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Dimensionless Form for the L-H Power Threshold

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ABSTRACT

The near separatrix region in tokamaks provides a new length scale parameter $x_0 = (-d \ln p_0/dx)^{-1}$ being of the order of several Larmor radii. Modified drift and interchange instabilities occur in this region due to longitudinal losses. The related turbulence gives rise to perpendicular transport and regulates the width x_0. The H-mode is set up when the shear flow stabilises these instabilities leading to the condition $\hat{\rho} = \rho_i/x_0 > c_1$ where c_1 is of order unity. The L-H power threshold depends strongly on the longitudinal loss mechanism. Classical thermal conduction provides a threshold scaling similar to the experimentally observed scaling, but the free-streaming loss yields a cubic dependence on B_0.

1. INTRODUCTION

Tokamak plasmas exhibit states of low (L) and high (H) confinement. Consequently, L-mode plasmas are characterised by higher amplitude turbulent activity than H-mode plasmas. Theory is still challenged to identify the underlying instability which is stabilised in the L-H transition. Following the transition the H-mode plasma develops steep temperature and density gradients near the plasma edge which are clearly destabilising for MHD perturbations. Therefore, theory has also to address the question why H-mode plasmas are stable. In this paper a possible explanation of the L-H power threshold being due to shear flow (SF) stabilisation of the drift and interchange modes is examined. Such modes can exist near the separatrix, where the magnetic field lines intersect the target plates. It is shown that this region introduces new physical effects and, therefore, new dimensional parameters.

The region where the physical effects of the open field lines become important includes part of the closed field line edge plasma as well as the scrape-off-layer (SOL) plasma. Due to the fast longitudinal losses just outside the last closed flux surface, i.e. the separatrix, on the time scale up to $L_{\|} / C_s$, where $2 L_{\|}$ is the distance along the field line between the target plates and C_s is the ion sound speed, steep gradients in the temperature and density build up. This strong gradient region is several ion gyro-radii wide. These gradients can drive resistive MHD and drift type instabilities. But they can also provide the strong shear flow stabilisation mainly due to the ExB drift [1,2]. It is assumed that this (SF) stabilisation of the interchange and drift waves gives the criterion for the L-H transition and further it will be shown that the criterion has a similar scaling to the experimental L-H power threshold. The stabilisation condition contains the Larmor radius as an essential parameter and it is therefore similar to FLR (Finite Larmor Radius) stabilisation.

The scientific method used is dimensional analysis, [3, 4], together with the mixing length arguments. This enables us to derive a rather general conclusion without knowing the specific details of the physical processes.

We suppose that the following physical picture applies: Near the separatrix there exists specific micro turbulence due to instabilities, which are connected with the SOL resistivity or the sheath resistance near the target plates, and this turbulence drives the local particle and the energy fluxes. Due to this transport (we speak here mainly about heat transport) there is a characteristic length scale for the temperature or the density near the separatrix x_0. If this scale length becomes less than some specific length connected with the ion Larmor radius - $f(\rho_i)$ - then the instabilities are stabilised. This effect decreases the local transport in this region and gives rise to the L-H transition. Furthermore we find the power scaling of the L-H transition follows from the condition $x_0 < f(\rho_i)$.

2. TURBULENT TRANSPORT COEFFICIENT $\chi_\perp$ FOR THE OPEN FIELD LINES REGION

Now we briefly discuss the derivation of the linear equations for the mutual drift and interchange modes [5] - [7], from which we define the dimensional parameters of the problem. One should emphasise here that it is not our main goal to get the complete solution of these equations but only to find the key dimensional parameters.

