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ABSTRACT

Sawtooth activity occurs in ohmically heated tokamak plasmas even at high values of edge safety factor. This is not consistent with the predictions of transport calculations in which the resistivity is classical and the thermal conductivity is uniform. In that case q falls to unity in the centre only if the edge q is less than 3.3. Sawteeth can persist to higher values of edge q if the neoclassical enhancement to resistivity is included or the thermal conductivity increases with radius. In JET, sawteeth persist to higher values of q than would be expected even using neoclassical resistivity demonstrating that the thermal conductivity increases with radius.

INTRODUCTION

It is generally accepted that for sawtooth activity to occur in tokamaks the central value of safety factor, q_0, must be close to or below unity. This is because the q = 1 surface gives an m = nq resonance for the m = 1, n = 1 instability. To achieve $q_0 \simeq 1$ the current profile must be sufficiently peaked, the peaking required increasing with the surface safety factor, q_a. A simple relation can be derived for the required current peaking as a function of q_a. Assuming for instance a current profile which is uniform inside some radius and zero outside then the condition $q_0 \leq 1$ can be written as a condition on the internal inductance, l_i

$$l_i \geq \ln(q_a) + 0.5$$

A similar relationship can be derived if the current profile can be represented as a parabola to some power. The resulting condition for $q_0 \leq 1$ is given by

$$l_i \geq \ln(0.76 + 0.89q_a)$$

derived to be exact at large q_a and to give $l_i = 0.5$ when $q_a = 1$.

For a given resistivity and an assumed thermal conductivity profile the value of q_a below which sawteeth should occur can be calculated. The experimental observations on sawteeth can then be used to test the assumptions on the resistivity and the thermal conductivity profile. Only ohmically heated plasmas are considered here.

TEMPERATURE AND CURRENT PROFILES

The power balance equation in an ohmically heated tokamak with Spitzer resistivity is

$$\frac{1}{r}\frac{d}{dr}(r\mathrm{K}\frac{dT}{dr}) = -\frac{E^2T^{3/2}}{\alpha} \qquad (1)$$

where K is the thermal conductivity and $\alpha/T^{3/2}$ is the resistivity. E is the electric field, this field being constant across the radius in steady state. The effect of radiation is assumed small and it is assumed that $T_i = T_e$.

If the thermal conductivity and the effective ion charge are uniform, Equation (1) can be rewritten as

$$\frac{1}{r}\frac{d}{dr}(r\frac{dT}{dr}) = -\beta T^{3/2} \qquad (2)$$

where

$$\beta = \frac{a^2T_0^{1/2}E^2}{\alpha\mathrm{K}}$$

the temperature has been normalised to its value on axis, T_0, and the radius normalised to the minor radius, a. Specifying the boundary condition T = 0 at the plasma edge Equation (2) can be solved numerically or analytically for the eigenvalue β yielding $\beta = 6.85$. The resulting temperature profile is shown in Figure 1 from which the current density and safety factor profiles can be calculated.

The safety factor used here is the cylindrical safety factor defined as

$$q_{cyl} = \frac{2\pi r^2B_\phi}{\mu_0 IR}(\frac{1+\kappa^2}{2})$$

where κ is the elongation and I the enclosed current. The ratio of edge to central safety factor is then given by

$$\frac{q_a}{q_0} = \frac{a^2}{2\int_0^a T^{3/2}rdr} \qquad (3)$$

which holds for plasmas with an elongation that does not vary with radius including those with a circular cross-section. When comparing with experiment, only low elongation discharges will be used which have a small variation of elongation with radius. For the temperature profile shown in Figure 1, Equation (3) gives

$$\frac{q_a}{q_0} = 3.3$$

Thus, if the resistivity were classical and the thermal conductivity uniform, sawteeth would not be expected for q_a above 3.3.

