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Turbulent Transport of Diluted Impurities

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Abstract:

Diluted impurity transport is analyzed in the presence of both neoclassical and turbulent transport processes. Comparison with impurity injection experiments in JET is carried out, with good qualitative agreement.

I. Introduction.

Various kinds of impurities are present in plasmas generated in magnetic confinement devices. When occurring in large concentration, impurities have detrimental effects on the machine performance. For a given electron density, whose upper value is determined by the machine operational limits, impurities replace the main ion species, thus reducing the fusion reactivity. Moreover impurities increase the radiation losses, causing a degradation of the energy confinement.

On the other hand, a small concentration of a selected impurity species is a useful investigation tool of the plasma transport properties. Provided that the concentration is sufficiently low, the impurity dynamics do not affect the bulk plasma evolution. In this conditions impurities can be considered as test particles, and their dynamics can be derived directly from the knowledge of local plasma properties. Conversely, measurements of the impurity motion gives indirect information on the local plasma properties.
The precise meaning of "low concentration" depends on the physics under study. For collisionless (collective) processes, such as plasma turbulence, the condition is simply

that the total impurity charge is much less than the total electron charge. Instead, for collisional processes, such as neoclassical transport, the stricter condition that the increment of the effective charge is much less than unity must be used, as discussed in the next sections.

It is the scope of this work to analyze the dynamics of diluted impurities in the presence of various types of both collisional and collisionless effects. It will be shown that the emerging picture is consistent with the experimental observations in various regimes of machine operations.

This work is organized as follows. In Sec. II the classic Hasegawa-Mima model for electrostatic turbulence is extended to accommodate an impurity species. In this way one can show that, at sufficient dilution, impurities behave as test particles. An important consequence is that their turbulent diffusivity can be much higher than the bulk particle diffusivity. In Sec. III the scaling laws of test particle diffusivity is briefly reviewed. The structure of the transport matrix in the more general case when neoclassical effects are included is discussed in Sec. IV. The complementarity of the additional turbulent effects with those proposed by Stringer is emphasized. Sec. V is devoted to the comparison with the experimental findings. Qualitative agreement with the analysis of Sec. IV is found. Final considerations and conclusions are drawn in Sec. VI.

II. The Hasegawa-Mima model with an impurity species.

In order to clarify the differences between transport of bulk particles and transport of diluted impurities we consider the extension of the well-known paradigm for electrostatic turbulence, the Hasegawa-Mima equation[1], when a small amount of impurities is included.

As it is customary, one starts from the Braginskii equations[2], assuming cold ions, adiabatic electrons and restricting the ion dynamics to the plane perpendicular to the equilibrium magnetic field. Two ion species are considered, a hydrogenoid main ion species and an impurity species of charge Z, whose densities are denoted as n_i and n_I, respectively. The continuity equations are therefore

$$\partial_t n_i + \nabla \cdot (n_i \mathbf{v}_i) = 0 \tag{1}$$

$$\partial_t n_I + \nabla \cdot (n_I \mathbf{v}_I) = 0 \tag{2}$$

Since the electrons are assumed adiabatic, the quasineutrality constraint takes the form:

$$n_i + Z n_I = n_o (1 + e\varphi / T_e) \tag{3}$$

where n_o is the equilibrium density, φ the electrostatic potential and T_e the electron temperature. Eqs. (1-3) are closed by an expression for the drifts $\mathbf{v}_i$ and $\mathbf{v}_I$ obtained from a low frequency expansion of the perpendicular momentum equations. Here we include the electric drift $\mathbf{v}_E = c\mathbf{E} \times \mathbf{B} / B^2 = (c/B)\hat{\mathbf{b}} \times \nabla\varphi$ and the polarization drifts $\mathbf{v}_{pi} = (1/\omega_i)\hat{\mathbf{b}} \times (d\mathbf{v}_E / dt)$ and $\mathbf{v}_{pI} = (1/\omega_I)\hat{\mathbf{b}} \times (d\mathbf{v}_E / dt)$ only:

$$\mathbf{v}_i = \mathbf{v}_E + \mathbf{v}_{pi} \tag{4}$$

$$\mathbf{v}_I = \mathbf{v}_E + \mathbf{v}_{pI} \tag{5}$$

Here ω_i and ω_I are the ion cyclotron frequencies and $\hat{\mathbf{b}}$ is the direction of the equilibrium magnetic field. The convective derivative is defined as $d/dt \equiv \partial_t + \mathbf{v}_E \cdot \nabla$. An equation for φ is obtained by adding Eq. (1) to Eq. (2) multiplied by Z and employing Eqs. (3-5):

$$n_o \partial_t (e\varphi / T_e) + \nabla \cdot (n_o \mathbf{v}_E) + \nabla \cdot [(n_i + A n_I)\mathbf{v}_{pi}] = 0 \tag{6}$$

where the impurity polarization drift has been eliminated in favour of $\mathbf{v}_{pi}$ (A is the atomic mass number). For diluted impurities, $A n_I / n_i \approx Z n_I / n_i << 1$ and the impurity density drops out of Eq. (6), to the leading order. The impurity dynamics is then given by Eq. (2) where the electric potential can be considered *prescribed* since its dynamics obeys other processes independent of the impurity dynamics. In the present simple example $n_i \approx n_o(1 + e\varphi / T_e)$ because the plasma is almost pure, and φ obeys the familiar Hasegawa-Mima equation, as one can derive from Eq. (6):

$$\partial_t(e\varphi / T_e) + \mathbf{v}_E \cdot (\nabla n_o / n_o) + \nabla \cdot \mathbf{v}_{pi} = 0 \qquad (7)$$

In two-dimensional models, collisional transport is associated with perpendicular friction which induces $\mathbf{F} \times \mathbf{B}$ radial flows. The leading collisional effect is the friction due to the differential diamagnetic velocities of the two ion species. This amounts to the addition of the following drifts to Eqs. (4-5):

$$\mathbf{v}_{Fi} = -(\nu_{iI} / \omega_i)(cT_e / eB)(\nabla n_i / n_i - \nabla n_I / Zn_I) \qquad (8)$$

$$\mathbf{v}_{FI} = -(\nu_{Ii} / \omega_I)(cT_e / eB)(\nabla n_I / Zn_I - \nabla n_i / n_i) \qquad (9)$$

where $\nu_{iI} = (Z^2 n_I / n_i)\nu_{ii}$ and $\nu_{Ii} = (Z^2 m_i / m_I)\nu_{ii}$ are ion-impurity and impurity-ion collision frequencies and ν_{ii} is the standard ion-ion collision frequency (heavy impurities such that $m_i / m_I << 1$ are assumed). In deriving Eq. (8-9) the ion and electron temperatures are taken equal and constant.
The fluxes associated with the drifts (8-9) are automatically ambipolar,

$$n_i \mathbf{v}_{Fi} + Zn_I \mathbf{v}_{FI} = 0 \ , \qquad (10)$$

which implies that Eq. (7), which governs turbulence, is unaffected by the inclusion of the frictional drifts.

In order to make progress we now assume that the correlation length of electrostatic turbulence λ_c is much smaller than the system size a. Then, at intermediate scalelengths l such that $\lambda_c << l << a$, the turbulent transport is diffusive. One can then average the turbulent impurity flux over the small scale fluctuations:

$$\langle n_I \mathbf{v}_I^{turb} \rangle = -D_{turb} \nabla \langle n_I \rangle \qquad (11)$$

where $\mathbf{v}_I^{turb}$ is the noncollisional part of the impurity drift and D_{turb} is the turbulent diffusivity, which is a functional of the fluctuation statistics. The (large scale) impurity transport equation (2) then becomes (the average symbol $\langle \cdot \rangle$ is omitted)

$$\partial_t n_I + \nabla\cdot(-D_{turb}\nabla n_I) - \nabla\cdot[(\nu_{Ii}/\omega_I)(cT_e/eB)(\nabla n_I/Z - n_I\nabla n_i/n_i)] = 0 \qquad (12)$$