The ion continuity equation can be rewritten as:

$$\frac{\partial n}{\partial t} + \mathrm{div}(n\vec{v}_{\perp i}) = -\frac{\partial}{\partial s}\left(\frac{j_{\parallel i}}{e}\right), \qquad (1)$$

where the perpendicular velocity is given by:

$$\vec{v}_{\perp i} = \frac{c}{B_0^2}\left[\vec{B}\times\left(-\vec{E} + \frac{\nabla p_i}{en} + \frac{Md\vec{v}_\perp}{edt} + \frac{\nabla\vec{\pi}_i}{en}\right)\right]. \qquad (2)$$

By linearising we obtain the following ion continuity equation:

$$-i\omega\cdot n' - \frac{c}{B_0}\frac{dn_0}{dx}\frac{\partial\phi'}{\partial y} + i(\omega+\omega_*)\cdot\left(\frac{n_0 Mc^2}{eB_0^2}\right)\cdot\Delta_\perp\phi' - \frac{c}{eB_0R}\frac{\partial p_i'}{\partial y} = -\frac{\partial}{\partial s}\left(\frac{j_{\parallel i}}{e}\right) \qquad (3)$$

where the $'$ denotes perturbed quantities. ϕ is the electrostatic potential and ω_* is defined later.

The linearised version of the electron continuity equation, where the inertia and the stress tensor terms can be omitted, reads:

$$-i\omega\cdot n' - \frac{c}{B_0}\frac{dn_0}{dx}\frac{\partial\phi'}{\partial y} + \frac{c}{eB_0R}\frac{\partial p_e'}{\partial y} = \frac{\partial}{\partial s}\left(\frac{j_{\parallel e}}{e}\right). \qquad (4)$$

In the following we apply the quasi-classical approximation for the transverse dependence of perturbations:

$$\varphi'(x,y,s) = \varphi'(s)\cdot\exp\left(-i\omega\cdot t + i\bar{k}_\perp \bar{r}_\perp\right).$$

Then we obtain from equations (3), (4) an ordinary differential equation of second order along the longitudinal co-ordinate "s", i.e. along the magnetic field line. Instead of solving this equation exactly as done in [9] we average equations (3) and (4) along "s" between s = 0 and s = $L_\parallel$, supposing that the perturbations do not vary strongly along "s" [8]. After this averaging we match the end values for the ion and electron currents in the equations (3), (4) to their values for the sheath region. For the sheath currents we introduce the usual expressions:

$$j_{\parallel i}\big|_{s=L_\parallel} = env_{T_i} / 2(\pi)^{1/2}$$

for the ion current, and

$$j_{\parallel e}\big|_{s=L_\parallel} = -env_{T_e} / 2(\pi)^{1/2} \exp(-e\phi / T_e)$$

for the electron current. Here v_{Ti} (v_{Te}) denote the ion (electron) thermal velocity. The linearised forms of these expressions are:

$$j_{\parallel i}\big|_{s=L_\parallel} = j_{0i}\frac{n'}{n_0} \tag{5}$$

and

$$j_{\parallel e}\big|_{s=L_\parallel} = j_{0e}\left[-\frac{n'}{n_0} + \frac{e\phi'}{T_{0e}}\right]. \tag{6}$$

j_{0i} and j_{0e} denote the absolute value of the equilibrium currents.

The total perturbed current on the plate is given by the sum:

$$j'_\parallel\big|_{s=L_\parallel} = j'_{\parallel i}\big|_{s=L_\parallel} + j'_{\parallel e}\big|_{s=L_\parallel}.$$

Equation (3) can be simplified by substitution of Eq. (4). After obvious normalisations equations (3) and (4) can be rewritten as:

$$\begin{aligned} &(\omega+\omega_*)\cdot(k_\perp\rho_i)^2\cdot\frac{e\phi'}{T_0} + \omega_{*g}\cdot\frac{n'}{n_0} = -i\nu\cdot\left[(1-\xi)\cdot\frac{n'}{n_0} + \xi\cdot\frac{e\phi'}{T_0}\right] \\ &\omega_*\cdot\frac{e\phi'}{T_0} - \omega\cdot\frac{n'}{n_0} + \frac{\omega_{*g}}{2}\cdot\frac{n'}{n_0} = -i\nu\cdot\left[(-\xi)\cdot\frac{n'}{n_0} + \xi\cdot\frac{e\phi'}{T_0}\right] \end{aligned} \tag{7}$$

It should be emphasised that the term $\omega_{*g}\cdot(n'/n_o)$ in the second equation of Eq. (7) is small compared to the other terms and can be omitted. But this term has to be retained in the first equation of Eq. (7).