The value of q_a below which sawteeth occur in JET can be determined from Figure 2 which shows how the sawtooth period varies with q_a in ohmically heated JET plasmas. Data from discharges with high radiation (more than 70% of the input power) are shown by open symbols and are not significantly different from the lower radiation discharges showing that radiation is not significantly affecting the sawtooth data. The sawtooth period reduces as q_a increases but sawteeth persist even up to $q_a \simeq 15$ as is also seen in sawtooth inversion radius data from JET [1]. This is clearly incompatible with the assumption of Spitzer resistivity and uniform thermal conductivity. The effect of relaxing these assumptions is examined separately below.

NON-UNIFORM THERMAL CONDUCTIVITY

The radial variation of the thermal conductivity can be represented reasonably by the form $K_0(1 + \gamma r^2)$. The effect of having a thermal conductivity increasing towards the plasma edge ($\gamma > 0$) is to lead to more peaked temperature and current profiles and hence to increase q_a/q_0. This can be seen in Figure 3 which shows the calculated value of q_a/q_0 as a function of γ. To explain the existence of sawteeth up to q_a of 15 with Spitzer resistivity requires a very high value of γ, greater than 23. A ratio of 24 between edge and central thermal conductivity is ruled out by local transport analysis which typically gives a ratio in the region of 1 to 5. An example of local transport analysis is [2] which gives the profile of thermal diffusivity, $\chi = K/n$, where n is the plasma density. The occurrence of sawteeth up to $q_a \simeq 15$ rules out Spitzer resistivity for JET plasmas.

NEOCLASSICAL RESISTIVITY

The neoclassical correction to the resistivity arising from the existence of trapped electrons can also be included leading to a more peaked current profile due to the increase of resistivity away from the plasma centre. This will be modelled using the expression given in [3] with the collisionality assumed everywhere small. In this case the neoclassical resistivity is given by

$$\eta_{neo} = \eta_{Spitzer}\left(\frac{(1-\varepsilon^2)^{1/2}(1+1.46\varepsilon^{1/2})}{(1-\varepsilon)^2}\right)$$

where ε is the local value of inverse aspect ratio reaching 0.4 at the edge of a JET plasma. The small correction depending on Z_{eff} is neglected here.

The increased resistivity away from the plasma centre leads to a more peaked current profile than with Spitzer resistivity. Assuming the thermal conductivity to be independent of radius the temperature profile calculated from the power balance is as shown in Figure 4. The profile derived using Spitzer resistivity is also shown for comparison. Equation (3) now gives

$$\frac{q_a}{q_0} = 8.6$$

As before the calculated value of q_a at which sawteeth are expected to cease is too small compared to the experimental value, however it seems likely that using neoclassical resistivity and non-uniform thermal conductivity together should give a more satisfactory result.

Figure 5 shows the variation of q_a/q_0 with γ for a plasma with neoclassical resistivity and, as before, $K = K_0(1 + \gamma r^2)$. To achieve sawteeth in a plasma with $q_a = 15$ requires $\gamma = 4$ and so the persistence of sawteeth up to $q_a \simeq 15$ in JET is evidence for both the neoclassical correction to the resistivity and a thermal conductivity increasing with radius. The ratio of edge to central thermal conductivity required to explain the observation of sawteeth at high q is

$$\frac{K_a}{K_0} = 5$$

which is consistent with typical results from local transport analysis.

As a check that these calculations give reasonable temperature profiles Figure 6 shows a plot of the temperature profile derived assuming neoclassical resistivity with $\gamma = 4$ compared to a typical experimental profile from a high q ohmic JET discharge which has no sawteeth. The experimental data is similar to the calculated profile.

CONCLUSIONS

The presence of sawteeth in ohmically heated discharges at high q($\simeq$15) is evidence of both neoclassical resistivity and a thermal conductivity which increases with radius.

JET data implies that in the absence of sawteeth the ohmically heated equilibrium reaches a ratio of $\frac{q_a}{q_0} \simeq 15$ while Spitzer resistivity and a uniform thermal conductivity would give 3.3. Inclusion of the neoclassical correction to the resistivity increases this ratio to 8.6, still below the experimental value.

