following code HAGIS (Hamiltonian GuIding Centre System). The TAE electromagnetic field input, together with associated equilibrium quantities are provided by the MHD code CASTOR [13] which computes the linear MHD spectrum for general axisymmetric plasma equilibria with finite pressure.

Recent analytical [14 - 17] and experimental [18] studies of Alfvén eigenmode spectra in tokamaks have shown that in high temperature plasmas multiple Kinetic Toroidal Alfvén Eigenmodes (KTAE's) appear, in addition to the TAE's. To compute the KTAE eigenfunction structure, the CASTOR code was generalised to include kinetic effects [19, 20]. Section 2 of the paper describes the spectra of TAE and KTAE modes and their computation for JET equilibria.

The code based on the Hamiltonian guiding centre formalism (HAGIS) is described in Section 3, and code validation tests are presented. Section 4 is devoted to the numerical study of passing and trapped fast particle orbit motion in the presence of TAE/KTAE perturbations of fixed amplitudes. The behaviour of the trapped particle population (which is relatively small in the case of an isotropic α -particles distribution), is discussed in the context of ICRH generated energetic ions, resonantly interacting with TAE/KTAE modes. A generalisation of the HAGIS code towards the self-consistent evolution of the TAE-amplitude and the α -particle distribution function is described in Section 5 and finally in section 6 a summary and conclusions are given.

2. TAE AND KTAE SPECTRA IN JET

Both types of weakly-damped discrete Alfvén eigenmodes, TAE and KTAE [1,14-17], are associated with the "gap" magnetic surfaces $r = r_m$, at which the condition

$$q(r_m) = (m - 1/2)/n \quad (1)$$

is satisfied. Here m and n are the poloidal and toroidal mode numbers, and $q(r)$ is the safety factor as a function of minor radius r . In contrast to TAE's, which can be described by ideal MHD, temperature effects in the layer surrounding $r = r_m$ are of major concern for KTAE's. At high plasma temperatures, the effect of Finite Larmor Radius (FLR), determined by the small parameter $(\kappa_{\perp} \rho_i)^2 \ll 1$ (ρ_i is the core ion Larmor radius), and the effect of a finite parallel perturbed electric field become comparable with the toroidal coupling effect, characterised by the small parameter $\varepsilon \ll 1$:

$$(\kappa_{\perp} \rho_i)^2 \sim \varepsilon \quad (2)$$

where $\varepsilon \approx 5 r_m/2R$, R being the major radius of the tokamak magnetic axis.

Competition between the two small parameters $(\kappa_{\perp} \rho_i)^2$ and ε resolves the singularity above the toroidicity-induced gap in the shear Alfvén continuum, and gives rise to a new discrete spectrum of weakly-damped KTAE modes [14-17].

The eigenfrequencies of the KTAE spectrum are nearly equally spaced with a distance

$$\Delta\omega/\omega \approx \varepsilon \lambda, \quad (3)$$

where λ is the non-ideal parameter, defined by [14 - 17]

$$\lambda = 4\rho_i \frac{mS}{r_m \varepsilon^{3/2}} \sqrt{\frac{3}{4} + \frac{T_e}{T_i}}. \quad (4)$$

Here $S \equiv r (dq/dr)/q$ is the magnetic shear, T_e and T_i are the electron and ion temperatures.

To compute the KTAE eigenfunction structures in realistic tokamak equilibria, we have developed a kinetic modification of the resistive MHD code CASTOR [19, 20]. Typical KTAE eigenfunctions for JET configurations are shown in Fig. 1. As is seen in Fig. 1, the KTAE eigenfunctions peak in a relatively narrow region around $r \approx r_m$. Away from the gap surface $r \approx r_m$, the KTAE structure is analogous to that of the TAE, with a radial scale length $\Delta^{\text{out}} \sim r_m/m$. Note that the orbit width of highly-energetic α -particles is comparable with, or larger than, $\Delta^{\text{out}} \sim r_m/m$ even for moderate values of m , $m \approx 3$ to 6. This indicates, that the effects of energy and momentum exchange between eigenmode and particles, are therefore independent of the inner eigenmode structure, and can be described by an identical set of equations for both TAE and KTAE [17].

To estimate the TAE-induced transport of fast α -particles, the most unstable TAE/KTAE modes have to be identified. For a JET high-performance discharge (#26087) α -particle driven TAE modes have been analysed [21], assuming equal densities of tritium and deuterium. TAE's were found to be stable due to the core ion Landau damping, but a modest decrease in plasma density could destabilise the TAE's, with the most likely candidates for the instability being the $n = 3$ modes. Recent experimental results in JET indicate, that the damping rates of KTAE-modes are typically much smaller than those of TAE [18]. Consequently we also consider in this paper the case of $n = 3$ KTAE modes, which are supposed to be driven unstable.

3. HAMILTONIAN FORMULATION OF GUIDING CENTRE DRIFT MOTION AND HAGIS CODE

The Alfvén perturbations, computed by the CASTOR code, are represented in the form of three components of the vector-potential $\tilde{\tilde{\mathbf{A}}}(\tilde{\mathbf{r}}, t)$ and the fluid velocity $\tilde{\tilde{\mathbf{v}}}(\tilde{\mathbf{r}}, t)$. It is convenient to represent an electromagnetic perturbation as input for the particle following code in the form of vector $\tilde{\tilde{\mathbf{A}}}$ and scalar $\tilde{\tilde{\Phi}}$ potentials

$$\tilde{\tilde{\mathbf{A}}}(\tilde{\mathbf{r}}, t) = \tilde{\tilde{A}}_{\psi_p} \nabla \psi_p + \tilde{\tilde{A}}_{\theta} \nabla \theta + \tilde{\tilde{A}}_{\zeta} \nabla \zeta, \tilde{\tilde{\Phi}} = \tilde{\tilde{\Phi}}(\tilde{\mathbf{r}}, t). \quad (5)$$

Here ψ_p, θ, ζ are the straight field line coordinates, introduced by White and Chance [8] with ψ_p representing the poloidal flux.

