JOINT EUROPEAN TORUS

JET–P(93)77

D.J. Ward

Transitions in Tokamak Edge Confinement

Transitions in Tokamak Edge Confinement

D.J. Ward

JET-Joint Undertaking, Culham Science Centre, OX14 3DB, Abingdon, UK

Preprint of a paper to be submitted for publication in
Nuclear Fusion (letters)
October 1993

ABSTRACT

It is shown that simple functional dependences of transport coefficients on plasma parameters can result in a transition in the particle and energy confinement near the edge of a tokamak where particle sources, and hence convection, are important. This is illustrated using cylindrical classical diffusion coefficients. The transition may be to a second steady state or to a regime where there is no steady solution, in which the density rises continuously, for instance. In practice such a density rise would be curtailed by high radiated power or by instabilities. Many experimental observations of L to H-mode transitions in tokamaks coincide with the non-steady solution rather than a bifurcation to a second steady state.

INTRODUCTION

The sudden improvement in particle and energy confinement observed at the transition from L-mode to H-mode in tokamaks [1 – 6] is usually described as a bifurcation from one steady state to another with an inevitable transient phase. There are many theories of the L to H-mode transition, often invoking stabilisation of turbulence by electric fields, rotation or by high shear near a separatrix, [7 – 12].

Experimental observations of H-modes are usually dominated by the transient phase, with the plasma density increasing continuously throughout the H-mode unless edge instabilities (elms) become important, [2,3]. Such behaviour can arise in simple models of edge transport in which the transport coefficients depend on plasma parameters (as is predicted for instance by classical or neoclassical transport theory). This is illustrated here using cylindrical classical diffusion coefficients. It is not, of course, suggested that the transport in a tokamak obeys classical theory, but studying the transition behaviour using physics based diffusion coefficients may give insight into the behaviour in the experiment.

SIMPLE TRANSPORT MODEL

Before discussing the effect of classical transport coefficients the model of the edge is described using constant values for the transport coefficients. In the edge region of a tokamak plasma (where neutral particles can penetrate) the steady state transport of particles and energy may be represented simply by

$$-Dn' + n\mathrm{v} = \Gamma \tag{1}$$

$$(-2\mathrm{K}T' + 5\Gamma T)A = P \tag{2}$$

The neutral particle flux Γ is balanced by a diffusive outflow with a diffusion coefficient D and an inward convection with velocity v. The heat flow P across a surface of area A is given by a conduction term with conductivity K and a convection term. The electron and ion temperatures are assumed equal. The primes represent radial derivatives.

Neutral particles are typically only important in the outer 20% of the minor radius of present large tokamaks and this region is considered here in slab geometry. The region under consideration is illustrated in Figure 1. P and Γ are taken to be constant across the edge region and are treated as the control variables.

In the simple problem where D, v and K are independent of plasma parameters, the solution of equations (1) and (2) is

$$n = n_0 \exp(\frac{\mathrm{v}x}{D}) + \frac{\Gamma}{\mathrm{v}}(\exp(\frac{\mathrm{v}x}{D}) - 1)$$

$$T = T_0 \exp(-\frac{5\Gamma x}{2\mathrm{K}}) - \frac{P}{5A\Gamma}(\exp(-\frac{5\Gamma x}{2\mathrm{K}}) - 1)$$

where n_0 and T_0 are the edge values of density and temperature, $x = 1 - \frac{r}{a}$, with a the minor radius.

Since P and Γ are considered as the control variables it is useful to consider solutions drawn in the P, Γ plane. Assuming that n at x=0.2 characterises the volume averaged density we will find it useful to plot contours of constant n(x=0.2) in the P, Γ plane. This is illustrated in Figure 2 and shows that density is simply a function of particle flux (as well as edge density which is assumed constant here). The parameters used for this plot are $D = 1m^2s^{-1}$, $\mathrm{v} = 5ms^{-1}$, $\mathrm{K} = 10^{19}m^{-1}s^{-1}$, $a = 1.4m$ and $R = 3m$. The boundary conditions used are $n_0 = 3 \times 10^{18}m^{-3}$ and $(\ln T)'_0 = 25m^{-1}$ (a temperature scrape-off length of 4cm). At a given value of input power, any value of density can be achieved by choosing the appropriate particle flux.