Non-uniform thermal conductivity allows more peaked temperature and current profiles. The experimental results could then be explained using Spitzer resistivity but the required ratio of edge to central thermal conductivity is unreasonably high (>24). With neoclassical resistivity the results are made consistent with experiment using a ratio of edge to central thermal conductivity of approximately 5, consistent with typical results from local transport analysis.

ACKNOWLEDGEMENTS

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REFERENCES

[1] Bartlett D V, Bindslev H, Brusati M et al, Proc. 13th European Conf. Control. Fusion Plasma Heating, (Schliersee,1986) Vol I 236

[2] Scott S D, Barnes C W, Grisham L R et al, Plasma Phys. Control. Nucl. Fusion Research (Proc 13th Int. Conf. Washington, 1990) Vol.1, IAEA, Vienna(1991) 235

[3] Hirshman S P and Sigmar D J Nucl. Fusion 21(1981) 1079

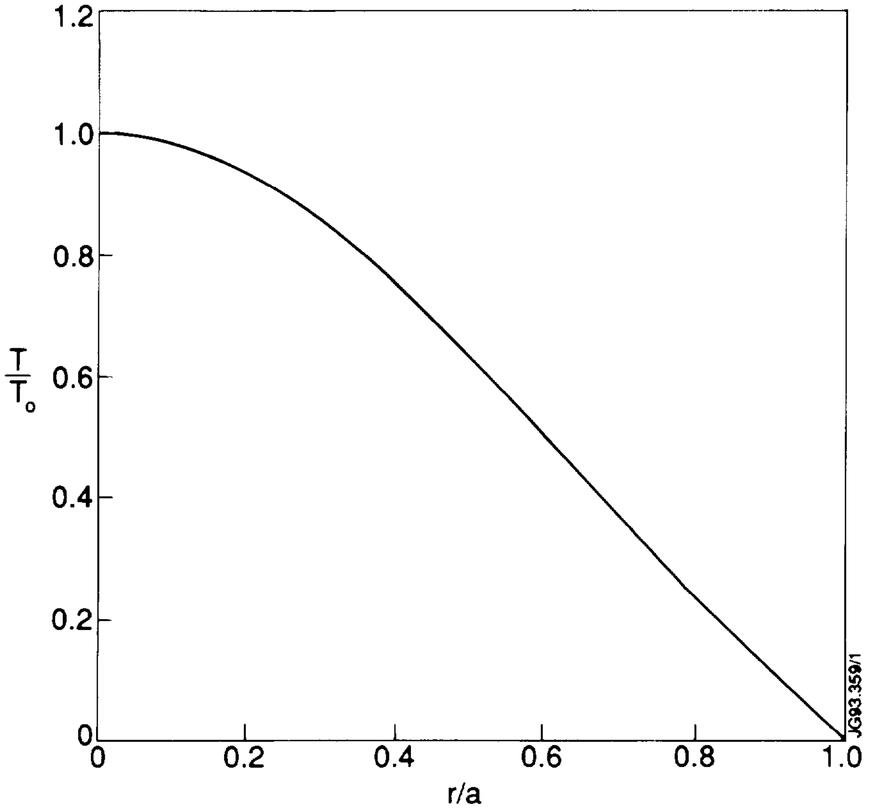

Figure 1: Calculated temperature profile for an ohmically heated tokamak with Spitzer resistivity and uniform thermal conductivity.