Here we introduce the notations:

$\omega_* = -\frac{2cT_0k_y}{eB_0n_0}\frac{dn_0}{dx}$ - the electron drift frequency (with the assumption that $T_i = T_e = T_0$);

$\rho_i^2 = \left(c^2T_0M / e^2B_0^2\right)$ - the square of the ion Larmor radius;

$\omega_{*g} = \frac{2cT_0k_y}{eB_0R}$ - the magnetic drift frequency;

$\nu = \frac{C_s}{L_\parallel}$ - characteristic frequency for the SOL plasma;

$C_s = (2T_0/M)^{1/2}$ - the ion sound velocity;

$\xi = \frac{j_{0\parallel e}}{j_{0\parallel i}}$ - the ratio of the equilibrium electron and ion current densities on the end plates. For the equilibrium we suppose that the total current is equal to zero but the local current densities can be different i.e. $\xi \neq 1$.

In the system Eq. (7) we neglect in contrast to Refs. [7], [8] the temperature gradient term and take into account only the density gradient term. Then this type of instability exists only if the current density near the end plates, i.e. in the region of the instability localisation, is non zero ($\xi \neq 1$; see (5), (6)). The instability due to the temperature gradient does not depend on the current density. The final result will then be determined by the combination of these two branches of the instability. In the case when the current density does not vanish the effect on the perturbation can be stabilising ($\xi < 1$) or destabilising ($\xi > 1$). If the current density on the target plates depends on the ion drift direction (towards the X-point or away from it) this then gives rise to a possible explanation for the dependence of the power threshold scaling on the magnetic drift direction.

The initial system (1) - (3) for non-electrostatic perturbations contains the following independent parameters [9]:

$$x_0, L_\parallel, R, \rho_i, C_s, \beta^*, \xi,$$

where x_0 denotes the pressure gradient length near the separatrix:

$$x_0 = \left(-\frac{dp_0}{p_0dx}\right)^{-1}, \tag{8}$$

$\beta^* = \left(L_\parallel^2 / Rx_0\right)\cdot\beta$ is the normalised beta. The parameter β^* appears in the more accurate solution of the system of equations (1), (3) and (4) (see Ref. [9]). $L_\parallel$ is the connection length.

From these combinations of parameters one can construct the following general expression for the turbulent transport coefficient:

$$\chi_\perp = \chi_{GB}\ f\left(\beta^*, \xi, L_\parallel / R, x_0 / R, \hat{\rho}\right). \tag{9}$$

Here

$$\chi_{GB} = \left(cT_0 / eB_0\right)\cdot\hat{\rho}\,, \tag{10}$$

is the Gyro-Bohm coefficient, defined with the normalised gyroradius:

$$\hat{\rho} = \rho_i / x_0\,, \tag{11}$$

and f is some dimensionless function of the dimensionless arguments.

In the next two sections the mixing length approximation is introduced, this contains additional information about the turbulent transport compared with the simple dimensional analysis and further reduces the number of the dimensional parameters in the transport coefficient $\chi_\perp$. This approximation has been successfully applied to turbulent transport caused by interchange instabilities in fluids. Consequently, the application to interchange instabilities in plasmas appears justified. In addition, the mixing length approximation is extended to other plasma instabilities, such as drift instabilities.