CLASSICAL TRANSPORT MODEL

The simple behaviour described above is, of course, a result of the simple model. In a physics based model of transport, the transport coefficients will depend on the plasma parameters, for instance in cylindrical classical theory D depends on $\frac{n}{T^{1/2}}$. Such dependences can lead to multiple solutions of the transport equations and so allow the possibility of transition from one state to another.

Since the physics underlying transport in a real tokamak is unknown, the transition properties of a model with classical diffusion coefficients is considered as an illustrative example. Classical transport is chosen since the underlying physical processes are well known, however the fact that classical transport coefficients are low compared to experimental values must be kept in mind as this will lead to considering regions of operating space not normally of interest.

For simplicity the boundary is assumed to be perfectly absorbing hence the boundary temperature is determined by the condition that heat is carried to the wall by particles striking the wall,

$$T_0 = \frac{P}{5A\Gamma}. \tag{3}$$

The factor 5 used in Equation (3) is not precise and is the subject of continuing work in edge plasma physics [13,14], however the actual value used does not change the following results in a qualitative way. Since P and Γ are assumed constant, convection dominates the whole edge region and the temperature is uniform. The absorbing boundary leads to the loss of any particle which comes within one Larmor radius of the edge making the edge density so low that it is of little importance in the solution. To avoid numerical difficulties, the edge density is assumed to be a constant (low) value, $n_0 = 10^{17} m^{-3}$. Again the following results are insensitive to this assumption. The form taken for the transport of particles is

$$-Dn' + nv = \Gamma$$

with the diffusion coefficient given by classical theory

$$D = \frac{4 \times 10^{-22} n}{T^{1/2} B^2}$$

where T is in eV. The pinch term used is not the classical one but is assumed to have a constant value typical of classical transport,

$$v = \frac{r}{200a} \ \mathrm{ms}^{-1}$$

Figure 3 shows a plot of n(x = 0.2) in the P, Γ plane for this transport model. The contours of constant n are now qualitatively different in that the density depends on both power and particle flux and the contours are confined to a restricted region of P, Γ space. This means that:

1. At a given power there is a minimum density attainable.

2. At a given power there are two values of particle flux which give the same density. Both are stable equilibria.

Figure 4 shows a plot of density against particle flux at a fixed power of 2W and there are two solutions in which the density is $3 \times 10^{19} m^{-3}$: one at $\Gamma = 1.1 \times 10^{16} m^{-2} s^{-1}$ and another at $\Gamma = 4.6 \times 10^{17} m^{-2} s^{-1}$. The particle confinement is approximately a factor of 40 higher in the former case. The edge density profiles for these two solutions are shown in Figure 5 and are rather similar. The temperature on the other hand is higher by a factor of 40 in the low Γ case. This represents a significant increase in energy confinement since the temperature in the interior plasma will also be higher. The improved confinement is due to the reduced particle diffusivity (a factor of 6 lower in the low Γ case) resulting in reduced heat convection.

There were several approximations made above. The assumptions of slab geometry with P and Γ not varying across the edge and that the density at r/a = 0.2 characterises the volume average density can all be removed in a more complete numerical solution of the transport model. Figure 6 shows the results from such a calculation where the boundary conditions are as before but now cylindrical geometry is assumed, the ionisation length of neutrals is taken to be 0.1a and the power is assumed to be deposited uniformly across the plasma. Thermal conduction now dominates the power balance over most of the plasma. The contours of constant volume averaged density in the P, Γ plane shown in Figure 6 are qualitatively the same as in Figure 3.

Having solved the transport model across the whole plasma the plasma energy is determined as a function of volume averaged density, as illustrated in Figure 7 for an input power of 4W. There are again two solutions with a density of $3 \times 10^{19}\mathrm{m}^{-3}$, one at high Γ with a stored energy of 45J, a confinement time of 11.3s and one at low Γ with an energy of 870J, a confinement time of 217s. Both the particle and energy confinement are a factor of 20 larger in the low Γ case.