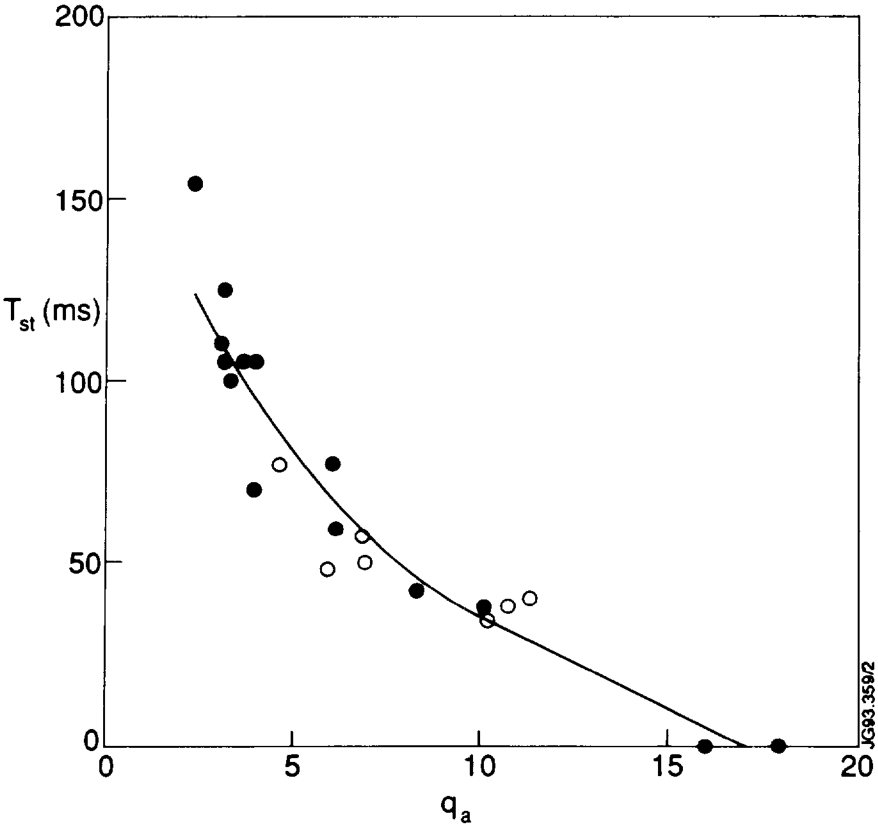

Figure 2: Sawtooth period against edge safety factor for ohmically heated JET discharges. Discharges with high radiated power are indicated by open symbols.

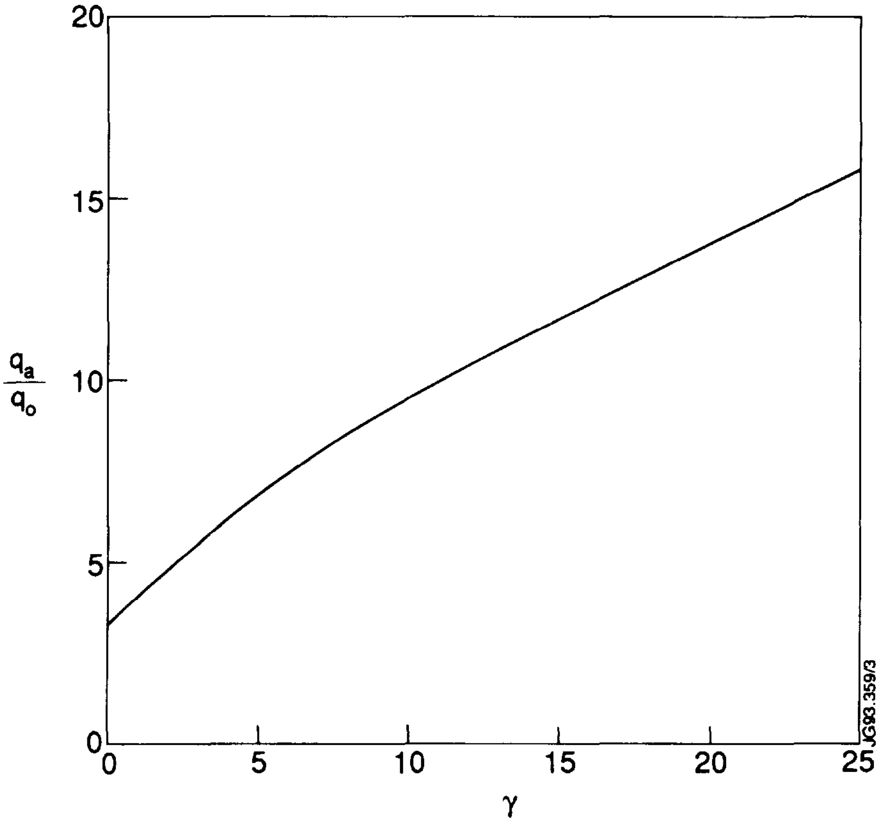

Figure 3: Calculated value of q_a/q_0 against γ where the thermal conductivity scales as $(1 + \gamma r^2)$ and Spitzer resistivity is assumed.