2.1 Resistive interchange instability

Let us start our discussion with the simplest case, namely that of the resistive interchange (RI) mode [5], [6]. For this instability branch the underlined terms have to be taken into account in Eq. (7):

$$(\omega + \omega_*)\cdot(k_\perp\rho_i)^2\cdot\frac{e\phi'}{T_0} + \underline{\omega_{*g}\cdot\frac{n'}{n_0}} = -i\nu\cdot\left[(1-\xi)\cdot\frac{n'}{n_0} + \xi\cdot\underline{\frac{e\phi'}{T_0}}\right],$$

$$\underline{\omega_*\cdot\frac{e\phi'}{T_0}} - \underline{\omega\cdot\frac{n'}{n_0}} = -i\nu\cdot\left[(-\xi)\cdot\frac{n'}{n_0} + \xi\cdot\frac{e\phi'}{T_0}\right].$$

The simplified equations give the result:

$$\gamma_{RI} = \frac{\omega_*\cdot\omega_{*g}}{\nu\cdot\xi}\,.$$

The growth rate increases as $(k_\perp)^2$ and its maximum is given by:

$$(\gamma)_{max} \approx C_s / (x_0 R)^{1/2}\ , \tag{12}$$

for the wave vector:

$$(k_\perp \rho_i)_{max} \approx \left(\frac{x_0}{L_\parallel} \cdot \frac{R}{L_\parallel} \right)^{1/4} . \tag{13}$$

Using the mixing length method($\chi_\perp = \left(\gamma / k_\perp^2\right)_{max}$) the following expression for the transverse heat conductivity $\chi_\perp$ (or for the diffusion coefficient) for the resistive interchange (RI) model is obtained

$$\chi_{\perp_{RI}} \approx \chi_{GB}\ f_{RI}\left(\beta^*, \xi, L_\parallel / R, x_0 / R, \hat{\rho}\right). \tag{14}$$

The function f_{RI} of Eq. (14) assumes the form

$$f_{RI}\left(\beta^*, \xi, L_\parallel / R, x_0 / R, \hat{\rho}\right) \propto L_\parallel / R, \tag{15}$$

for the double-null divertor when ω_{*g} is approximately constant and equals

$$f_{RI}\left(\beta^*, \xi, L_\parallel / R, x_0 / R, \hat{\rho}\right) \propto \left(L_\parallel / R\right) \cdot \beta^* \tag{16}$$

for the single-null configuration, see Ref. [9] (in both cases it is assumed that $\xi \sim 1$ and the ξ dependence is not taken into account).

From both these expressions it is seen that in the mixing length approximation for the interchange instability, neglecting the drift effects, the dimensionless function "f_{RI}" does not depend on the parameters, $\hat{\rho}$ and x_0/R.

Hence the following model for the function

$$f_{RI}\left(\beta^*, \xi, L_\parallel / R\right) \propto \left(L_\parallel / R\right) \cdot \left(\beta^*\right)^{\alpha}\ , \tag{17}$$

will be used from now on, to cover both single-null and double-null configurations where α is some fitting parameter of order unity. In this case the thermal diffusivity scales as:

$$\chi_{\perp RI} \propto \frac{L_\parallel^{1+2\alpha} n^{\alpha} T_0^{3/2+\alpha}}{R^{1+\alpha} x_0^{1+\alpha} B_0^{2(1+\alpha)}}\ . \tag{18}$$

2.2 Drift instability

For the drift instability again the underlined terms of Eq. (7) have to be taken into account.

$$\underline{(\omega+\omega_*)\cdot(k_\perp\rho)^2\cdot\frac{e\phi'}{T_0}}+\omega_{*g}\cdot\frac{n'}{n_0}=-i\nu\cdot\left[\underline{(1-\xi)\cdot\frac{n'}{n_0}}+\xi\cdot\frac{e\phi'}{T_0}\right],$$

$$\underline{\omega_*\cdot\frac{e\phi'}{T_0}}-\underline{\omega\cdot\frac{n'}{n_0}}=-i\nu\cdot\left[(-\xi)\cdot\frac{n'}{n_0}+\xi\cdot\frac{e\phi'}{T_0}\right].$$