Two things have been demonstrated using a transport model with classical diffusion coefficients. Firstly, there are two equilibria at a given power and density, one with high confinement and one with low confinement. As both are stable equilibria it is tempting to suggest that these are analogous to L and H modes although no explanation of how a transition from one to the other might occur has been given. Secondly, there is a minimum value of density attainable at a given power or, conversely, with a chosen density, there is a critical power which must not be exceeded. Above the critical power the density can no longer be held at its chosen value but must increase with a time evolution which depends on the control of Γ. The scaling of the critical power and the evolution of density at higher powers are discussed below.

CRITICAL POWER

The reason for the minimum in the density is seen clearly by deriving asymptotic solutions for density. At low Γ the inward pinch and the outward diffusion balance

$$Dn' = n\mathrm{v}$$

which becomes

$$n' = 2.5 \times 10^{21} B^2 T^{1/2} \mathrm{v}$$

and with negligibly small edge density yields

$$n(0.2) = 5.6 \times 10^{29} \frac{B^2 \mathrm{v} P^{1/2} a}{\Gamma^{1/2} A^{1/2}}$$

The density becomes large at low Γ because the resulting high temperature leads to a low diffusion coefficient whilst the pinch velocity is maintained constant.

At high Γ on the other hand the outward diffusion balances the inward flux of neutrals,

$$-Dn' = \Gamma$$

then

$$nn' = 2.5 \times 10^{21} B^2 T^{1/2} \Gamma$$

hence

$$n(0.2) = 1.1 \times 10^{15} \frac{BP^{1/4}\Gamma^{1/4}a^{1/2}}{A^{1/4}}$$

The density becomes large at high Γ because of the strong particle source.

An approximate analytic solution can be constructed as the sum of the asymptotic limits,

$$n(0.2) = 5.6 \times 10^{29} \frac{B^2 \nu P^{1/2} a}{\Gamma^{1/2} A^{1/2}} + 1.1 \times 10^{15} \frac{B\Gamma^{1/4} P^{1/4} a^{1/2}}{A^{1/4}}$$

The minimum density occurs when $\dfrac{\partial n}{\partial \Gamma} = 0$ that is when

$$\Gamma = 1.1 \times 10^{20} \frac{B^{4/3} \nu^{4/3} P^{1/3} a^{2/3}}{A^{1/3}}$$

which leads to the minimum density

$$n(0.2) = 1.6 \times 10^{20} \frac{B^{4/3} \nu^{1/3} P^{1/3}}{A^{1/3}}$$

Conversely, the maximum power at which a density n is attainable is

$$P_c = \frac{n^3 A}{4.3 \times 10^{60} B^4 \nu a^2}$$

For instance at a density of $3 \times 10^{19} m^{-3}$ in a tokamak with $B = 3T, a = 1.4m$ and $A = 166m^2$ the critical power is 1.3W. This very low power is a result of the very low transport assumed but if the diffusion coefficient were increased by a factor of 10^3 to a level comparable to experimental values, the critical power would become 1.3MW, similar to experimental H-mode threshold powers.

The critical power to attain H-mode in tokamaks is still uncertain as different experiments report different scalings [3,5,6,15]. It is reported that the critical power increases with n and B, a possible form for the scaling being

$$P \sim nBA.$$

It is difficult to derive a similar scaling using classical diffusion coefficients even if the pinch velocity varies with plasma parameters. It is not surprising that the experimental scaling, particularly the dependence on B, is completely different from that derived here since the transport in a tokamak does not scale in the same way as classical transport. The classical case is outlined here to show how a simple physics based model can lead to a transition in behaviour and what form the transition takes.

In the model described above the minimum density occurs because of the dependence of the diffusion coefficient on $n/T^{1/2}$. At low values of particle flux the edge temperature rises decreasing the diffusion coefficient. Since the pinch velocity was maintained the particle confinement increases and the density increases. More generally the minimum density in this model persists if the ratio of pinch velocity to diffusion coefficient increases with temperature

$$\frac{\mathrm{v}}{D} \propto \frac{T^{\alpha}}{n}$$

with $\alpha > 0$.