Starting from Morozov and Solov'ev and Littlejohn results [7, 22], one can derive the dimensionless guiding centre Lagrangian L

$$L = (\rho_{\parallel} I + \psi + \tilde{\tilde{A}}_{\theta}) \dot{\theta} + (\rho_{\parallel} g - \psi_p + \tilde{\tilde{A}}_{\zeta}) \dot{\zeta} + \mu \dot{\xi} - H + (\delta \rho_{\parallel} + \tilde{\tilde{A}}_{\psi_p}) \dot{\psi}_p. \quad (6)$$

where $\rho_{\parallel} = v_{\parallel}/\omega$ is the parallel gyroradius, ξ the gyroangle, ψ the toroidal flux, and the functions $I = I(\psi_p)$, $g = g(\psi_p)$ and $\delta = \delta(\psi_p, \theta)$ determine the magnetic field in a general magnetic configuration through

$$\tilde{\tilde{\mathbf{B}}} = \delta(\psi_p, \theta) \nabla \psi_p + I(\psi_p) \nabla \theta + g(\psi_p) \nabla \zeta. \quad (7)$$

The Hamiltonian H for this system is the total energy, given by

$$H = \frac{1}{2} \rho_{\parallel}^2 B^2 + \mu B + \Phi + \tilde{\tilde{\Phi}}. \quad (8)$$

The canonical momenta for the guiding centre Hamiltonian approach are

$$\begin{aligned} P_{\theta} &= \rho_{\parallel} I + \psi + \tilde{\tilde{A}}_{\theta}, \\ P_{\zeta} &= \rho_{\parallel} g - \psi_p + \tilde{\tilde{A}}_{\zeta}. \end{aligned} \quad (9)$$

It is appropriate to represent the perturbed vector potential in the form

$$\tilde{\mathbf{A}} = \alpha(\psi, \theta, \zeta) \bar{\mathbf{B}} . \quad (10)$$

Substituting the components of $\alpha \bar{\mathbf{B}}$ into the equations of motion we obtain the same expressions for $\dot{\theta}$, $\dot{\zeta}$, $\dot{\psi}_p$ and $\dot{\rho}_{\parallel}$, as used by White and Chance [8].

Based on this system of equations the HAGIS code has been developed to follow particles in an arbitrary plasma geometry in the presence of time-dependent perturbations. The input data to HAGIS consist of the equilibrium field, the perturbed field, and the particle's initial conditions. The perturbed magnetic field is modelled by taking $\alpha \bar{\mathbf{B}}$ as the perturbed vector potential, with α calculated to match the radial component of the perturbed magnetic field. The initial conditions, together with the equations of the orbit motion, are then used to advance the particle's location and parallel velocity in time.

To validate the HAGIS code, several tests have been performed to compare particle trajectories with those predicted by analytic theory. The first tests were comparisons of particle trajectories in an equilibrium magnetic field. The most sensitive particle orbits, such as a very fat banana orbit close to the trapped/passing boundary and the pinch orbit were found to conserve energy with accuracy 1.3×10^{-7} and 3.8×10^{-5} per orbit, respectively. With a field perturbation present the analytic invariant $E - \omega P_{\phi}/n$ is conserved for α -particles in JET with an accuracy 7×10^{-6} per orbit.

4. FAST PARTICLE LOSSES AND STOCHASTIC DIFFUSION DUE TO TAE/KTAE IN JET

In this section we consider losses and stochastic diffusion of passing and trapped fast particles in the presence of $n = 3$ TAE and KTAE modes with fixed amplitude. Typically, the amplitude of the Alfvén perturbations in the calculations is in the range $\delta \bar{B}_r / B_0 \sim 10^{-5}$ to 10^{-2} .

Two types of TAE/KTAE induced α -particle losses can be identified for typical JET equilibria.

Firstly, in the presence of an Alfvén wave α -particles close to the prompt loss boundaries may lose toroidal angular momentum, or gain or lose energy, and cross over into the prompt loss region. The time scale of such TAE-induced prompt losses is of the order of the bounce time, i.e. $t \sim 20 \mu\text{s}$. Since these losses arise from the distortion of the flux surface, they scale linearly with the amplitude $\delta \bar{B}_r / B_0$. Secondly, above a certain threshold $(\delta \bar{B}_r / B_0)_{\text{crit}}$, the orbit motions of the resonant α -particles become stochastic due to the overlap of orbit resonances. If the stochasticity becomes global, i.e. the stochastic orbit region covers almost all magnetic surfaces, additional losses of resonant α -particles arise due to orbit stochastic diffusion into a loss boundary. These losses scale as $(\delta \bar{B}_r / B_0)^2$.

To study stochastic diffusion effects we first estimate the stochasticity thresholds. For the case of passing α -particles Fig. 2 shows the evolution of the particle orbit islands in the reference frame $\zeta - \omega t/n$ for increasing TAE amplitudes. In this case just one $n = 3$ TAE mode is considered. All resonant passing particles have $v_{\parallel} = v_A$, but are initially located at different radii. Three islands at $q \approx 1.1, 1.3$ and 1.5 can be seen at moderate TAE amplitudes (e.g. $\delta\bar{B}_r / B_0 \geq 10^{-3}$). Because of the low shear in the central plasma region, the resonances are well separated radially. As a consequence, the resonances overlap at a relatively large value of $\delta\bar{B}_r / B_0$,

$$2 \times 10^{-3} < \delta\bar{B}_r / B_0 < 4 \times 10^{-3} \quad (11)$$

for the JET discharge considered (#26087). If several TAE/KTAE modes are included, a lower stochasticity threshold is obtained, e.g. for three $n = 3$ TAE in the same equilibrium as described above the threshold decreased to $\delta\bar{B}_r / B_0 \sim 10^{-3}$ per mode. To demonstrate the appearance of α -orbit stochastic behaviour in the presence of multiple Alfvén eigenmodes, one can consider the temporal evolution of the quantity $\langle \Delta P_{\phi}^2 \rangle = \langle P_{\phi}^2 \rangle - \langle P_{\phi} \rangle^2$ ($\langle \rangle$ denotes the ensemble average, $P_{\phi} = RM_{\alpha} V_{\zeta\alpha} - (e_{\alpha}/c) \psi(r)$, where e_{α} , M_{α} , $V_{\zeta\alpha}$ are the charge, mass and toroidal velocity of the α -particles). 500 α -particles are launched with initial energy $E = 2.5$ MeV and toroidal angular momentum, $P_{\phi} = 3.6 \times 10^{-19}$ kg m² s⁻¹ and they are randomly distributed in pitch, poloidal and toroidal angles. No resulting α -particle stochastic diffusion has been found for the TAE modes with the amplitude $\delta\bar{B}_r / B_0 \sim 5 \times 10^{-4}$, but stochastic diffusion of the order of $D \sim 0.3$ m²/sec appears for fast α -particles at the TAE-amplitude $\delta\bar{B}_r / B_0 \sim 3 \times 10^{-3}$, see Fig. 3 (a).

Fig. 3 (b) shows analogous calculations for the particle diffusion coefficient in the presence of an $n = 3$ KTAE-spectrum (the frequencies and the eigenmode structure are displayed in Fig. 1 (a) - (d)). The stochasticity threshold due to KTAE's is nearly the same as the threshold due to TAE's. For the same perturbation amplitude, $\delta\bar{B}_r / B_0 \sim 3 \times 10^{-3}$, and α -population as in the preceding TAE-case, the KTAE-induced diffusion is estimated as $D \sim 0.5$ m²/sec.