Similarly, one can derive an equation for the main ion species. In this case, one cannot use an equation like (11), because the evolution of the electric potential is not independent of the dynamics of the main ion density. In fact, φ is strongly correlated with n_I through Eq. (3). Solving for n_i and recognizing that the electric drift is dominant one obtains

$$\langle n_i \mathbf{v}_i^{turb}\rangle = ZD_{turb}\nabla\langle n_I\rangle \qquad (13)$$

and

$$\partial_t n_i + \nabla\cdot(ZD_{turb}\nabla n_I) - \nabla\cdot[(\nu_{iI}/\omega_i)(cT_e/eB)(\nabla n_i - n_i\nabla n_I/Zn_I)] = 0 \qquad (14)$$

In Eq. (12,14) one can recognize the collisional diffusive fluxes (the diagonal terms in the transport matrix) and the pinch terms. One can also note that while the turbulent impurity diffusive flux can be substantial, there is no similar term in Eq. (14). The only turbulent contribution to Eq. (14) is a small pinch term which is needed to maintain the overall ambipolarity. Clearly this occurs because electrons are assumed adiabatic: no overall particle transport is allowed in this example, but only ambipolarity-preserving relative motion between the main ion species and the impurity.

This example is interesting because it proves the principle that impurities can experience effective diffusivities which differ from those of the main ion, although the basic dynamics is the same, $\mathbf{E}\times\mathbf{B}$ drift. Conversely, when analyzing experimental data on trace impurity diffusivity, no direct inference about bulk density transport can be made.

III. Brief review of test particle transport in two-dimensional $\mathbf{E}\times\mathbf{B}$ turbulence.

We now review some results about the test particle diffusivity. We limit the discussion to the evaluation of the diffusivity due to $\mathbf{E}\times\mathbf{B}$ (incompressible) turbulence, which is the dominant effect.

Eq. (2) can be formally solved with the method of characteristics and then averaged over the fluctuation ensemble. One can conveniently write the result in the form:

$$\langle n_I \rangle = \int g(\mathbf{x}-\mathbf{x}',t) n_o(\mathbf{x}') d\mathbf{x}' \tag{15}$$

where

$$g(\mathbf{x}-\mathbf{x}',t) = \langle \delta[\mathbf{x}-\mathbf{x}'-\mathbf{v}(t)] \rangle \tag{16}$$

is the particle propagator ($\mathbf{v}(t)$ is the instantaneous particle velocity) and $n_o(\mathbf{x})$ the initial distribution. The goal of the theory is to compute the propagator or at least some of its moments. The result will in general depend on the details of turbulence. However, at sufficiently long scalelengths, $l >> \lambda_c$, and timescales, $t >> \tau_c$, where τ_c is the turbulence correlation time, the Central Limit Theorem ensures that the propagator is Gaussian:

$$g(\mathbf{x}-\mathbf{x}',t) \propto \exp(-|\mathbf{x}-\mathbf{x}'|^2 / D_{turb} t) \tag{17}$$

where D_{turb} is the turbulent diffusivity. Indeed inserting Eq. (17) in Eq. (15) and taking the time derivative one recovers a standard diffusion equation for the average density, with a flux of the form (11).

D_{turb} is in general a functional of the turbulence features, i.e. λ_c, τ_c and the r.m.s. velocity $\mathrm{v_E}$. Upon introducing a dimensionless parameter, the Kubo number $K = \mathrm{v_E}\tau_c / \lambda_c$, one can write:

$$D_{turb} = \mathrm{v_E} \lambda_c f(K) \tag{18}$$

The various asymptotic forms of the function $f(K)$ found in the literature have been assessed numerically in Ref. [3] (see Fig. 1).

