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Comments on ‘Relativistic Effects on Microwave Reflectometry’

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Comments on "Relativistic effects on microwave reflectometry" [Phys. Fluids B 4, 3460 (1992)]

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Abstract

New simple approximate relativistic expressions for the Hermitian part of the dielectric tensor, the real part of the refractive index and the locations of cutoffs have recently been proposed [Phys. Fluids B 4, 3460 (1992)]. It was suggested that this approximation would be of use to reflectometry as well as to a number of other applications. There are, however, significant limitations in the accuracy of this approximation, which are pointed out here.

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Investigations of relativistic effects in reflectometry [2,3,4] have revealed that in high temperature fusion plasmas it is essential to take relativistic effects into account when reconstructing density profiles from X–mode reflectometry. For the analysis of reflectometry data as well as for other applications it is thus desirable to have accurate and readily evaluated relativistic expressions for dielectric properties of plasmas. Several authors have sought simple approximations to the relativistic expressions for the Hermitian part of the dielectric tensor, the locations of cutoffs and the real part of the refractive indices. One of the most recent expressions is published in [1]. As is the case with an earlier simple approximation [5], the accuracy of the new expression varies and it is important that its limitations be documented.

In an ω_p^2/ω^2 versus ω_c/ω (CMA) diagram the new expression for the R–cutoff (upper X–mode cutoff) is a straight line parallel to the cold R–cutoff, whereas the R–cutoff predicted with the fully relativistic expression [6, expression 79] is curved (see also Figure 3 in [4]). For a given frequency and magnetic field the relativistic effects increase the R–cutoff density by an amount $\Delta n_{\text{Cut.}}$ relative to the cutoff density found in a cold plasma. In Figure 1 we plot the ratio of the density increase, $\Delta n_{\text{Cut. New app.}}$, calculated with the new approximation, to the increase, $\Delta n_{\text{Cut. Rel.}}$, obtained with the fully relativistic expression. It is seen that the new expression predicts the density increase very well near the second harmonic of the cyclotron frequency, but near the cyclotron frequency the new approximation considerably overestimates the density increase.

In [1] a plot was presented of the refractive indices obtained with the cold theory, with the new approximation and with the fully relativistic theory as functions of ω_p^2/ω^2 for $\omega_c/\omega = 0.8$ and $\theta = \angle(\mathbf{k}, \mathbf{B}) = 90°$. For these values of ω_c/ω and θ the new approximation fits the relativistic curve very well and the present calculations confirm this result, as may be seen from Figure 2 (b). However, calculations for other values of ω_c/ω show that the new approximation can be considerably in error: see, for example, calculations for $\omega_c/\omega = 0.95$, 0.65 and 0.48 in Figure 2. For propagation at oblique angles to the magnetic field the new approximation can be in even greater error as seen from the plots (e) and (f) in Figure 2.

It was suggested in [1] that the new approximation may be applied to the analysis of reflectometry data. To check this we have repeated the wide band X–mode reflectometry simulation presented in [4, Section 6] but now using the new approximation to analyze the simulated reflectometry data. In this simulation a model plasma with given magnetic field, density and temperature profiles is assumed. The reflectometric phase shift as a function of probing frequency is calculated for this plasma taking relativistic effects into account. The density profile is then reconstructed from the phase shift function using the new approximation for the dielectric properties. In the present simulation the plasma is modelled by

$$
\begin{aligned}
n_e &= (n_{e0} - n_{el})\left(1 - (r/a)^2\right)^{p_n} + n_{el} \\
T_e &= (T_{e0} - T_{el})\left(1 - (r/a)^2\right)^{p_T} + T_{el} \\
B &= (B_0 R_0)/(R_0 + r)
\end{aligned}
$$

with the following parameters: $R_0 = 3$ m, $a = 1.2$ m, $n_{e0} = 5.01 \cdot 10^{19}\text{m}^{-3}$, $p_n = 0.5$, $n_{el} = 1 \cdot 10^{17}\text{m}^{-3}$, $T_{e0} = 0.1, 5.1, 10.1, 15.1$ keV, $p_T = 1$, $T_{el} = 100$ eV, $B_0 = 2.8$ T. $r = R - R_0$ is the minor radius. The result of the simulation is presented in Figure 3. Although the reconstructed density profiles obtained with the new approximation are notably more accurate than those obtained with the cold model (compare Figure 3 with [4, Figure 6 b]), the new approximation still introduces significant errors. Reconstruction using accurate relativistic expressions always results in accurate reproduction of the model density profile [4].

We conclude that while in most cases the new approximation predicts the value of the refractive index and the locations of cutoffs more accurately than the cold approximation, the accuracy of the new approximation does have significant limitations. In X–mode at frequencies above the R–cutoff (where relativistic effects are most important) the accuracy displayed by the new approximation at $\omega_c/\omega = 0.8$, $\theta = 90°$ (see [1] and Figure 2 b) appears to be obtainable only in the ranges $0.75 < \omega_c/\omega < 0.85$ and $\omega_c/\omega < 0.25$. For propagation at oblique angles to the magnetic field the new approximation can be considerably in error even with ω_c/ω limited to these ranges (see Figure 2 f).

The new approximation presented in [1] can also be applied to O–mode. For this mode the accuracy of the new approximation is generally very good except for propagation at oblique angles to the magnetic field in the vicinity of the cyclotron frequency. A simple approximation to the relativistic dielectric properties for O–mode radiation similar to that presented in [1] was given in [4]. This approximation predicts the location of the O–mode cutoff with slightly greater accuracy (approximately half the error) than the new approximation discussed here.