In the study of the stochastic behaviour of trapped fast particles we concentrate mainly on the case of ICRH-generated fast ions. This case is of particular interest in accordance with the experimental observation of KTAE-activity, driven by ICRH-produced fast ions in JET [18]. As an example, we consider discharge #34188, where seven KTAE modes have been excited by ICRH ($P \sim 7$ MW). The CASTOR code identified the KTAE-spectrum, and these KTAE-frequencies agree well with the measured mode frequencies.

In the numerical simulation, the effect of RF-heating is assumed to produce a distribution of fast particles with v_{\parallel} close to zero at the radius of the ICRH resonance. We assume furthermore that the distribution is a Gaussian in velocity space, consistent with a Stix

distribution function [23]. The effective temperature of the ICRH-generated fast ion tail in the shot #34188 was estimated to be 1 MeV. The minimum energy of the ions considered is 300 KeV.

ICRH-generated ions of such high energies interact with Alfvén eigenmodes through the trapped particle resonance, which can be written for the simplest case as [24, 25]

$$\omega = n\langle\omega_{Dh}\rangle + l\omega_{bh}, \quad l = 0, 1, 2 \dots \quad (12)$$

where ω_{bh} is the bounce frequency and $\langle\omega_{Dh}\rangle$ the toroidal drift frequency of fast ions.

In the case of multiple Alfvén eigenmodes and for $\omega_{bh} \sim n \langle\omega_{Dh}\rangle$, in the JET case considered, the structure of the resonances becomes more complicated than Eq. (12). Therefore, the resonances should be analysed numerically. Fig. 4 shows the KTAE-induced energy changes of ICRH-generated trapped particles, whose orbit tips lie on the ICRH-resonance surface. A population of 2500 particles was followed for 1 ms in the presence of seven KTAE modes with amplitudes $\delta\tilde{B}_r / B_0 = 4 \times 10^{-4}$. The fine structure of the resonances yields a threshold for the onset of stochasticity in the range

$$10^{-4} \leq \delta\tilde{B} / B_0 \leq 4 \times 10^{-4}. \quad (13)$$

This can be seen from comparison of Figs. 5(a) and 5(b), where trapped fast ion orbits are shown for these two cases.

The stochastic diffusion of trapped fast ions due to KTAE-modes can be seen from Fig. 6, where a non-zero gradient $\partial \langle\Delta P_\phi^2\rangle / \partial t$ occurs for $\delta\tilde{B}_r / B_0 \sim 4 \times 10^{-4}$.

Finally, α -particle losses are analysed by an extended Monte-Carlo simulation including 50.000 α -particles. The α -particles have a slowing down distribution with $1.5 \text{ MeV} \leq E \leq 3.5 \text{ MeV}$, a radial distribution $\sim (1-\psi_p)^3$ and a random distribution in pitch angle, poloidal angle and toroidal angle. Results of these Monte-Carlo simulations predict that in the presence of three $n = 3$ TAE-modes with $\delta\tilde{B}_r / B_0 = 10^{-3}$ per mode, an additional 1% of α -particles are lost during 0.4 ms. For the expected JET fusion power these α -losses cannot lead to a significant degradation of global confinement, but these losses are unfavourable for the first wall or divertor due to the high localisation of the losses at the outboard mid-plane of the tokamak.

5. GENERALISATION TOWARDS SELF CONSISTENT PARTICLE-WAVE EVOLUTION

In the non-linear self-consistent problem, the TAE/KTAE eigenmode structure is assumed to be invariant during the wave evolution, with only the wave amplitude and phase varying. Each eigenmode has therefore just two degrees of freedom, namely the amplitude A_s and the phase β_s , so that the perturbation can be represented in the form

$$\tilde{\mathbf{B}} = \sum_s A_s(t) \hat{\mathbf{B}}_s(\vec{r}) \exp[-i(\beta_s(t) + \omega_s t)], \quad (14)$$

where s is the index of the eigenmode with frequency ω_s and $\hat{\mathbf{B}}_s(\vec{r})$ is the fixed linear TAE/KTAE eigenfunction, as computed by CASTOR. To include the wave evolution into the Hamiltonian equations for energetic particles, the wave Lagrangian is introduced

$$L_w = \sum_{\substack{\text{core} \\ \text{plasma}}} \left[\frac{mv^2}{2} + e(\vec{A} \cdot \vec{v} - \phi) \right] + \frac{1}{2\mu_0} \int \left(\frac{E^2}{c^2} - B^2 \right) dx^3. \quad (15)$$

Taking into account Eq. (14), this wave Lagrangian in the approximation $\dot{\beta}_s \ll \omega$ can be reduced to the form

$$L_w = \sum_s \frac{A_s^2 \dot{\beta}_s}{2\mu_0 \omega_s} \int \hat{\mathbf{B}}_s(\vec{r}) \hat{\mathbf{B}}_s^*(\vec{r}) dx^3. \quad (16)$$

Lagrange's equations then determine the wave evolution

$$\dot{A}_s = -\frac{i\omega_s}{2A_s \overline{W}_s} \sum_{\text{particles}} \left\{ \rho_{\parallel} B^2 \tilde{\alpha}_s - \tilde{\phi}_s \right\}, \quad (17)$$

$$\dot{\beta}_s = -\frac{\omega_s}{2A_s^2 \overline{W}_s} \sum_{\text{particles}} \left\{ \rho_{\parallel} B^2 \tilde{\alpha}_s - \phi_s \right\},$$

where \overline{W}_s is the normalised eigenmode energy E_s for unit amplitude, i.e.

$$E_s = A_s^2 \overline{W}_s. \quad (18)$$

For an initial seed amplitude and phase of the TAE/KTAE perturbation all α -particles are followed by HAGIS code for a time Δt . After Δt the amplitude and phase of the waves are updated taking into account contributions from all particles. Thus, HAGIS allows the self-consistent wave particle interaction to be treated, enabling the simultaneous evolution of plasma modes and energetic particles.

Details of the self-consistent results will be presented in a future paper, where an extension of the method discussed above using the δf -algorithm [26] will be described.