Not surprisingly, in the quasilinear regime, when $K << 1$, the diffusivity is found to agree with the analytic prediction, $f(K) \approx K$. In the large Kubo number regime $K >> 1$ the work of Gruzinov et al.[4] predicts a power law:

$$f(K) \sim K^{-\alpha} \quad , \qquad K >> 1 \tag{19}$$

where $\alpha = 3/10$. The numerical assessment[3] shows qualitative agreement with Ref. [4], in the sense that the diffusion is found to be dominated by a small population of particles wandering in fractal-like trajectories over many turbulence correlation lengths. However the measured exponents turns out somewhat smaller $\alpha = 0.2 \pm 0.04$. (The error corresponds to three standard deviations). It is not clear what is the origin of the discrepancy.

One possibility is that the value $\alpha = 3/10$ is not universal. Indeed the exponent computed in Ref. [4] is obtained by constructing a model of the turbulent velocity field and showing that the statistical problem posed by the diffusion in that specific field belongs to the same universality class as certain percolation problems. The authors are therefore able to compute the exponent α from the (exactly known) exponents of the percolation problem. However, in order to show that $\alpha = 3/10$ is universal one must also show that the chosen model of the velocity field is representative of the general class of two-dimensional turbulent fields. Although plausible, this connection is not clear from the analysis of Ref. [4].

Another possibility is that $\alpha = 3/10$ is universal, but that there are corrections to scaling not small enough when evaluated at the Kubo numbers used in the simulations. This possibility is difficult to assess in the absence of a more detailed analytic theory.

In any case, the experimental data seem to suggest that the Kubo number is not much bigger than unity. For the purpose of estimating the role of the various terms occurring in the transport matrix it is therefore sufficient to compute the turbulent diffusivity at the upper bound given by Eq. (18), by taking $f(K) = f(K_{\max}) = 1$ and $D_{turb} = v_E \lambda_c \approx c\varphi / B$. By assuming adiabatic electrons, $e\varphi / T_e \approx \tilde{n} / n_o$, one can relate, at least qualitatively, the test particle turbulent diffusivity to the density fluctuations

$$D_{turb} \approx (cT_e / eB)(\tilde{n} / n_o) \tag{20}$$

The density fluctuation level $\tilde{n}/n_o$ is commonly measured in various machines. It usually grows from less than 1% at the center to up to 50% at the plasma edge. For most of the plasma radius, this increase is only partially compensated by the decrease of the temperature towards the edge. Thus the turbulent diffusivity increases with minor radius.

IV. Extension to neoclassical theory.

Unlike, the classical transport discussed in Sec. II, which is due to the perpendicular friction, neoclassical transport is linked to the friction parallel to the magnetic field. Detailed calculations, which in certain regimes require the use of kinetic theory, are beyond the scope of this work. Here we maintain the same assumption as in the previous sections, that the plasma is composed by two ion species, one of which is a trace impurity, and that the electrostatic turbulence is governed by an autonomous equation and is small scale.

Following the Stringer viewpoint[5], we allow for an explicit dependence of the fluxes on the radial electric field, which is determined by the *overall ambipolarity constraint.* Assuming again constant temperatures, the radial particle fluxes possess the following general structure:

$$\Gamma_i = n_i D_i[-(1/n_i)(dn_i/dr) + eE_r/T_e] + ZD_I^{turb}(dn_I/dr) - D_{bulk}^{turb}(dn_e/dr) \quad (21)$$

$$\Gamma_I = n_I D_I[-(1/n_I)(dn_I/dr) + ZeE_r/T_e] - D_I^{turb}(dn_I/dr) \quad (22)$$

$$\Gamma_e = n_e D_e[-(1/n_e)(dn_e/dr) - eE_r/T_e] - D_{bulk}^{turb}(dn_e/dr) + \Gamma_e^{magn} \quad (23)$$

The form of the neoclassical transport coefficients can be taken as $D_\alpha = A_{ncl}\rho_\alpha^2 \nu_\alpha$, with $\alpha = i$, I or e, where ρ_α is the Larmor radius, ν_α is the collision frequency and A_{ncl} is a neoclassical enhancement factor which depends on the collisionality regime.

