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Effect of Temperature Gradient on Plasma Sheath

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In the plasma sheath the electron current is severely limited by a retarding electrostatic potential and this implies that only fast plasma electrons reach the material surface. These fast electrons have a much longer mean free path than thermal electrons and consequently their last collision will have occurred well away from the surface. If the temperature gradient is such that the temperature at these last collisions is different from that at surface, the sheath potential will be modified. This effect can lead to large modifications of the sheath potential and could also lead to substantial errors in the interpretation of probe signals.

When a plasma is in contact with a solid surface an electrostatic sheath is formed. This arises from the greater mobility of the electrons as compared to the ions. The electrostatic potential difference, φ_o, across the sheath is such as to equalise the electron and ion currents by retarding the electrons. The electron density at the solid is thus reduced by a factor exp $- e\varphi_o/T$, and this factor has to balance the natural electron-ion flux ratio which is ~ $(m_i/m_e)^{1/2}$. Thus roughly

$$\frac{e\varphi_o}{T} \sim \ell n\left(\frac{m_i}{m_e}\right)^{1/2} , \tag{1}$$

and for a hydrogen plasma $e\varphi_o/T$ is typically ~ 3. It is seen, therefore, that the initial energy of electrons which reach the solid is several times the thermal energy.

If there is a temperature gradient away from the solid, the electrons arriving at the surface will have a velocity distribution which is characterised by the plasma temperature some distance from the surface. For a given electron velocity the characterising temperature will be that at the position of the last collision. Thus, this temperature is that at a distance of a mean free path from the surface. Since the mean free path increases as v^4, the characterising temperature is a function of the electron velocity.

It is seen, therefore, that it is possible for the fast electrons which overcome the sheath potential to have a velocity distribution quite different from that of the local Maxwellian and to have a higher effective temperature.

A full calculation is extremely complicated and we shall make use of various simplifications. First of all let us consider the simplest model.

The electrons with velocity v had their last collision at a distance $x = \lambda$ from the surface where λ is the mean free path. Now λ is a function of v and we can take $\lambda = \lambda_o\left(v / v_{T_o}\right)^4$ where λ_o is the mean free path of a particle with the thermal velocity v_{To} corresponding to the temperature T_o at the surface. At this distance, x, the plasma temperature is T(x). Thus the electron distribution function at the surface has an exponential factor

$$f \sim \exp - \frac{\frac{1}{2}mv^2}{T(x(v))} \tag{2}$$

where, for large velocities,

$$x(v) \sim \lambda_o \left(\frac{v}{v_{T_o}} \right)^4 . \tag{3}$$

We might expect that it would be possible to use a simple expansion for T(x), $T(x) = T_o + T'_o x$. However for large velocities relations (2) and (3) would give

$$f \sim \exp - \frac{\frac{1}{2}mv^2}{T_o + T'_o \lambda_o v^4 / v_{T_o}^4} .$$

and it is seen that integrals over the velocity distribution would diverge since the exponential approaches a finite value at large v.

The reason is clear. With this approximation the contribution to the electron current would be dominated by electrons from a distance larger than the linear expansion for T would allow. Thus although this approach fails, it brings out the importance of the remote plasma. It is also evident that the use of probe characteristics to determine the electron temperature using conventional theory could lead to error.

Before proceeding to a calculation of the effect of a temperature gradient, it is possible to obtain some insight from a more heuristic treatment.

The sheath potential will be determined by an effective temperature T_{eff} such that, from relation (1),

$$\frac{e\varphi_o}{T_{eff}} \sim \ell n \left(\frac{m_i}{m_e} \right)^{1/2} . \tag{4}$$

The electrons which reach the surface will have a velocity characterised by

$$\tfrac{1}{2}mv^2 \sim e\varphi_o \tag{5}$$

and will have had their last collision at a distance

$$x \sim \lambda \sim \lambda_o \left(\frac{v}{v_{T_o}} \right)^4 . \tag{6}$$

Combining relations (5) and (6) we have the position x which characterises the effective temperature, thus

$$x \sim \lambda_o \left(\frac{2e\varphi_o}{T_o} \right)^2 . \tag{7}$$

Using relation (7) to eliminate φ_o in relation (4) we obtain a relation for the characterising value of x at which $T(x) = T_{eff}$

$$T(x) \sim \tfrac{1}{2} \left(\frac{x}{\lambda_o} \right)^{1/2} \frac{T_o}{\ell n(m_i / m_e)^{1/2}} . \tag{8}$$