6. CONCLUSIONS

Toroidal Alfvén Eigenmodes and Kinetic Toroidal Alfvén Eigenmodes have been calculated with the CASTOR code for JET equilibria. A Hamiltonian guiding centre particle following code HAGIS has been developed and applied to study fast ion orbits in the presence of

TAE/KTAE modes with amplitudes $10^{-5} \leq \delta\tilde{B}_r / B_0 \leq 10^{-2}$. A fairly high α -orbit stochasticity threshold was found for a single $n=3$ TAE in the low-shear JET equilibrium, $3 \times 10^{-3} < \delta\tilde{B}_r / B_0 < 6 \times 10^{-3}$. For several (three or four) TAE/KTAE modes stochasticity threshold decreases to $\delta\tilde{B}_r / B_0 \sim 10^{-3}$ in the same plasma configuration. Stochastic diffusion coefficients for TAE/KTAE perturbations with amplitude 3×10^{-3} were found to be of the order of $D \sim 0.3 \div 0.5 \text{ m}^2/\text{Sec}$. Monte-Carlo simulations including 50.000 α -particles have shown, that for $n = 3$ TAE's with amplitudes $\delta\tilde{B}_r / B_0 = 10^{-3}$ global orbit stochasticity barely appears in a JET low-shear equilibrium. The main α -loss mechanism was found to be due to the TAE-induced conversion of resonant counter-going circulating α -particles to large trapped orbits (near the trapped/passing boundary). An additional 1% of the α -particles are lost due to TAE's in this case.

ICRH-generated fast ion tails in the presence of KTAE-modes were found to display stochastic orbit behaviour at $10^{-4} < \delta\tilde{B}_r / B_0 < 4 \times 10^{-4}$. The threshold of stochasticity is slightly below those for passing particle case due to more fine structure of trapped particle resonances and larger number of KTAE modes considered. The HAGIS code has been generalised to incorporate self-consistent particle-wave evolution.

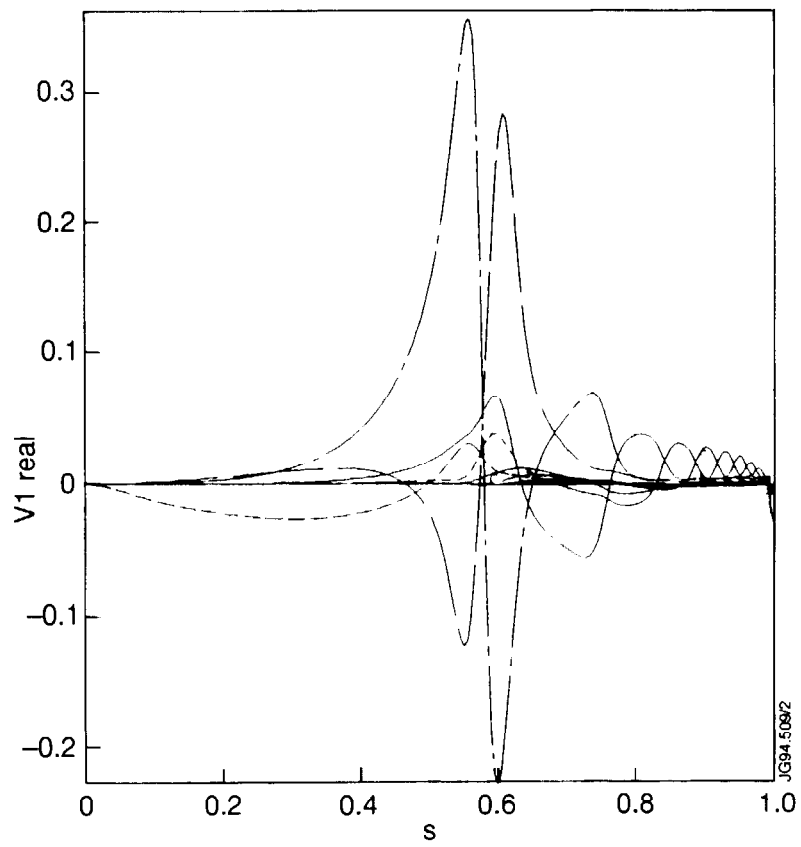
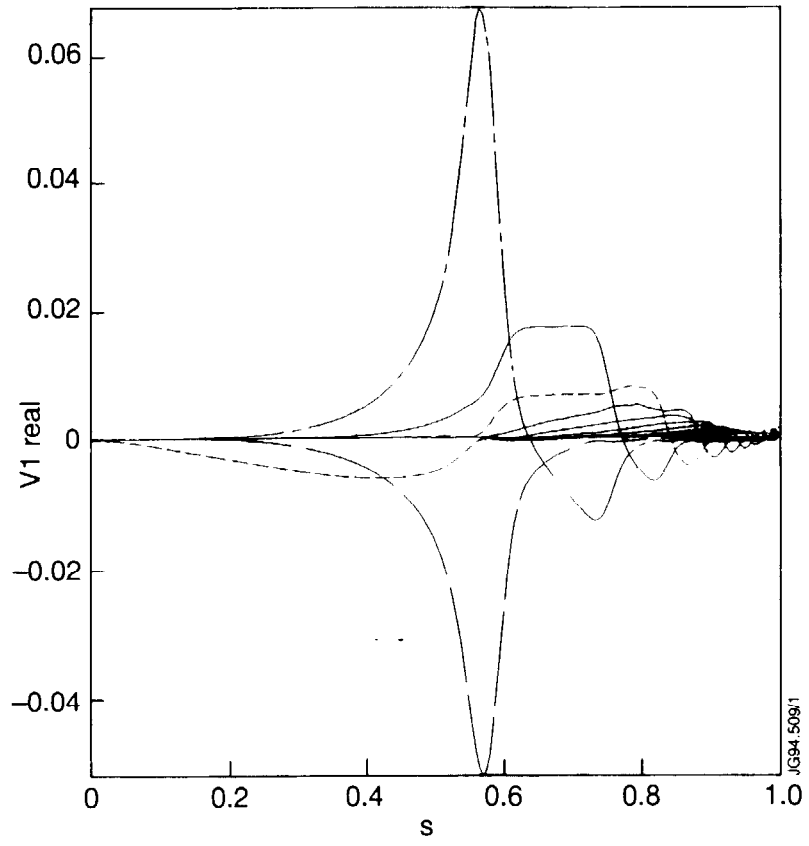
ACKNOWLEDGMENTS

We would like to thank Dr A. Fasoli for many stimulating discussions on TAE experiments. The UKAEA authors were jointly funded by the U.K. Department of Trade and Industry and EURATOM.