The growth rate is

$$\gamma=\frac{\nu\cdot(\xi-1)}{(k_\perp\rho_i)^2}. \tag{19}$$

The maximum growth rate for the-drift instability is given by:

$$(\gamma)_{max}\approx\frac{C_s}{L_\parallel}\cdot\left(\frac{L_\parallel}{x_0}\right)^{2/3}, \tag{20}$$

and occurs for the wave vector, [7]:

$$(k_\perp\rho_i)_{max}\approx(x_0/L_\parallel)^{1/3}. \tag{21}$$

Using the mixing length expression $\chi_\perp=\left(\gamma/k_\perp^2\right)_{max}$ we find according to Refs. [7], [8]:

$$\chi_{\perp Dr}=\chi_{GB}\ f_{Dr}\left(\beta^*,\xi,L_\parallel/R,x_0/R,\hat{\rho}\right), \tag{22}$$

where χ_{GB} is given by (10), and the function f_{Dr} is found to be of the form

$$f_{Dr}\left(\beta^*,\xi,L_\parallel/R,x_0/R,\hat{\rho}\right)\propto\left(\frac{L_\parallel}{R}\right)^{1/3}\cdot\left(\frac{x_0}{R}\right)^{-1/3}. \tag{23}$$

Here we insert the transverse wave number given by (21). The dimensionless function f_{Dr} does not depend on β^* and $\hat{\rho}$.

The resulting heat conductivity (22) scales as:

$$\chi_{\perp Dr}\propto\frac{L_\parallel^{1/3}T_0^{3/2}}{x_0^{4/3}B_0^2}. \tag{24}$$

The ratio between the resistive interchange and drift instability thermal diffusion coefficients is:

$$\frac{\chi_{\perp RI}}{\chi_{\perp Dr}} = \frac{f_{RI}}{f_{Dr}} \approx \left(\frac{L_{\parallel}}{R}\right)^{2/3} \left(\frac{x_0}{R}\right)^{1/3} g(\beta^*) \,, \tag{25}$$

where $g(\beta^*)$ is a function of β^*, which depends on the divertor geometry. The quantity (25) is of the order of unity for a double-null divertor and smaller (for small β^*) for a single-null configuration.

Thus the dimensional analysis together with the mixing length argument give the following general expression for the turbulent transport for the mutual influence of interchange and drift instabilities;

$$\chi_{\perp} = \chi_{GB} \cdot f\left(\beta^*, \xi, L_{\parallel} / R, x_0 / R\right) \approx \chi_{GB} \cdot f\left(\beta^*, x_0 / R\right). \tag{26}$$

The last expression is correct if $\xi \sim 1$ and the weak dependence on $L_{\parallel}/R \sim q$ is omitted. Together with two limiting cases for the pure interchange and pure drift scalings from expression (26) conclusions about the power threshold scaling can be derived.

We adopt an ansatz for the function $f(\beta^*, x_0 / R)$ in form of a power dependence in β^* and in x_0 / R such that

$$\chi_{\perp} = \chi_{\perp GB} \cdot (\beta^*)^{\alpha} \cdot (x_0 / R)^{\delta}, \tag{26a}$$

where α and δ are constants. Their values can be specified by comparison of the combined drift-interchange turbulent transport with experiments.

3. THE CRITERION FOR STABILITY

As a criterion for stability we apply the condition:

$$V_0' > \gamma_{max} \,, \tag{27}$$

where $V_0' = dV_0 / dx$ is the shear velocity. This condition has been derived for both the interchange and drift modes [10]. It has a clear physical meaning. The perturbations have no time to develop fully and are suppressed due to the velocity shear drag.