For any particular temperature profile T(x), relation (8) determines x and T(x) and hence T_{eff}. Knowing T_{eff}, relation(1) gives the sheath potential φ_o,

$$\frac{e\varphi_o}{T_{eff}} \sim \ell n \left(\frac{m_i}{m_e} \right)^{1/2} .$$

If, for example, the temperature rises to an upstream value T_∞ over a length l, the effective temperature will become T_∞ if, using relation (8),

$$\frac{\lambda_o}{\ell} \geq \left(\frac{T_o}{T_\infty} \frac{1}{\ell n(m_i / m_e)^{1/2}} \right)^2 . \tag{9}$$

For $T_\infty = 3T_o$, this would require only that the mean free path, λ_o, for a thermal particle be such that $\lambda_o / \ell \gtrsim 1\%$.

We shall now turn to a simple, but more careful, analysis.

Simple analysis

We need to calculate the electron current density, j_e, at the surface. Defining a positive current to be a current toward the surface, j_e is given by

$$j_e = e \int_{0}^{-\infty} f(0, v)\, v\, dv \,, \tag{10}$$

where f(x, v) is the electron distribution function, and we note that the velocities are negative. Since, in the absence of collisions, f is constant along a particle trajectory we can approximate f(0, v) by

$$f(0, v) = f\big(x, v'(x)\big) \tag{11}$$

up to the distance of the "last collision", the velocity $v'(x)$ being related to the velocity v at the surface by the energy conservation equation

$$\tfrac{1}{2} m v^2 = \tfrac{1}{2} m v'^2(x) - e\varphi(x) \,, \tag{12}$$

the potential being taken to be zero at x = 0.

Now, in general we need to know $\phi(x)$ and n(x) in addition to T(x). However the physics will be clearer if we neglect the weak role of n(x) and further assume that the essential change of φ to be that across the sheath . The more general case will be considered later. Thus putting $\varphi = \varphi_o$ outside the sheath, substituting equation (11) into equation (10), and using equation (12)

$$j_e = e \int_{-\left(\frac{2e\varphi_o}{m}\right)^{1/2}}^{-\infty} f\,(x, v')\, v' dv' \,. \tag{13}$$

Then, taking $x = \lambda(v')$ where λ is the mean free path, equation (13) becomes

$$j_e = e \int_{-\left(\frac{2e\varphi_o}{m}\right)^{1/2}}^{-\infty} f\big(\lambda(v'), v'\big)\, v' d v' \,. \tag{14}$$

The distribution function is taken to be locally Maxwellian, that is

$$f(x,v') = n(x)\left(\frac{m}{2\pi T(x)}\right)^{1/2} \exp - \frac{\frac{1}{2}mv'^2}{T(x)} ,$$

so that, putting $n = n_o$,

$$f(\lambda(v'),v') = n_o\left(\frac{m}{2\pi T(\lambda(v'))}\right)^{1/2} \exp - \frac{\frac{1}{2}mv'^2}{T(\lambda(v'))} \tag{15}$$

and substitution of equation (15) into equation (14) gives

$$j_e = n_o e\left(\frac{m}{2\pi}\right)^{1/2} \int_{-\left(\frac{2e\varphi_o}{m}\right)^{1/2}}^{-\infty} \frac{1}{T^{1/2}(\lambda(v'))} \exp - \frac{\frac{1}{2}mv'^2}{T(\lambda(v'))} v'dv'$$

and so, writing $\varepsilon = \frac{1}{2}mv'^2 / T_o$

$$j_e = - n_o e\left(\frac{T_o}{2\pi m}\right)^{1/2} \int_{\frac{e\varphi_o}{T_o}}^{\infty} \left(\frac{T_o}{T(\varepsilon)}\right)^{1/2} \exp - \frac{\varepsilon}{T(\varepsilon)/T_o} d\varepsilon \tag{16}$$

where $T(\varepsilon)$ is the temperature at $x = \lambda(\varepsilon)$. Since the density has been taken to be constant, we have $\lambda(v') = (v'/v_{To})^4 \lambda_o$ and

$$\lambda(\varepsilon) = 4\,\varepsilon^2\,\lambda_o , \tag{17}$$

λ_o being the mean free path at $T = T_o$.