As in Stringer, we have allowed for a possible *intrinsically non ambipolar* electron flux Γ_e^{magn} due to magnetic fluctuations or occasional strong magnetic events (such as ELMs). We have also included a contributions from the electrostatic turbulence, one for the impurities, with diffusivity D_I^{turb} and one for the bulk with diffusivity D_{bulk}^{turb}. As we have seen in Sec. II, D_I^{turb} and D_{bulk}^{turb} need not be equal or even of the same order, although both are due to the same kind of turbulence. Note also that the poloidal flow has been set to zero, since its role is simply additive to the electric field, without affecting the fluxes. The Ware pinch has also been ignored.

The intrinsic ambipolarity of the turbulent terms in Eqs. (21-23) implies that the radial electric field is determined by the same mechanism as in Stringer. In particular, in the absence of magnetic effects, the impurity pinch is neoclassical. Ignoring the electron collisional contribution ($D_e \to 0$) and eliminating the electric field using the ambipolarity condition $\Gamma_i + Z\Gamma_I = \Gamma_e$ one obtains:

$$\Gamma_I = n_I D_I Z_w^{-1}[-(1/n_I)(dn_I/dr) + (Z/n_i)(dn_i/dr) + Z\Gamma_e^{magn}/(n_i D_i)] - D_I^{turb}(dn_I/dr) \quad (24)$$

where $Z_w \equiv 1 + Z^2 n_I D_I/(n_i D_i)$ is a coefficient of order one. Note that $Z_w \to 1$ when the incremental effective charge $\delta Z_{eff} \equiv Z^2 n_I / n_e$ is much less than unity. In this limit the selected impurity species behaves as a test particle even in respect of collisional theory. Indeed in this limit the radial electric field is determined by the ambipolarity condition involving only the bulk species.

In Eq. (24), the three terms in the square bracket are proportional to the neoclassical impurity diffusivity, the neoclassical impurity pinch and the outward "anomalous" pinch due to the magnetic events. All these terms are present in Stringer's work. The last term is the test particle flux due to electrostatic turbulence.

In addition one could introduce a turbulent pinch if the nature of electrostatic turbulence allows it. Mechanisms for the anomalous impurity pinch, relying on compressibility effects, have been proposed by Cowley[6] and Isichenko et al. [7]. Smolyakov and Yushmanov[8] have considered modifications to D_I^{turb} due to drift orbits effects. These modifications would not alter the form of Eq. (24). Moreover the regimes considered in Ref. 8 are unlikely to occur in usual experiments.

The relative importance of the various terms occurring in Eq. (24) is discussed in the next section, for typical machine operation regimes.

V. Interpretation of the experiments.

In laser blow-off experiments, a small amount of impurities is injected in the plasma and the evolution of the initial distribution is followed with spectroscopic techniques[9]. In order to discuss the various scenarios, it is convenient to introduce a pinch parameter P:

$$P = -aV_{\mathrm{pinch}} / D \qquad (25)$$

where a is the minor radius, V_{pinch} is the total pinch velocity:

$$V_{\mathrm{pinch}} = V_{\mathrm{neo}} + V_{\mathrm{magn}} = D_I Z_w^{-1}[(Z/n_i)(dn_i/dr) + Z\Gamma_e^{magn}/(n_i D_i)] \qquad (26)$$

and D is the total diffusivity:

$$D = D_{\mathrm{neo}} + D_{\mathrm{turb}} = D_I Z_w^{-1} + D_I^{turb} \quad . \qquad (27)$$

From Eq. (18) one can see that in steady state ($\Gamma_I = 0$) the pinch parameter is the ratio of the minor radius to the impurity profile scalelength:

$$P = -a(1/n_I)dn_I/dr \qquad \text{(in steady state)} \qquad (28)$$

Since a is also a measure of the equilibrium density scale length, P is also a local measure of the relative peaking of the impurity profile in steady state: $P \approx (d\ln n_I/dr)/(d\ln n_i/dr)$.