From the expression (2) it is seen that the flow is generated mainly by the radial electric field and by the pressure gradient. It is reasonable to assume that the electrical potential is in the order of T_0/e. Then

$$V_0 \approx \frac{c}{B_0} \frac{d\phi_0}{dx} \approx \frac{c}{B_0} \frac{1}{en_0} \frac{dp_0}{dx} \approx \frac{c}{B_0} \frac{T_0}{ex_0} \,.$$

This gives the following estimate for the shear flow:

$$V_0' \approx \frac{C_s}{x_0} \cdot \frac{\rho_i}{x_0} \; .$$

Combining the condition (27) with the expression for maximum interchange growth rate (16) the following stability condition for the resistive interchange is found:

$$\hat{\rho}_{RI} > \left(\frac{x_0}{R} \right)^{1/2} \; . \tag{28}$$

This condition can also be directly derived by the Finite-Larmor-Radius (FLR) stabilisation of interchange instabilities. For typical tokamak edge parameters ($B_0 \sim 3T$, $Ti \sim 400eV$ and $R =$ 300cm for JET) x_0 is about one centimetre and ρ_i about half a millimetre which shows that the condition (28) is realistic.

The same stabilisation argument applied to the drift instabilities, gives after inserting expression (20):

$$\hat{\rho}_{Dr} > \left(\frac{x_0}{L_\parallel} \right)^{1/3} = \left(\frac{R}{x_0} \frac{R^2}{L_\parallel^2} \right)^{1/6} \left(\frac{x_0}{R} \right)^{1/2} \approx \left(\frac{x_0}{R} \right)^{1/2} \; . \tag{29}$$

Clearly criteria (28) and (29) are very similar for realistic values of x_0, R and $L_\parallel$. When the stability threshold is satisfied, the maximum growth rates (see Eq. (9), (20)) are stabilised. But shorter wave-length perturbations will still be unstable. In order to stabilise all perturbations conditions (28) and (29) have to be more strict, i.e.

$$\hat{\rho} > c_1 \; , \tag{30}$$

where the constant c_1 is of order unity. Therefore condition (30) will be used from now onwards and this is our main hypothesis for the derivation of the L-H threshold scaling. Strictly the stability threshold applies for the linear phase. But the more stringent criterion should suppress the instabilities in the nonlinear phase, too. Recently, the nonlinear diamagnetic stabilisation of resistive ballooning turbulence has been demonstrated by computer simulations [11]. In conclusion, condition (30) with c_1 of order unity is a reasonable hypothesis.

4. THE GRADIENT LENGTH X_0 AND THE SCALING FOR THE L-H THRESHOLD

The gradient length x_0 can be estimated by the (simplified) energy balance equation near the separatrix:

$$\frac{\partial}{\partial t}T_0 = \frac{\partial}{\partial x}\chi_\perp \frac{\partial}{\partial x}T_0 - \frac{1}{\tau_\parallel}T_0 \,.$$

Here the $\tau_\parallel$ is the characteristic time of the longitudinal losses. Integrating this equation and inserting the total power which diffuses into the SOL as a boundary condition gives the following balance relations:

$$\frac{P}{n_0 S} \approx \frac{\chi_\perp T_0}{x_0} \approx \frac{x_0 T_0}{\tau_\parallel} \,. \tag{31}$$

Here P denotes the total power which diffuses across the separatrix and S the tokamak surface $S \approx 4\pi^2 aR$ for the simple case of a circular cross-section tokamak with minor radius a. From (31) it follows that the product x_0T_0 can be obtained as a function of P, once $\tau_\parallel$ is known. The longitudinal losses are described by two different models. In the first regime the losses are due to classical longitudinal thermal conduction, i.e. if $\lambda < L_\parallel$:

$$\tau_\parallel^{th} = \frac{L_\parallel^2}{\chi_\parallel} \propto \frac{L_\parallel^2 n_0}{T_0^{5/2}} \tag{32}$$

and the second one is due to free-streaming flow, i.e. if $\lambda > L_\parallel$:

$$\tau_\parallel^{fs} = \frac{L_\parallel}{C_s} \propto \frac{L_\parallel}{T_0^{1/2}} \,. \tag{33}$$

Here λ is the particle mean free path.