In this approximation the ion current does not depend on the temperature distribution and it is convenient to calculate the ion current by equating it to (minus) the electron current, j_{ec}, in the constant temperature, $T = T_o$, case. Thus from equation (16)

$$j_{ec} = - n_o e\left(\frac{T_o}{2\pi m}\right)^{1/2} \exp - \frac{e\varphi_{oc}}{T_o} \tag{18}$$

where φ_{oc} is the constant temperature sheath potential. For a zero current sheath, $j_i + j_e = 0$ and equation (18) gives

$$j_i = n_o e \left(\frac{T_o}{2\pi m}\right)^{1/2} \exp - \frac{e\varphi_{oc}}{T_o} . \tag{19}$$

The equation for the sheath potential in the varying temperature case is now obtained by substituting equations (16) and (19) into $j_i + j_e = 0$ to give

$$\int_{\frac{e\varphi_o}{T_o}}^{\infty} \left(\frac{T_o}{T(\varepsilon)}\right)^{1/2} \exp - \frac{\varepsilon}{T(\varepsilon)/T_o} d\varepsilon = \exp - \frac{e\varphi_{oc}}{T_o} . \tag{20}$$

For a given φ_{oc} equation (20) can be solved to find the φ_o corresponding to the temperature variation T(x). It is clear from equation (20) that for higher values of upstream T(x), and therefore $T(\varepsilon)$, the exponent will be smaller and the integrand will be larger. Since the right hand side of the equation is constant this will require that the lower limit of the integral be raised, and means that the sheath potential, φ_o, will be higher.

The solution can be represented conveniently by introducing an effective temperature. This is defined as the uniform temperature which would give the same potential φ_o as the actual temperature. For this uniform case the quantity on the left hand side of equation (20) would be

$$\left(\frac{T_{eff}}{T_o}\right)^{1/2} \exp - \frac{e\varphi_o}{T_{eff}}$$

and equating this to the quantity on the right hand side of equation (20) gives

$$\left(\frac{T_{eff}}{T_o}\right) = \frac{\frac{e\varphi_o}{T_o}}{\frac{e\varphi_{oc}}{T_o} + \frac{1}{2} \ln \frac{T_{eff}}{T_o}} . \tag{21}$$

Given the value of $e\varphi_{oc}/T_o$ for the reference case and the value of φ_o obtained from equation (20), equation (21) is an implicit equation for T_{eff}/T_o.

An example

To illustrate the behaviour we will take a model temperature profile which characterises a rise in temperature from T_0 to T_∞ with a scale length l,

$$T = T_0 + \left(T_\infty - T_0\right)\tanh\frac{x}{\ell}\,.$$

This expression is inserted into equation (20) with $x = \lambda(\varepsilon)$, using equation (17) for $\lambda(\varepsilon)$. The resulting equation is

$$\int_{\frac{e\varphi_0}{T_0}}^{\infty} \frac{1}{\left(1+\left(\frac{T_\infty}{T_0}-1\right)\tanh 4\frac{\lambda_0}{\ell}\varepsilon^2\right)^{1/2}} \exp - \frac{\varepsilon}{1+\left(\frac{T_\infty}{T_0}-1\right)\tanh 4\frac{\lambda_0}{\ell}\varepsilon^2}\, d\varepsilon = \exp - \frac{e\varphi_{oc}}{T_0}$$

and this equation can be solved numerically to give $\varphi = \varphi\,(T_\infty/T_0, \lambda_0/l, e\varphi_{oc}/T_0)$. Contours of constant T_{eff}/T_0 for a typical case are given in figure 1. It is seen that if the mean free path of a thermal particle is greater than 1% of the length characterising the temperature change, the sheath is dominated by the distant temperature. This is in agreement with the heuristic result of relation (9).

Effect of φ(x)

We now return to the question of the influence of $\varphi(x)$. In the above calculation the entire change in φ was taken to be within the sheath. More generally there is a potential variation in the upstream plasma. Basically this arises from the electron force balance relation

$$\nabla p_e = ne\nabla\varphi. \tag{22}$$

where p_e is the electron pressure.

To explore the effect of this potential gradient we retain the assumption of constant density. Equation (22) then gives

$$e\left(\varphi(x) - \varphi_0\right) = T(x) - T_0\,. \tag{23}$$

Inclusion of this x dependence of φ leads to two effects in the calculation. In the first the additional potential difference seen by electrons at each position retards their motion toward the surface. This will reduce somewhat the sheath potential required to restrain them and will therefore reduce the effective temperature. Mathematically this appears as a change in the limit of integration. The other effect is that the electrons are slowed and consequently carry a smaller current to the surface. This again reduces the sheath potential and the effective temperature. In the calculation it appears as a modified Jacobian.