BEHAVIOUR ABOVE CRITICAL POWER

The behaviour of the transport model once the critical power is exceeded depends on the way in which the control parameters are controlled. For instance the density may be controlled by feedback on the particle flux and, as the power is increased, the particle flux must be varied as shown in Figure 3 to maintain a constant density. Above a critical power there is no value of Γ which will allow the chosen density so the way in which Γ is controlled then becomes important.

With perfect control over Γ but with a feedback system which assumes $\frac{\partial n}{\partial \Gamma} > 0$, the density will rise continuously once the critical power is exceeded. As Γ is reduced the equilibrium density increases and the actual density increases as rapidly as the fuelling source allows.

In a real experiment the control over Γ is not as direct because neutrals from the wall fuel the plasma, hence the density behaviour would be somewhat different. With a minimum value of Γ given by the residual outflux from the wall, for instance, the plasma would move to a new equilibrium at an increased density.

CONCLUSIONS

Transitions in confinement occur naturally in models of edge transport with transport coefficients which depend on plasma parameters, illustrated here by a model with classical diffusion coefficients. The transitions may take the form of a bifurcation to a second equilibrium or to a non-steady state in which, for example, the density increases continuously. The latter is the more natural in the transport model discussed here.

In the illustrative model discussed here there is a minimum density achievable in steady state at a given input power. Conversely if a given density is required, there is a critical power which must not be exceeded. The behaviour at higher powers depends on how the particle flux is varied in response to the increasing density but the likely result is a continuous density rise and an increased energy confinement time. In this model the improvement in energy confinement is a result of reduced heat convection.

Using an approximate analytic solution of the transport model, a scaling law is derived for the critical power. The critical power increases with density but reduces with magnetic field strength. The observed H-mode threshold power on the other hand increases with increasing field strength. Such discrepancies are not surprising since it is known that transport in tokamaks does not scale in the same way as classical transport.

It is possible that experimentally observed H-mode transitions are of this type, that is the transition may result from the dependence of the transport coefficients on plasma parameters rather than on a critical condition on some instability being reached. Observations of H-modes seem to conform more to transitions to a non-steady state rather than to a new equilibrium. Steady state H-modes are usually only achieved by generating instabilities in the edge plasma.

ACKNOWLEDGEMENTS

I would like to thank Drs. R.J. Bickerton, T.E. Stringer, J.A. Wesson and M. Keilhacker for helpful comments.

REFERENCES

[1] WAGNER, F., et al., Phys. Rev. Letts. 49(1982) 1408

[2] BURRELL, K.H., et al., Phys. Rev. Lett. 59(1987) 1432

[3] ASDEX TEAM, Nucl. Fusion 29(1987) 1959

[4] TANGA, A., et al., Nucl. Fusion 27(1987) 1877

[5] ODAJIMA, K., et al., Plasma Phys. Control. Nucl. Fusion Research (Proc. 11th Int. Conf. Kyoto,1986) Vol.1, IAEA, Vienna(1987) 151

[6] BURRELL, K.H., et al., Plasma Phys. Control. Nucl. Fusion Research (Proc. 12th Int. Conf. Nice,1988) Vol.1, IAEA, Vienna(1989) 193

[7] HINTON, F.L., et al., Plasma Phys. Control. Nucl. Fusion Research (Proc. 10th Int. Conf. London,1984) Vol.2, IAEA, Vienna(1985) 3

[8] HAHM, T.S., DIAMOND, P.H., Phys. Fluids 30(1987) 133

[9] SHAING, K.C., et al., Plasma Phys. Control. Nucl. Fusion Research (Proc. 12th Int. Conf. Nice,1988) Vol.2, IAEA, Vienna(1989) 13

[10] SHAING, K.C., CRUME, E.C.,Jr., Phys. Rev. Lett. 63(1989)2369

[11] ITOH, S-I., ITOH, K., J. Phys. Soc. Jpn. 59(1990) 3815

[12] BIGLARI, H., et al., Phys. Fluids B2(1990) 1

[13] CHODURA, R., Proc. 19th Euro. Conf. Control. Fusion Plasma Phys. (Innsbruck,1992) Vol II 871

[14] RIEMANN, K.U., Contrib. Plasma Phys. 32(1992) 231

[15] NARDONE, C., et al, Proc. 18th Euro. Conf. Control. Fusion Plasma Phys. (Berlin,1991) Vol I 377