With Eq. (31) the heat balance equation implies:

$$x_0^2 = \chi_\perp \cdot \tau_\parallel \,. \tag{34}$$

The marginal condition $\hat{\rho} \simeq 1$ can be rewritten as:

$$T_0 \propto x_0^2 B_0^2 \,. \tag{35}$$

If our suppositions are valid then the dependence (35) between the temperature on the separatrix T_0, its pressure gradient length x_0 and the magnetic field B_0 should give the marginal stable case.

4.1 Scaling for the resistive interchange instability

Making use of the results (35) and (17) gives the transverse heat conductivity:

$$\chi_{\perp RI} = \frac{L_{\parallel}}{R}\chi_{GB}\left(\beta^*\right)^{\alpha} \propto \frac{L_{\parallel}}{R}B_0 x_0^2\left(\frac{L_{\parallel}^2 x_0 n_0}{R}\right)^{\alpha} .$$

Inserting this result into Eq. (34) and taking into account relation (35) gives the scaling of x_0 with B_0, and then using Eq. (31) the scaling of P with B_0.

a) The free-streaming loss case Eq. (33) gives the following scaling for the L H power threshold:

$$\frac{P}{n_0 S} \propto B_0^3 \frac{n_0^{\frac{4\alpha}{1-\alpha}}}{L_{\parallel}}\left(\frac{L_{\parallel}^2}{R}\right)^{\frac{4(1+\alpha)}{1-\alpha}} . \qquad (36)$$

For any value of α the L-H power threshold scales strongly, i.e. as B_0^3, with the magnetic field.

b) The classical loss case Eq. (32) gives the following scaling:

$$\frac{P}{n_0 S} \propto \frac{x_0 T_0}{\tau_{\parallel}} \propto \frac{B_0^7 x_0^8}{L_{\parallel}^2 n_0} \propto \frac{B_0^7}{L_{\parallel}^2 n_0}\left[\frac{L_{\parallel}^{\frac{8(3+2\alpha)}{5-\alpha}} n_0^{\frac{8(1+\alpha)}{5-\alpha}}}{R^{\frac{8(1+\alpha)}{5-\alpha}} B_0^{\frac{32}{5-\alpha}}}\right] . \qquad (37)$$

For the single-null divertor configuration ($\alpha = 1$) we obtain:

$$\frac{P}{n_0 S} \propto \frac{n_0^3}{B_0}\left(\frac{L_{\parallel}^2}{R}\right)^4 , \qquad (38)$$

and for the double-null configuration ($\alpha = 0$):

$$\frac{P}{n_0 S} \propto \left(\frac{L_{\parallel}^2}{R}\right)^{8/5} n_0^{3/5} L_{\parallel}^{-2/5} B_0^{3/5} . \qquad (39)$$

4.2 Scaling for the drift instability

a) Free-steaming losses

Inserting the perpendicular heat transport coefficient (24) into (31) together with (33) and taking into account criterion (35) we obtain:

$$\frac{P}{n_0 S} \propto B_0^3 L_{\parallel}^3 . \qquad (40)$$

Again there occurs a strong, i.e. cubic, dependence on magnetic field.

b) Classical losses.

For the classical losses the result is:

$$\frac{P}{n_0 S} \propto n_0^{1/2} L_{\parallel}^{3/2} B_0 \, . \tag{41}$$

4.3 Combined drift-interchange instability

For the general form of the perpendicular heat conductivity, Eqs. (26) and (26a), there are two free constants, namely α and δ. In order to define their values two additional constraints are required.

a) In the collisionless limit the free-streaming parallel losses yield a cubic dependence for any value of α and δ.

$$\frac{P}{n_0 S} \propto B_0^3 f(n_0,\alpha,\delta), \tag{42}$$

where the function f assumes the form

$$f(n_0,\alpha,\delta) = n_0^{-4\alpha/(\alpha+\delta-1)} \tag{42a}$$

b) For classical parallel losses in the collisional limit we find

$$\frac{P}{n_0 S} \propto \frac{B_0^{(7\alpha+7\delta-3)/(\alpha+\delta-5)}}{n_0^{(9\alpha+3)/(\alpha+\delta-5)}}. \tag{43}$$

This result is discussed in detail in the next section.