REFERENCES

- [1] C.Z. Cheng, L. Chen, M.S. Chance, *Annals of Physics* **161**, 21 (1985).
- [2] D.T. Sigmar, C.T. Hsu, R. White, C.Z. Cheng, *Phys. Fluids* **B4**, 1506 (1992).
- [3] K.L. Wong, R.J. Fonck, S.F. Paul, D.R. Roberts, E.D. Fredrickson, R. Nazikain, H.K. Park, H. Bell, N.L. Bretz, R. Budny, S. Cohen, G.W. Hammett, F.C. Jobes, D.M. Meade, S.S. Medley, D. Mueller, Y. Nagayama, D.K. Owens and E.J. Synakowski, *Phys. Rev. Lett.* **66**, 19874 (1991).
- [4] W.W. Heidbrink, E.J. Strait, E. Doyle, G. Sager and R. Snider, *Nuclear Fusion* **31**, 1635 (1991).
- [5] K.L. Wong, J.R. Wilson, Z.Y. Chang, G.Y. Fu, E. Fredrickson, G.W. Hammett, C. Bush, C.K. Phillips, J. Snipes and G. Taylor, *Plasma Physics Controlled Fusion* **36**, 879, 1994.
- [6] S. Ali-Arshad and D.J. Campbell, *Plasma Physics Controlled Fusion* **37**, 715, 1995.
- [7] A.I. Morozov and L.S. Solov'ev, *Reviews of Plasma Physics*, v.2, Consultants Bureau, 1966.
- [8] R.B. White and M.S. Chance, *Phys. Fluids* **27**, 2455, 1984.
- [9] H.L. Berk, B.N. Breizman and Huanchun Ye, *Phys. Rev. Letters* **68**, 3563, 1992.
- [10] H.L. Berk, B.N. Breizman and M. Pekker, "Basic Principles Approach for Studying Nonlinear Alfvén Wave-Alpha Particle Dynamics", in *Phys. of High Energy Particles in Toroidal Systems*, AIP Conf. Proc. 311, edited by T. Tajima and M. Okamoto, (Amer. Inst. of Physics, New York, 1994), p.18.
- [11] G.Y. Fu and W. Park, *Phys. Rev. Letters*, **74**, 1594, 1995.
- [12] Y. Wu, R.B. White, Y. Chen and M.N. Rosenbluth, *Nonlinear Evolution of the Alpha Particle Driven Toroidicity - induced Alfvén Eigenmode*. Submitted to *Nuclear Fusion* 1995.
- [13] G.T.A. Huysmans, J.P. Goedbloed and W. Kerner, *Phys. Fluids* **B5**, 1545, 1995.
- [14] R.R. Mett and S.M. Mahajan, *Phys. Fluids* **B4**, 1885, 1992.
- [15] J. Candy and M. Rosenbluth, *Phys. of Plasmas* **1**, 356, 1994.
- [16] H.L. Berk, R.R. Mett and D.M. Lindberg, *Phys. Fluids* **B5**, 3969, 1993.
- [17] B.N. Breizman and S.E. Sharapov, *Plasma Phys. Contr. Fus.*, **37**, 1057 (1995).
- [18] A. Fasoli, J.B. Lister, S. Sharapov, S. Ali-Arshad, G. Bosia, D. Borba, D.J. Campbell, N. Deliyannis, J.A. Dobbing, C. Gormezano, H.A. Holties, G.T.A. Huysmans, J. Jacquinet, A. Jaun, W. Kerner, P. Lavanchy, J.-M. Moret, L. Porte, A. Santagiustina, L. Villard, *Overview of Alfvén Eigenmode Experiments in JET*, Submitted to *Nuclear Fusion* 1995.

- [19] J.W. Connor, R.O. Dendy, R.J. Hastie, D. Borba, G.T.A. Huysmans, W. Kerner and S. Sharapov, Proceedings of XX1 EPS Conf. on Contr. Fus. Plasma Phys., v. 18B Part III, p. 616, 1994.
- [20] S. Sharapov, L. Appel, D. Borba, T.C. Hender, G.T.A. Huysmans, W. Kerner, S.D. Pinches, Bull. Amer. Phys. Soc. **39** 1566, 1994 (see also JET Report JET-P(94)61, p. 105, 1994).
- [21] W. Kerner, D. Borba, G.T.A. Huysmans, F. Porcelli, S. Poedts, J.P. Goedbloed and R. Betti, Plasma Phys. Contr. Fusion **36**, p.911, 1994.
- [22] R.G. Littlejohn, J. Plasma Phys., **29**, 111, 1983.
- [23] T.H. Stix, Nuclear Fusion, **15**, 737 1975.
- [24] H. Biglari, F. Zonca, L. Chen, Phys. Fluids B **7**, 2385, 1992.
- [25] G.Y. Fu, C.Z. Cheng, Phys. Fluids B **4**, 3772, 1992.
- [26] S. Parker and W.W. Lee, Phys. Fluids B**5**, 77, 1993.



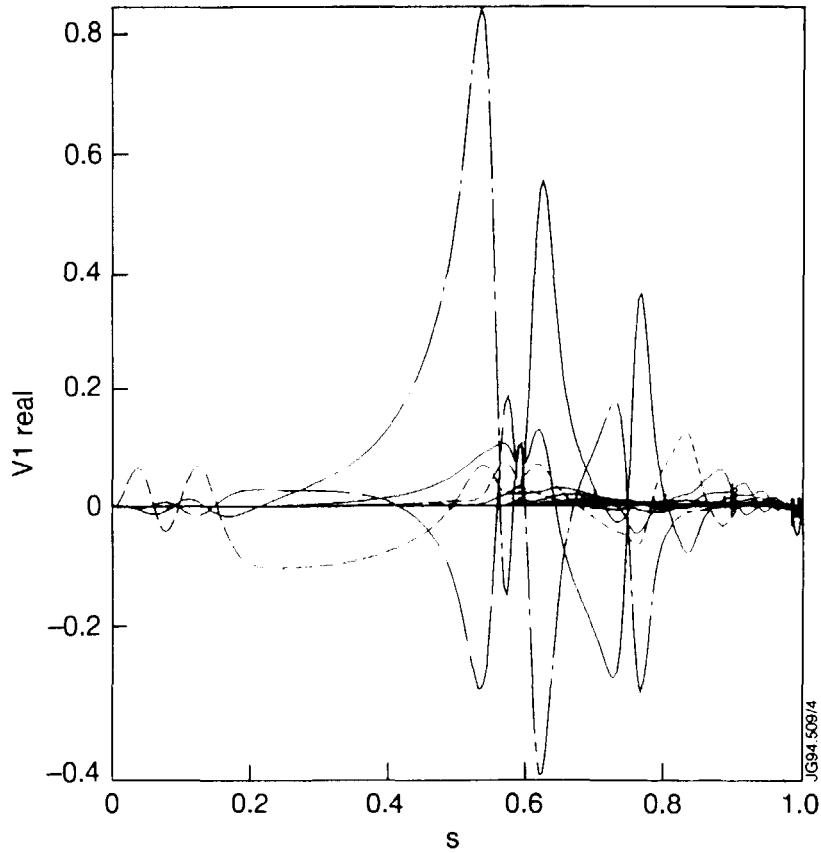
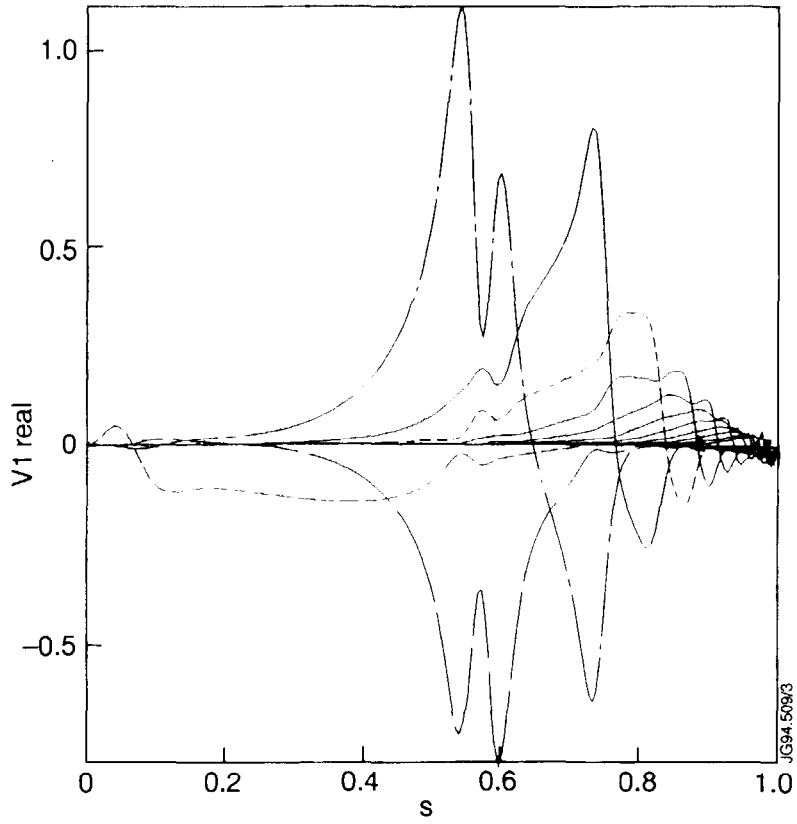