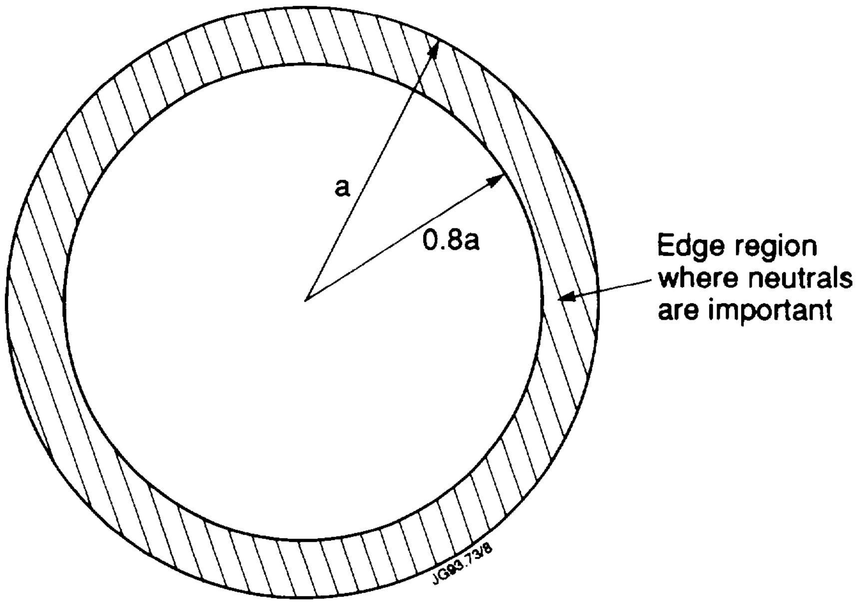

Figure 1: Schematic showing the edge region of the plasma which is considered here.

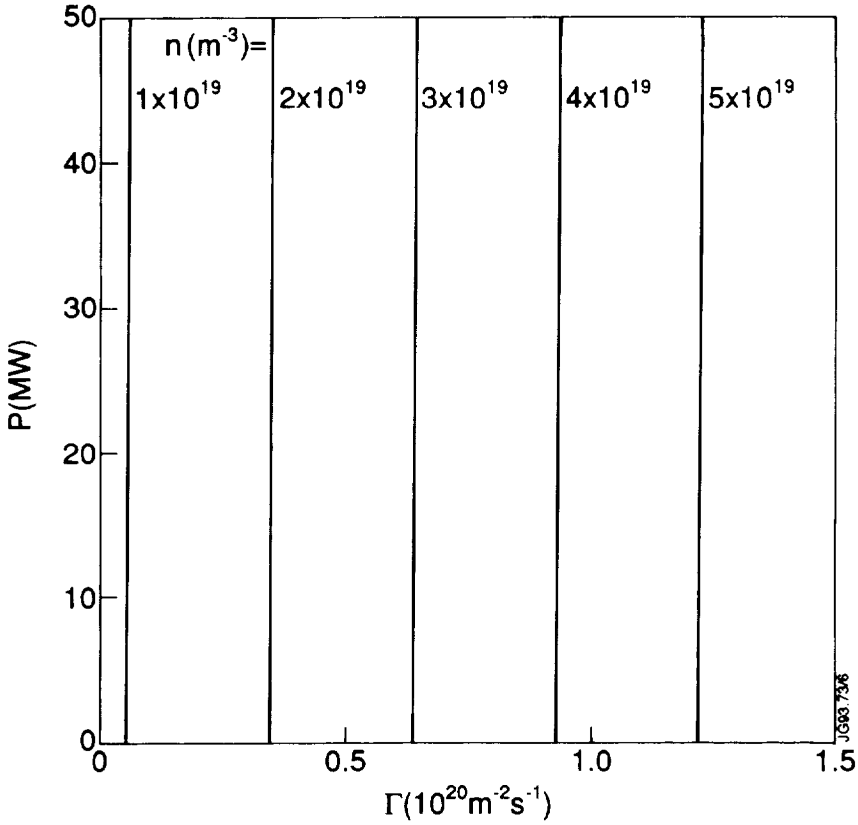

Figure 2: Contours of constant density at $x = 0.2$ plotted in the P, Γ plane.