We first estimate the various contributions to Eq. (25) coming from Eq. (24) in a good H-mode discharge. We start noting that the neoclassical impurity diffusivity is of the order of the bulk ion neoclassical thermal conductivity. In a good H-mode, the latter is not much smaller than the actual (anomalous) conductivity and diffusivity. One can therefore assume $D_{\mathrm{turb}} / D_{\mathrm{neo}} \approx 2 \div 5$. Also, we neglect magnetic activity, $\Gamma_e^{magn} \approx 0$. This leads to:

$$P_{\text{H-mode}} \approx (0.2 \div 0.5) Za (d \ln n_i / dr) \qquad (29)$$

In Ni injection experiments in JET[9] $P \sim 10^2$ at the edge (see Fig. 2). This is consistent with Eq. (29), since typically the ionization number is $Z \approx 20$ and $a(d \ln n_i / dr) > 10$, because of the density pedestal. Thus one can see how Eq. (29) predicts strong impurity accumulation in quiescent H-mode plasmas.

In Ohmic and L-mode operations, the turbulent diffusivity greatly exceeds the collisional one, $D_{\text{turb}} / D_{\text{neo}} >> 1$, especially at the edge where the fluctuation level is measured to be $e\varphi / T_e \approx \tilde{n} / n \approx 0.1 \div 0.5$. Moreover the density scalelength is of order of the minor radius, which implies that the pinch velocity is an order of magnitude lower than in the H-mode. One therefore obtains

$$P_{\text{L-mode}} \approx Z D_{\text{neo}} / D_{\text{turb}} \sim 1 \qquad (30)$$

Therefore there is little or no impurity accumulation in L-mode plasmas on the basis of Eqs. (28,30). This results is also consistent with the experimental findings[9].

Finally, during occasional strong magnetic activity (ELM's), Γ_e^{magn} is not negligible. The direction of the electric field can be reversed and the high Z impurities are pumped out as discussed in Stringer[5].

VI. Conclusions.

The transport of diluted impurities has been revisited, including both collisional (classical or neoclassical) effects and collisionless/turbulence effects.

A key ingredient of the theory is the fact that turbulent test particle diffusivity is independent of the bulk particle transport properties, and usually much larger than the bulk (electron) diffusivity. This has been shown explicitly in Sec. II, where an extreme model with zero electron transport but finite test particle transport has been constructed.

The fact that impurities have a higher turbulent diffusivity implies that they would normally diffuse out, where it not for the presence of a pinch. In this work only

collisional pinches have been considered, although mechanisms for anomalous pinches are theoretically possible and have been indeed proposed. The reason of this limitation is that it does not seem strictly necessary to invoke an anomalous pinch to explain the observations. The neoclassical impurity pinch velocity can be very high because it is proportional to the charge number and to the bulk gradients which in certain regimes can be very high (edge plasmas in H-mode discharges).
The comparison with the experiments is mainly limited by the uncertainties in the direct experimental knowledge of the turbulence parameters. We had therefore to employ quantities inferred from other measurements, such as ratios of bulk diffusivities or thermal conductivities. Therefore, at this stage, the comparison is regarded as only "qualitative", but the results are encouraging.