Fig. 1: Kinetic TAE modes with $n=3$ of different quantum numbers p in JET discharge #26087, showing $\text{Re } V_\psi$ versus $s \equiv \sqrt{\psi_p / \psi_p(\text{edge})}$.

(a) $p = 0$, $\omega/\omega_A = 0.6768$, $\gamma\omega = 0.16\%$; (b) $p = 1$, $\omega/\omega_A = 0.6901$, $\gamma\omega = 0.275\%$; (c) $p = 2$, $\omega/\omega_A = 0.7170$, $\gamma\omega = 0.36\%$; (d) $p = 3$, $\omega/\omega_A = 0.7365$, $\gamma\omega = 1.26\%$; $\omega_A = v_A$ (o)/R.

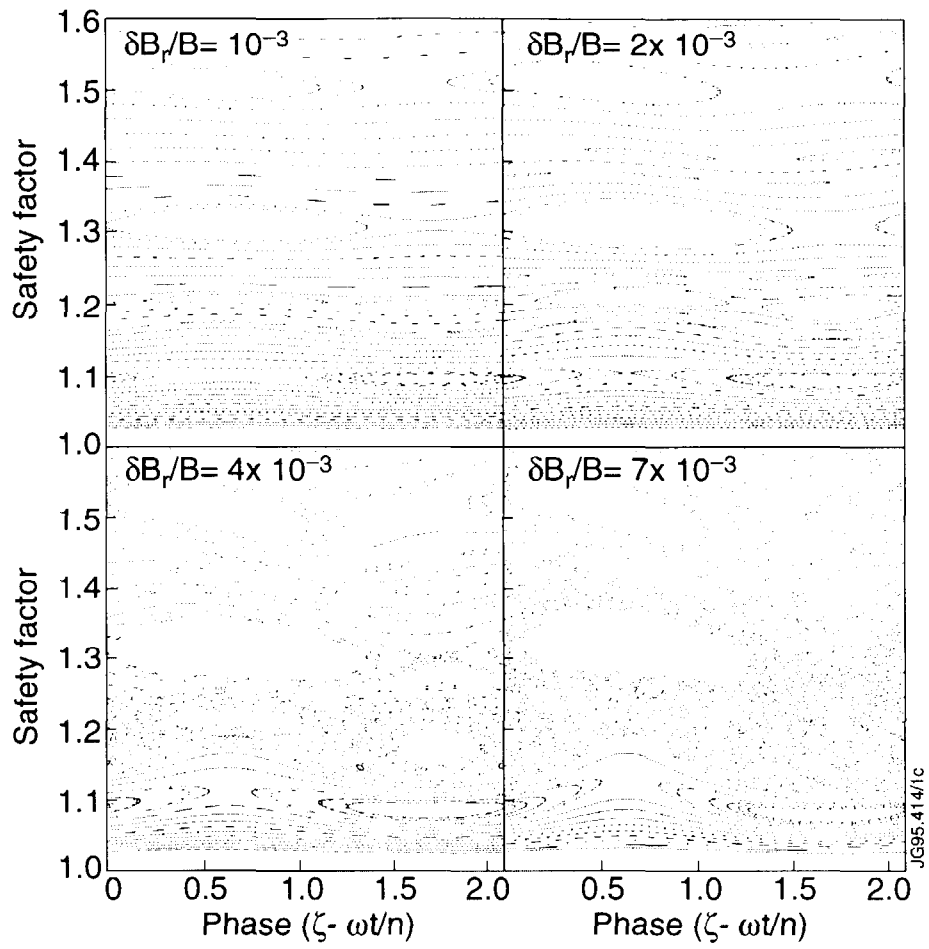


Fig. 2: Particle orbit islands plotted versus the TAE phase $\zeta - \omega t/n$.