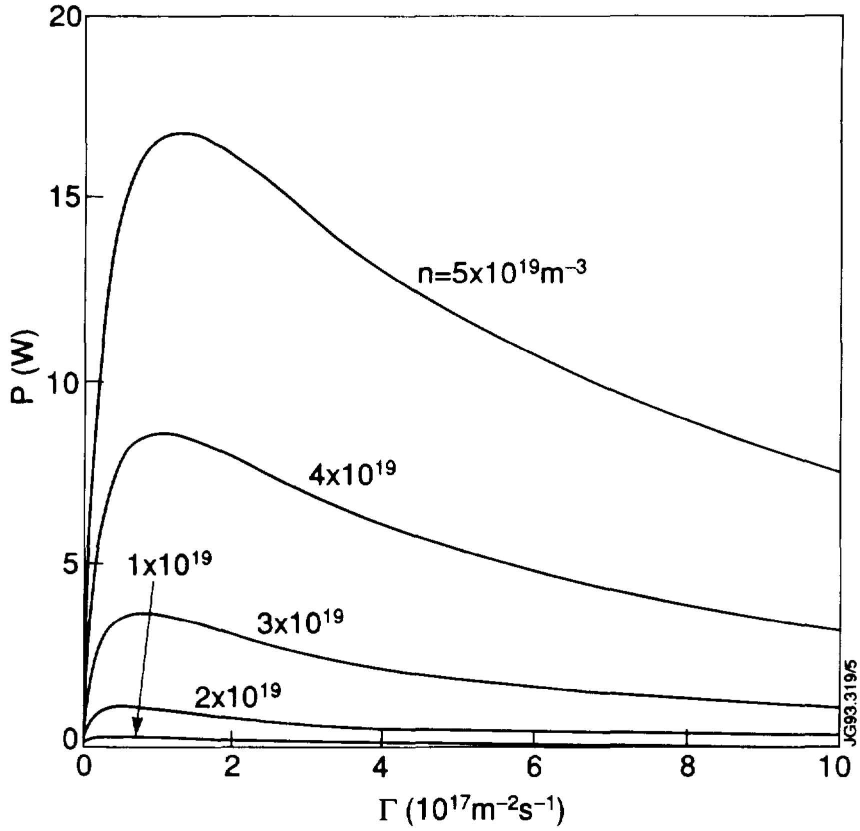

Figure 3: Contours of constant density in the P, Γ plane using classical diffusion coefficients.