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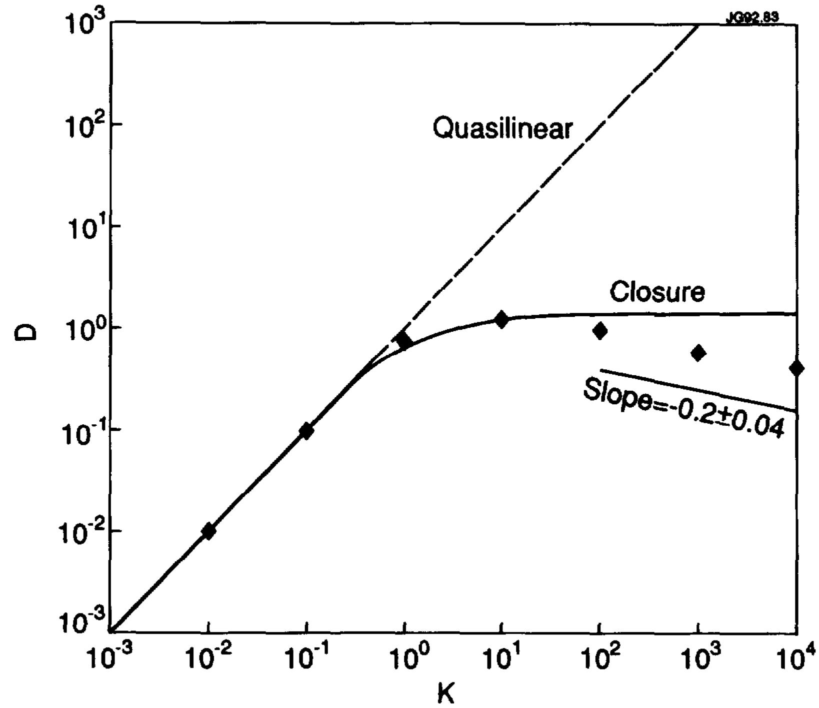

Fig. 1 Test particle diffusivity (normalized to $v_E\lambda_c$) vs. Kubo number $K \equiv v_E\tau_c/\lambda_c$. Comparison between analytic theories and numerical results (from Ref. 3).

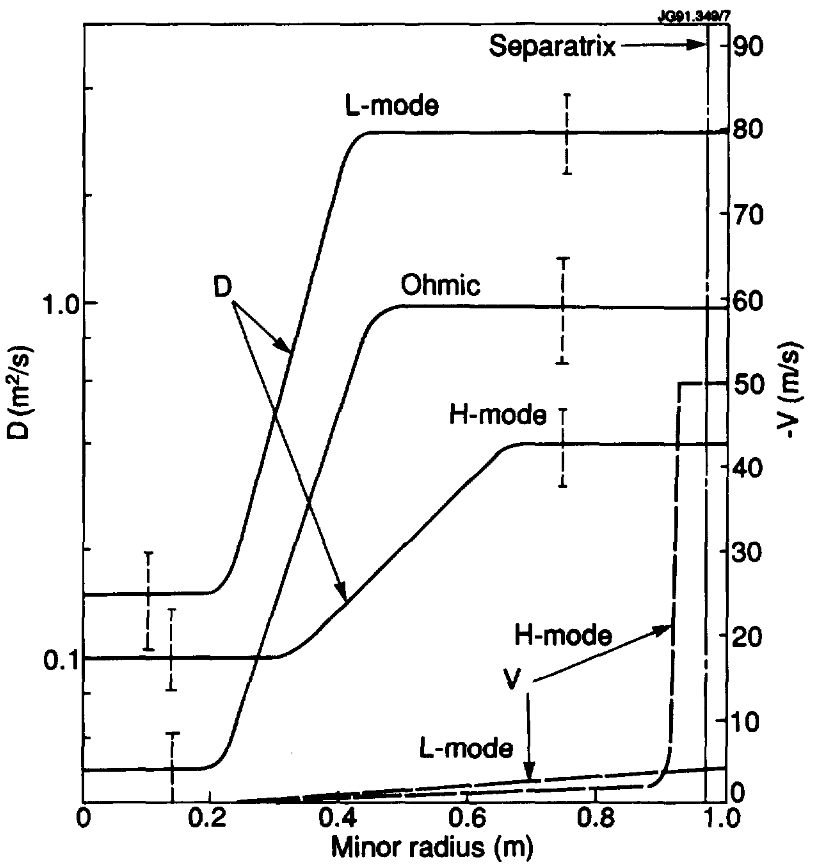

Fig. 2 Measured impurity diffusivity and pinch velocity (from Ref. 9).