Using equation (23), the conservation of electron energy given by equation (12) now takes the form

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}mv'^2(x) - e\varphi_o - T(x) + T_o \,. \qquad (24)$$

With the mean free path relation

$$x = \lambda_o\left(\frac{v'}{v_{T_o}}\right)^4$$

equation (24) gives

$$v\,dv = \left(1 - \frac{4\lambda_o}{T_o}\frac{dT}{dx}\frac{v'^2}{v_{T_o}^2}\right)v'dv'$$

and, again using equation (11) with a Maxwellian distribution function, substitution into equation (10) leads to

$$j_e = n_o e\left(\frac{m}{2\pi}\right)^{1/2}\int_{v_c'}^{-\infty}\frac{1}{T^{1/2}(x(v'))}\exp\left(-\frac{\frac{1}{2}mv'^2}{T(\lambda(v'))}\right)\left(1 - \frac{4\lambda_o}{T_o}\frac{dT}{dx}\frac{v'^2}{v_{T_o}^2}\right)v'dv' \qquad (25)$$

where dT/dx is an implicit function of v' and the limit v_c' is obtained by putting v = 0 in equation (24), that is

$$\tfrac{1}{2}mv_c'^2 - e\varphi_o - T\left(x(v_c')\right) + T_o = 0 \,.$$

Alternatively the integration can be carried out over the variable x. Then

$$j_e = n_o e\left(\frac{1}{2\pi m}\right)^{1/2}\int_{x_c}^{\infty}\frac{1}{T(x)^{1/2}}\exp\left(-\frac{\frac{1}{2}(x/\lambda_o)^{1/2}T_o}{T(x)}\right)\left(\frac{1}{4}\frac{T_o}{(\lambda_o x)^{1/2}} - \frac{dT}{dx}\right)dx \qquad (26)$$

with

$$\tfrac{1}{2} m v_{T_o}^2 \left(\frac{x_c}{\lambda_o}\right)^{1/2} - e\varphi_o - T(x_c) + T_o = 0 .$$

Using equation (25) or (26) we can calculate j_e for a given T(x). Then, using equation (19) for j_i and putting $j_i + j_e = 0$ gives the value of φ_o. Equation (21) can then be used to calculate T_{eff}.

This calculation has been carried out for the tanh temperature distribution of the previous section and the results are shown in figure 2. It is seen that the basic pattern is similar to that of figure 1 but that, as expected, the values of T_{eff} are somewhat reduced.

A similar, but more complicated, analysis can be made for the case of non-constant density.

Summary

It is seen that the combined effect of the dominance of fast electrons and the rapid decrease in collision frequency with velocity leads to a significant modification of the sheath. A similar effect will modify the current-voltage characteristics of Langmuir probes and would lead to an overestimate of the temperature at the probe.

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Reference

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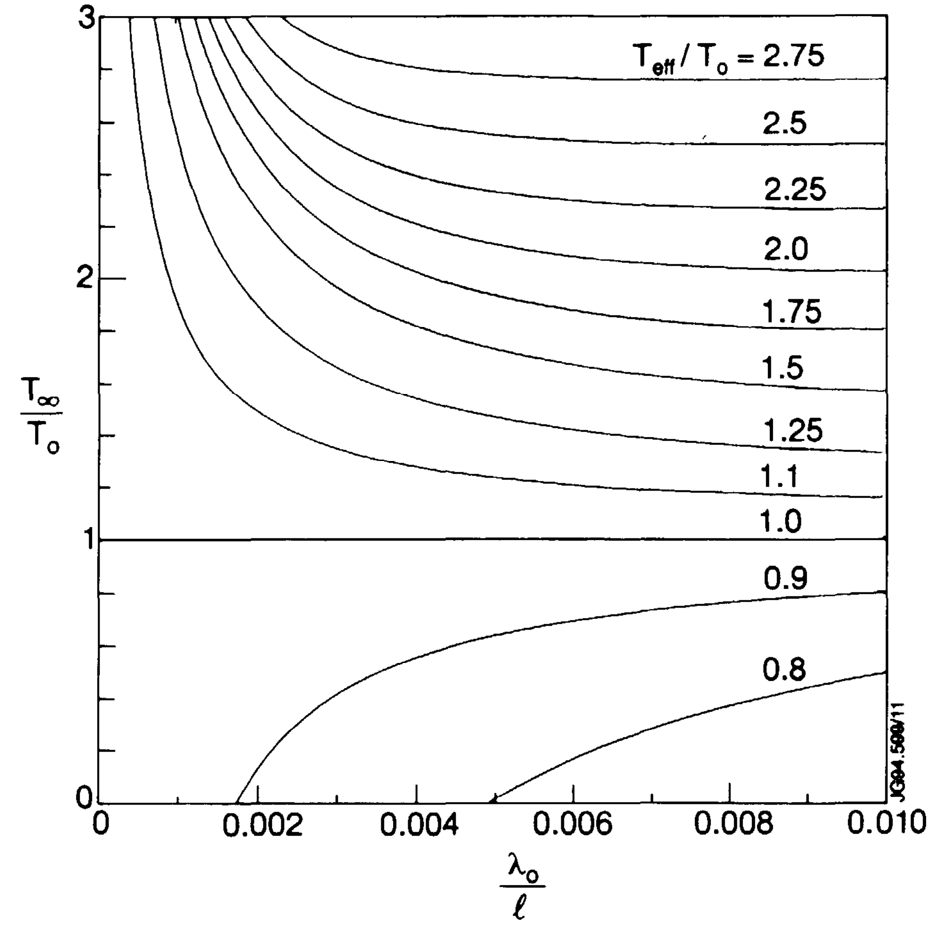