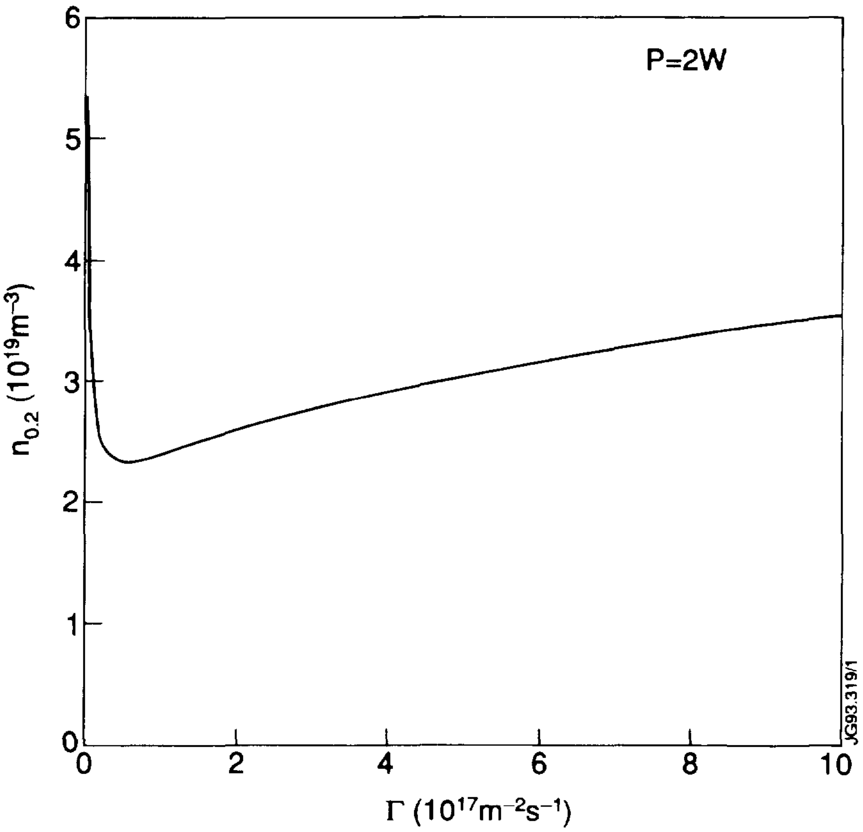

Figure 4: Variation of density with particle flux with an input power of 2W in the model with classical diffusion coefficients.

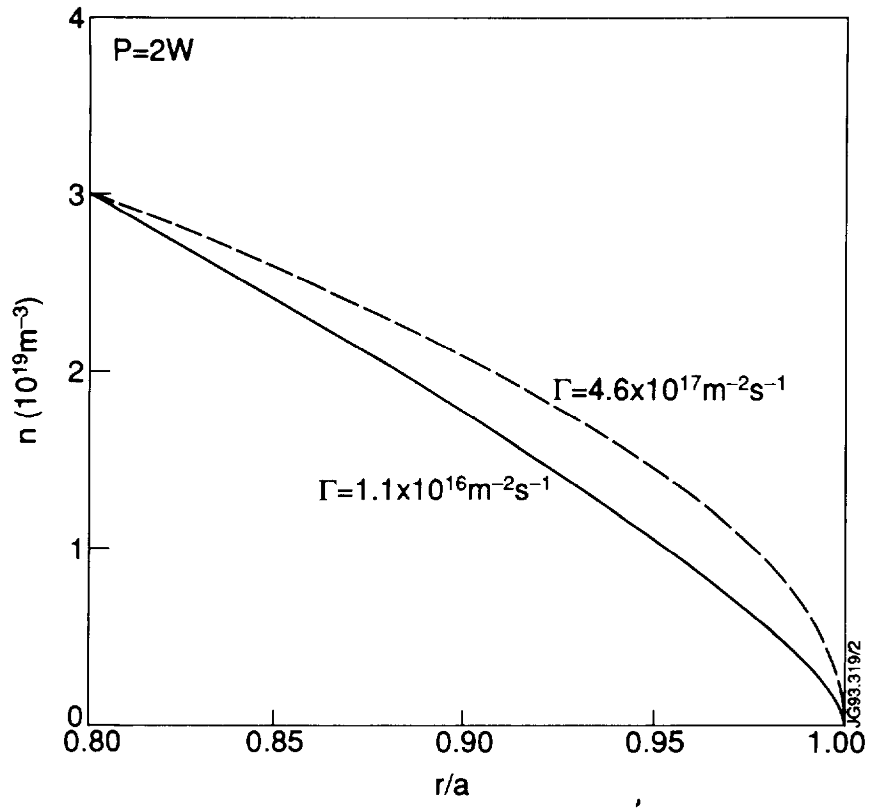

Figure 5: The same density at x = 0.2 (r = 0.8a) can be achieved with two values of particle flux.