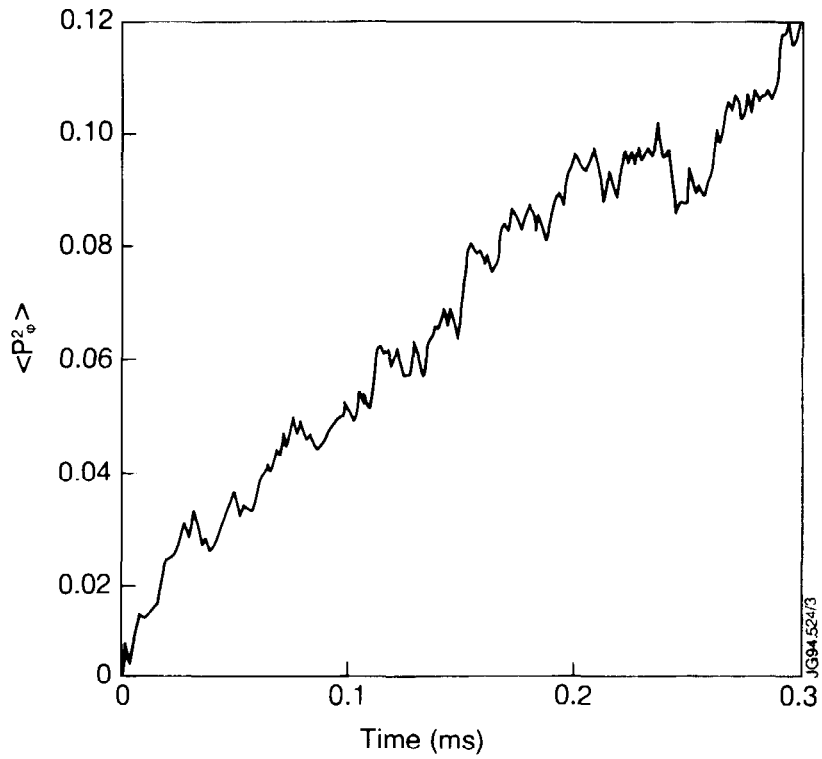
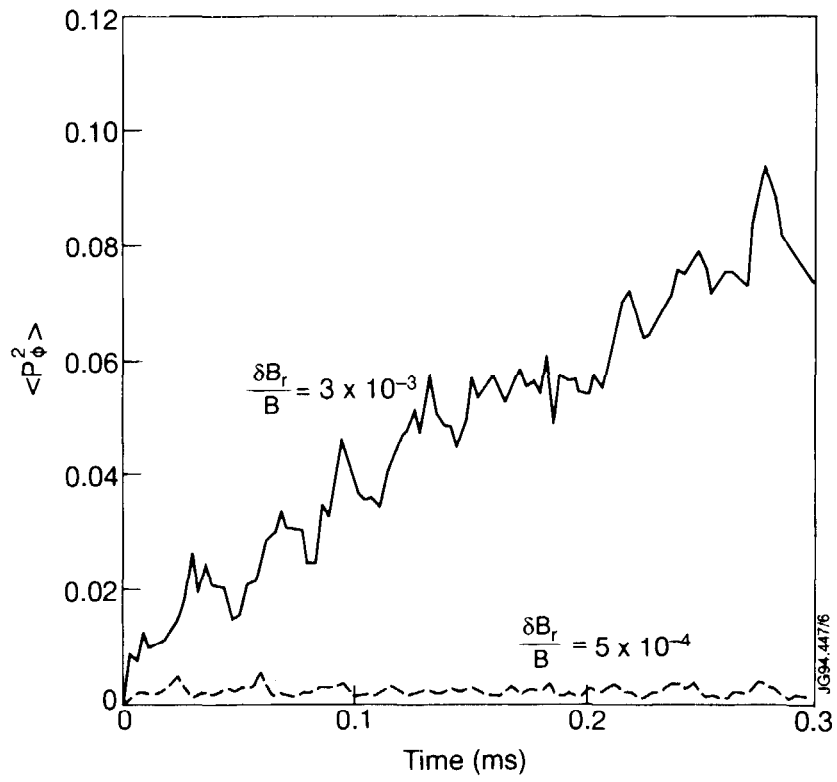


Fig. 3: Variation of $\langle \Delta P_{\phi}^2 \rangle \times 10^{40} \text{ (kg m}^2 \text{ s}^{-1})^2$ with time (a) for the TAE case (b) for the KTAE case ($\delta \bar{B}_r / B_0 = 4 \times 10^{-3}$).

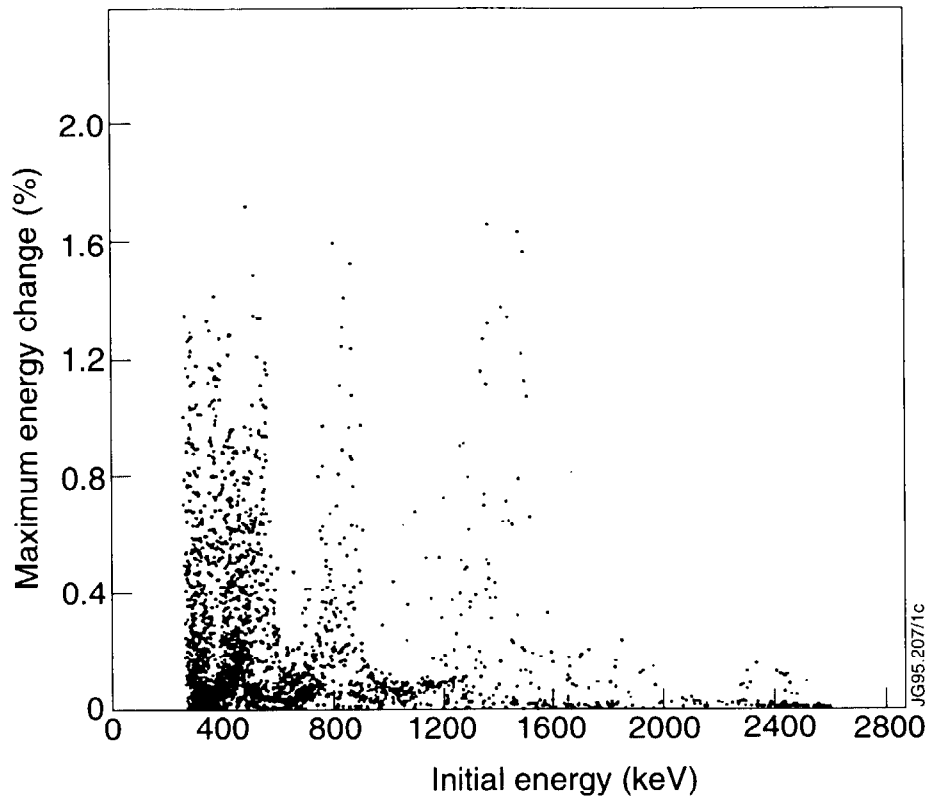


Fig. 4: Trapped particle resonances for ICRH-generated fast ions in the presence of seven ($n = 3$) KTAE-modes.