Fig. 1: Contours of equal effective temperature ratio in the (T_∞/T_0, λ/l) plane for $e\varphi_{0c}/T_0 = 3$. T_0 is the plasma temperature at the surface, λ_0 is the mean free path of a thermal electron at the surface, and l is the characteristic length of the temperature variation.

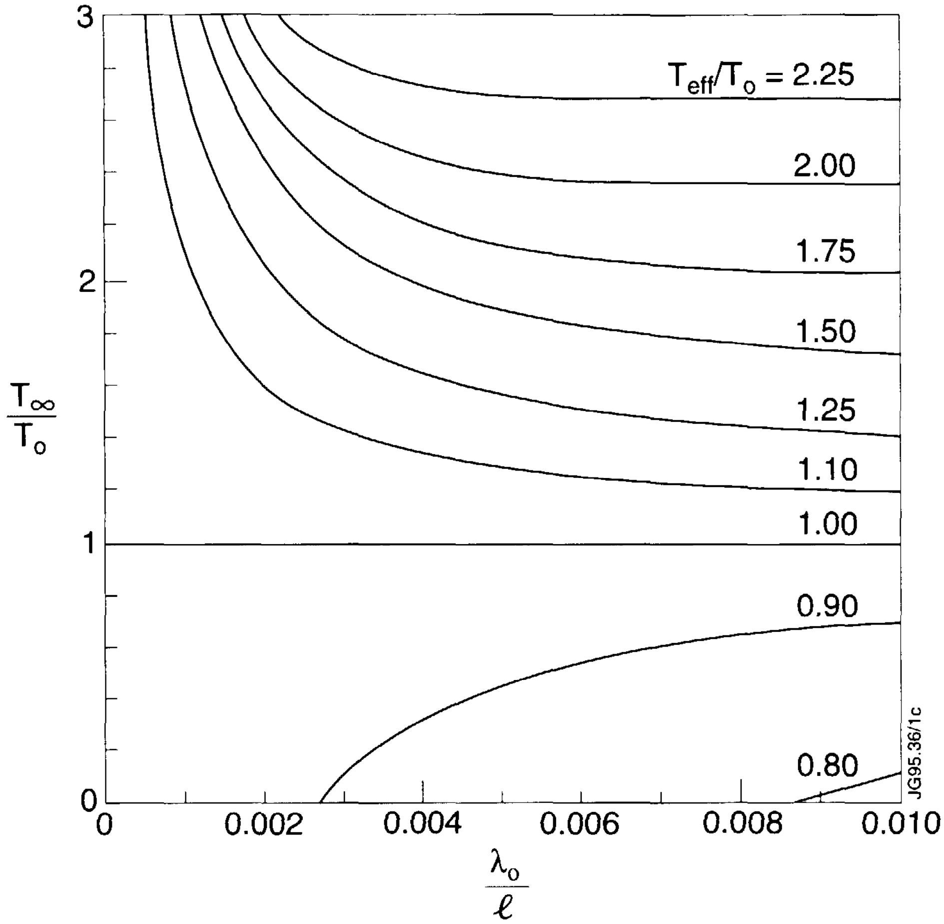

Fig.2: Contours of equal effective temperature ratio in the (T_∞/T_O, λ/ℓ) plane for $e\varphi_{OC}/T_O = 3$ including the effect of the potential variation outside the sheath.