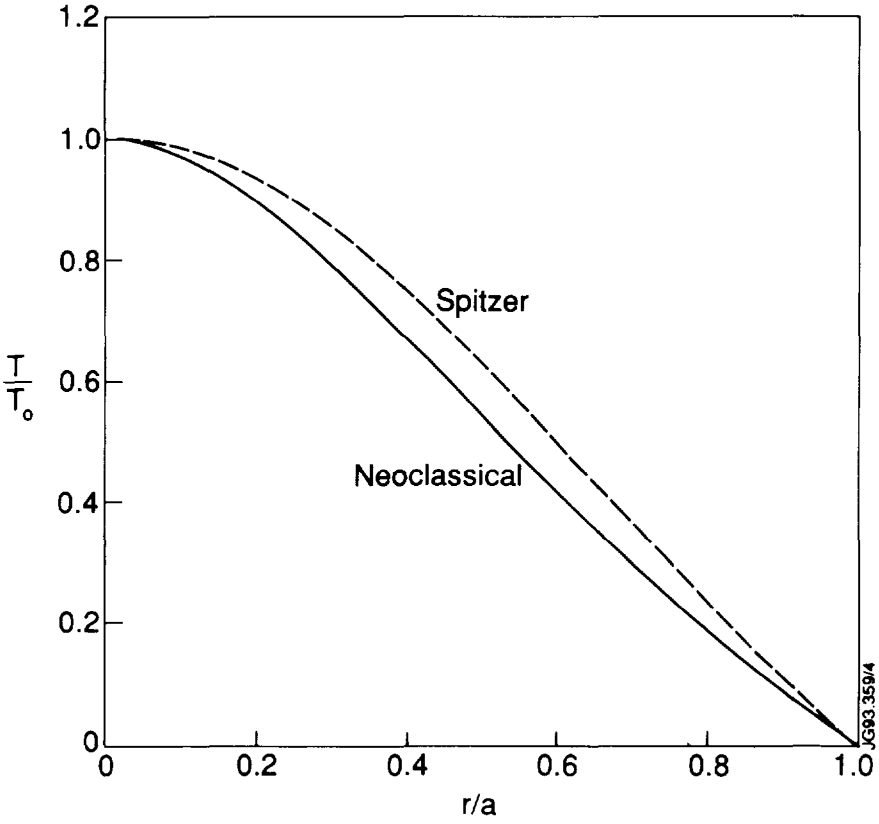

Figure 4: Calculated temperature profile for an ohmically heated tokamak with neoclassical resistivity and uniform thermal conductivity. The Spitzer case is shown for comparison.