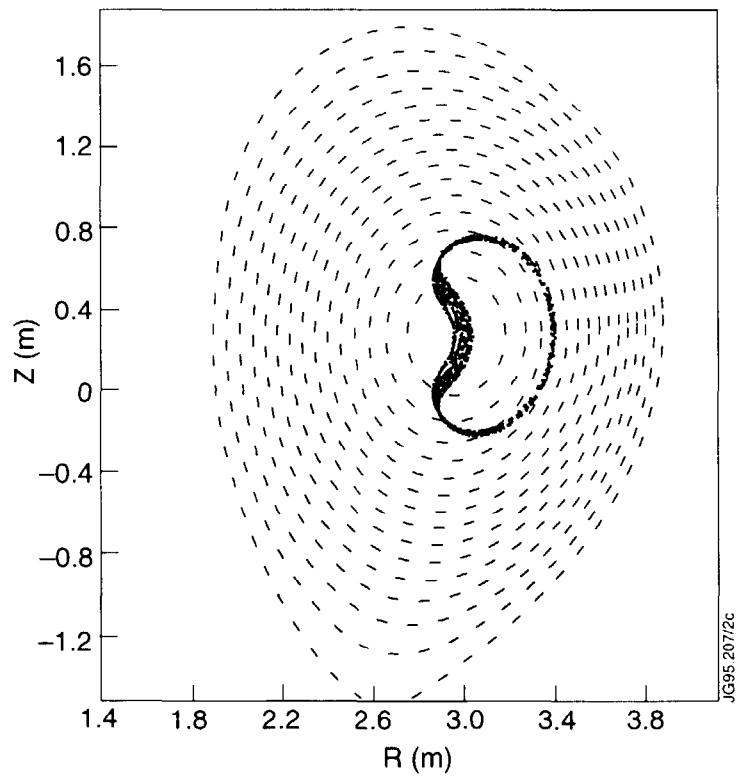
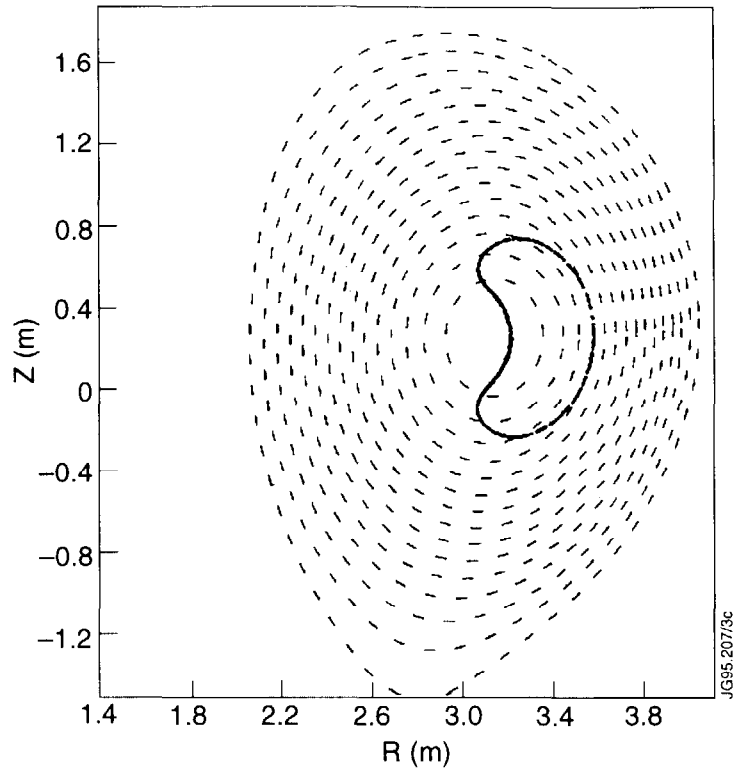


Fig. 5: Onset of stochasticity of trapped particle orbits (ICRH-case) (a) $\delta\bar{B}_r / B_0 = 10^{-4}$, (b) $\delta\bar{B}_r / B_0 = 4 \times 10^{-4}$.

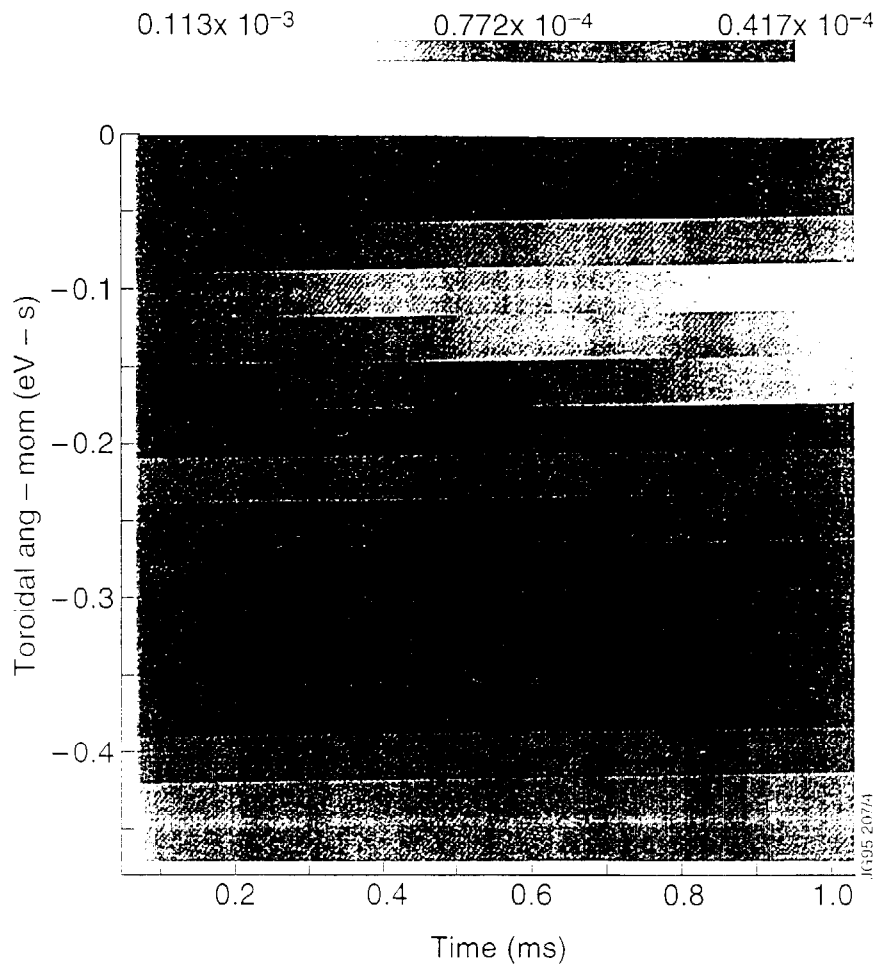


Fig. 6: Temporal evolution of $\langle \Delta P_{\phi}^2 \rangle$ (eV-sec)² in the case of ICRH-generated fast ions, for different initial P_{ϕ}