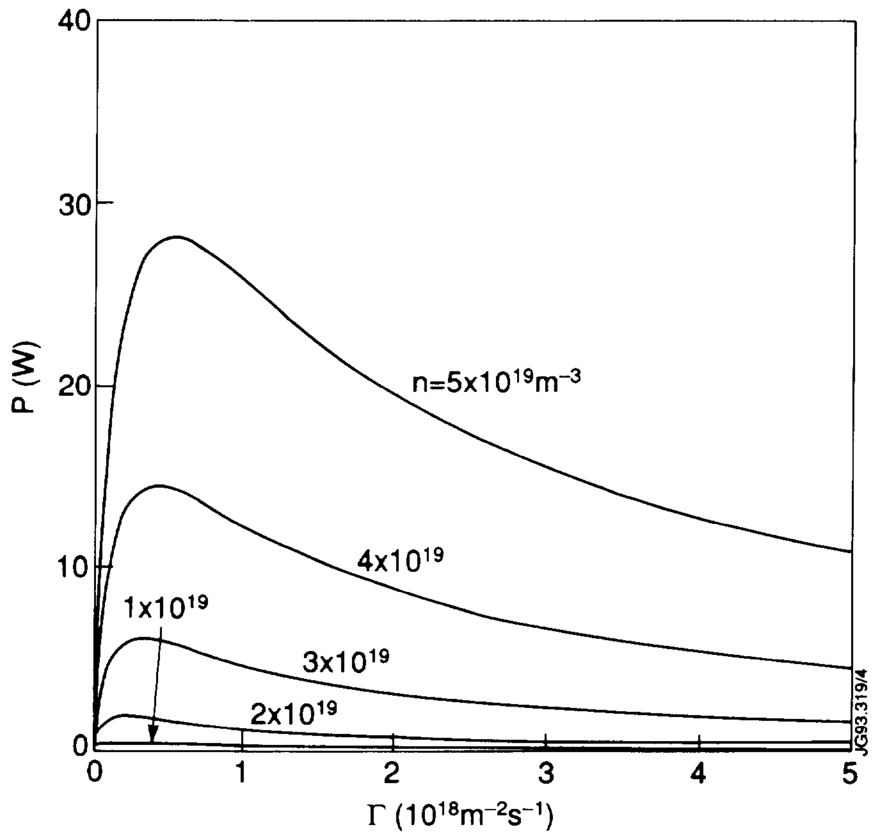

Figure 6: Contours of constant volume averaged density in the P, Γ plane derived solving the classical transport model across the whole plasma.

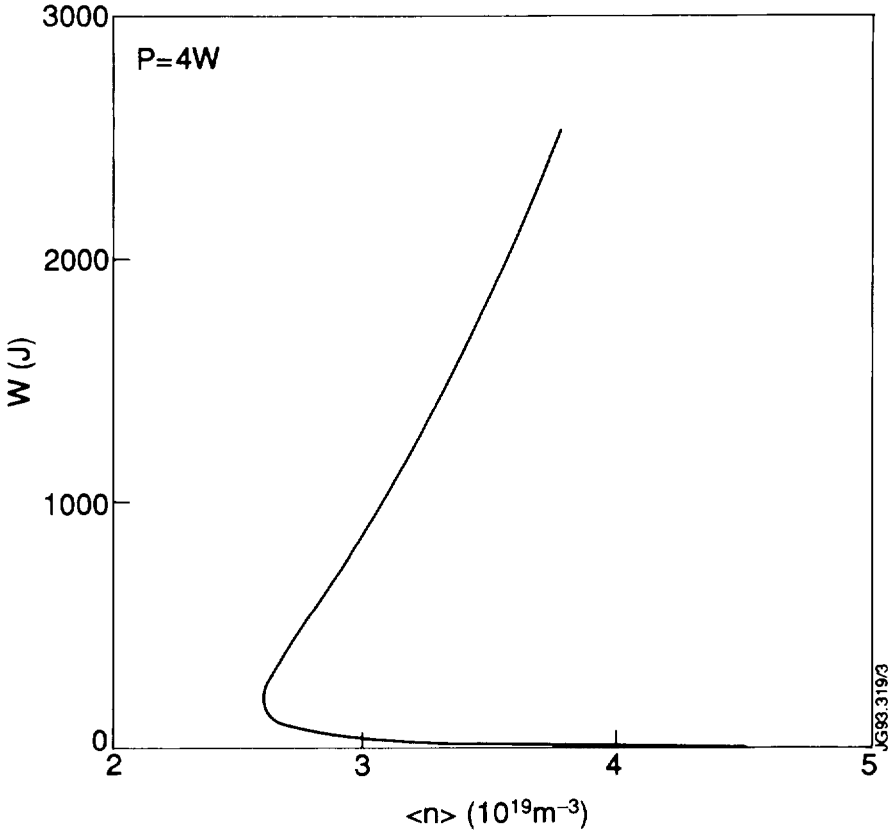

Figure 7: Plot of plasma energy against volume averaged density in the more complete classical calculation. The input power was 4W.