5. DISCUSSION

Preliminary studies by the ITER data base working group [12] of the scaling of the power threshold with plasma parameters have shown that

$$P > 0.04 n_0 B_0 S \tag{44}$$

gives a reasonable fit to the multimachine threshold data base. Where S is the surface area in m^2, n_0 the average density in 10^{20} m^{-3}, B_0 the toroidal field in tesla, with P the power in MW.

Eq. (44) is not dimensionally correct and in a more recent article [13] two dimensionally correct forms have been presented that give a reasonable fit to the data. Both forms assume that the linear B_0 dependence is correct, and the first form also takes the quadratic dependence on length scale as correct and adjusts the n_0 dependence:

$$P > 0.025 n_0^{0.75} B_0 S. \tag{45}$$

In the second form the n_0 dependence is assumed to be correct and the length scale is adjusted to yield a dimensionally correct result:

$$P > 0.4 n_0 B_0 R^{2.5}. \tag{46}$$

In deriving the above two forms it was also assumed, as in the present paper, that atomic physics is unimportant and that the dimensionless plasma physics parameters β, ρ^* and ν^* are the only significant parameters.

Comparing the results of Sec. 4 with the above experimental scaling expressions the following observations can be made: The strong, cubic dependence of the threshold power on B_0 for the free streaming case (42), which is characteristic of this case, is not apparent in the present data. However in the classical loss case for the shear stabilisation of drift waves there is a simple linear dependence of the threshold power on the toroidal field Eq. (41), and for the shear stabilisation of the resistive interchange instability one can also recover the linear scaling with B for $\alpha = -{}^1/_3$ in Eq. (37).

It is very improbable that the pure drift or interchange instability is realised in experiments. It is more likely that both effects occur simultaneously. In that case the generalised scaling (26a) should be applied to fit the experimental data, as presented in (48), (49), more accurately.

From Eq. (43) in the classical parallel loss case, for a linear dependence of the threshold power on the magnetic field, the coefficients α and δ must satisfy the condition:

$$3\alpha + 3\delta = -1. \tag{47}$$

Naturally, this condition contains the drift instability scaling (41) as a special case. The next constraint is found by fitting the density dependence for the scalings (45) and (46).

The density dependence in the scaling (45) is reproduced, if the condition

$$3\alpha + 3\delta = -7 \tag{48}$$

is satisfied.

From (47), (48) we obtain $\alpha = -1/2$ and $\delta = 1/6$. Then the transport coefficient has the form:

and for the combined drift-interchange instability both of the dimensionally correct experimental scalings Eqs. (45) and (46) can be obtained through small changes to the parameters α and δ. In the free streaming case the stabilisation of both modes leads to a cubic dependence of the power threshold on the toroidal field. Although this is clearly stronger than the experimental dependence on B in the medium to high range of densities, it may be the explanantion of the low density H-mode limit since it is at low densities that the free streaming limit will be satisfied.

Considering the case of SOL drift type instabilities the dependence of the L-H threshold on the net current onto the target plates provides the possibility for control. It follows from Eq. (7b) that for an ion current in the instability region exceeding the corresponding electron current, i.e. $\xi < 1$, the plasma becomes more stable. Consequently the condition for L-H threshold is more favourable, i.e. lower. If the ion drift is in the direction towards the X-point then the ion current onto the target plates just outside the separatrix should be larger than in the case when the ion drift direction is away from the X-point. In contrast, the electrons stick close to the separatrix. This fact can explain the difference in the observed L-H transition power threshold.

The presented model allows to derive conclusions on detached divertor plasmas [14]. In this case a cold layer develops in front of the target plates, which suppresses the currents onto the target plates. It is concluded that the SOL drift instability will be absent but the SOL interchange instabilities become more severe due to the absence of line-tying of the magnetic field. As a consequence the L-H threshold should increase.

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