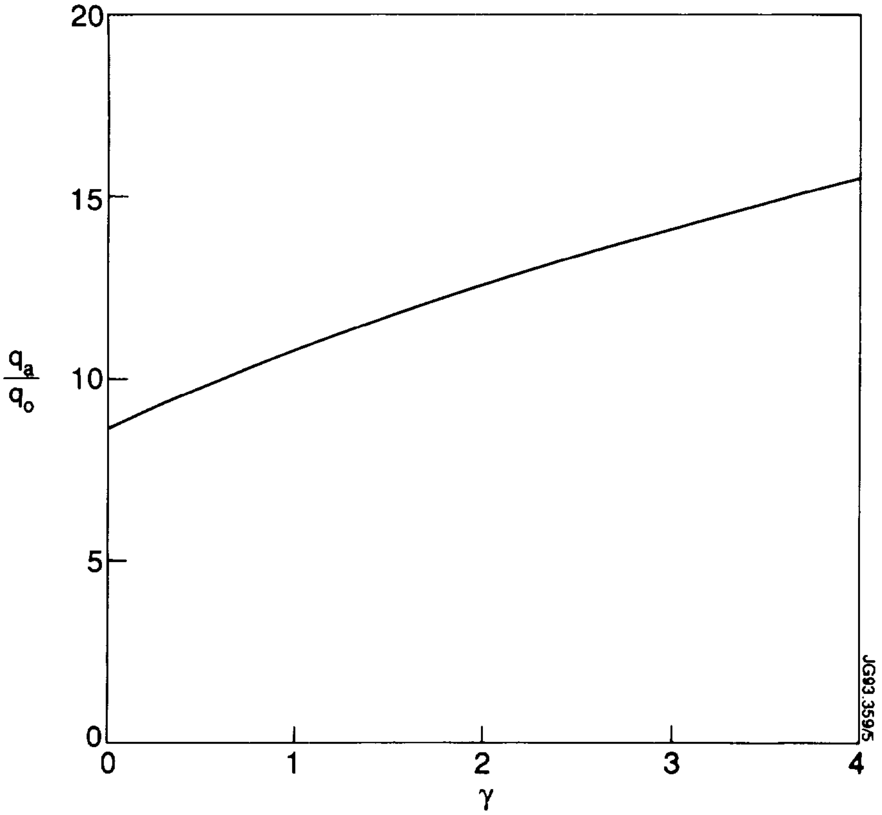

Figure 5: Calculated value of q_a/q_0 against γ where the thermal conductivity scales as $(1 + \gamma r^2)$ and neoclassical resistivity is assumed.

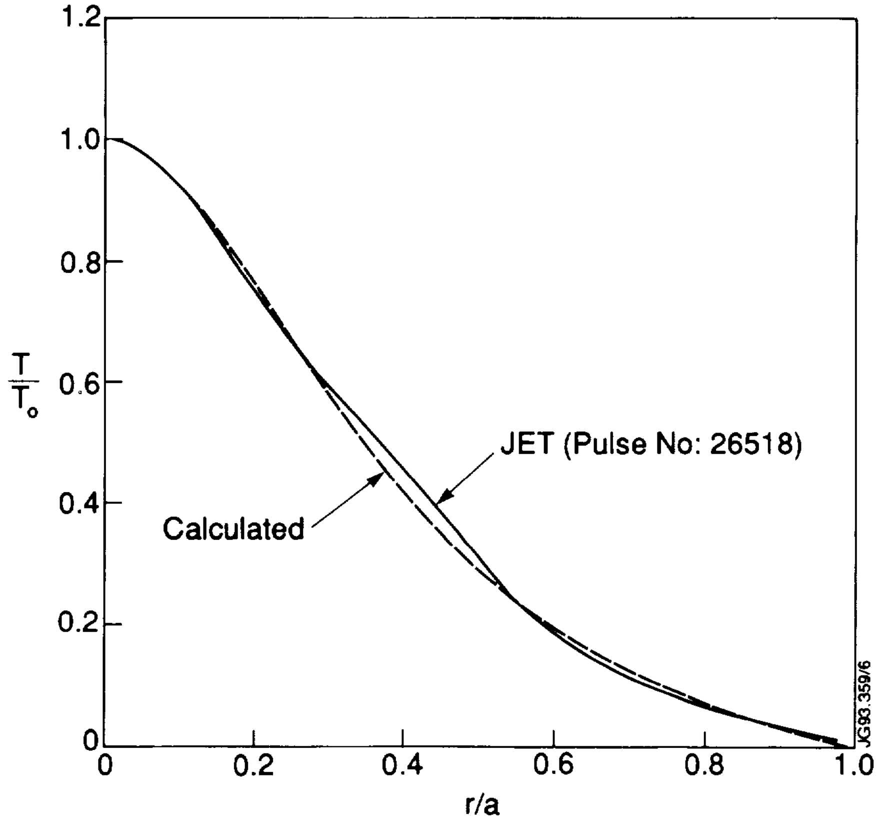

Figure 6: Comparison of the calculated temperature profile using neoclassical resistivity with a typical experimental profile at high q. There were no sawteeth in the experiment.