Although the fully relativistic expressions for the cutoffs [6] are more complicated than the new expressions they are, in practice, readily evaluated. For the evaluation of refractive indices and the dielectric tensor the *weakly relativistic approximation* [7] is an excellent approximation at temperatures below 20 keV and readily lends itself to numerical evaluation. If computation time is important in a particular application (e.g. analysis of reflectometry data), then a multidimensional spline fit created for a limited parameter range relevant to that application may be quite acceptable and preferable to a less accurate simple approximation.

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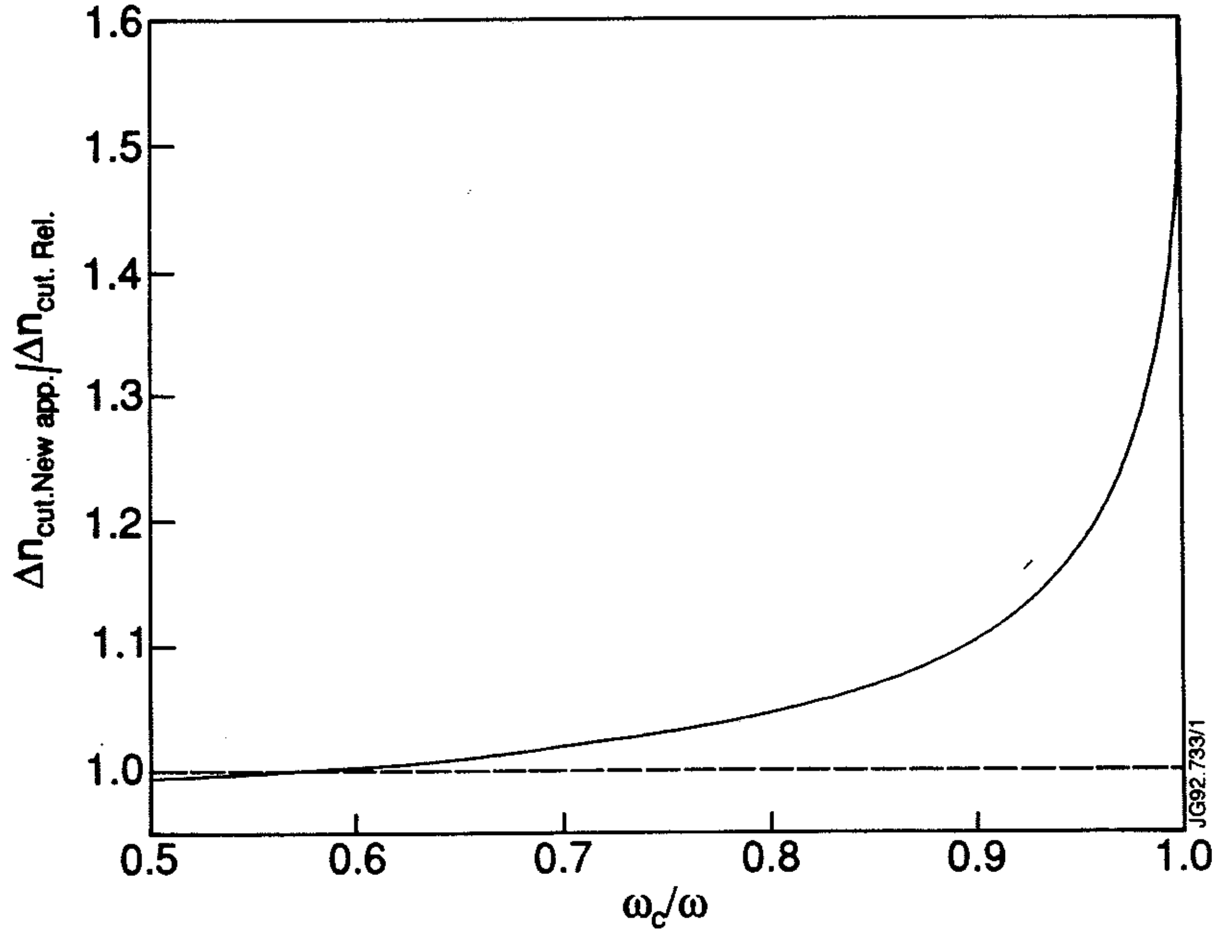
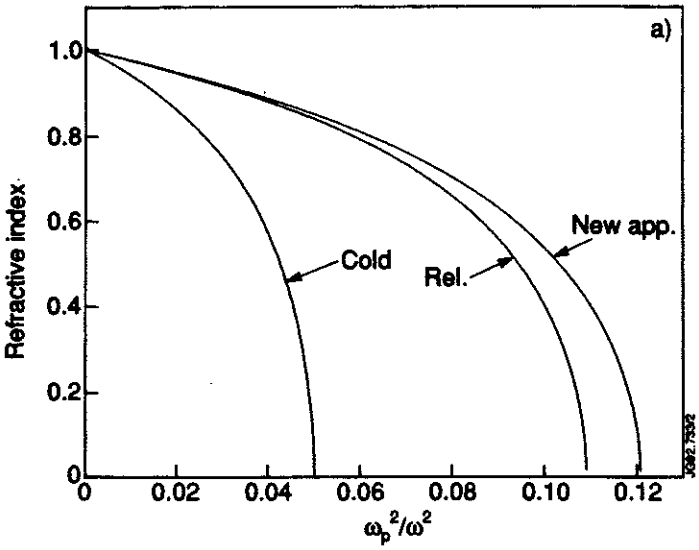
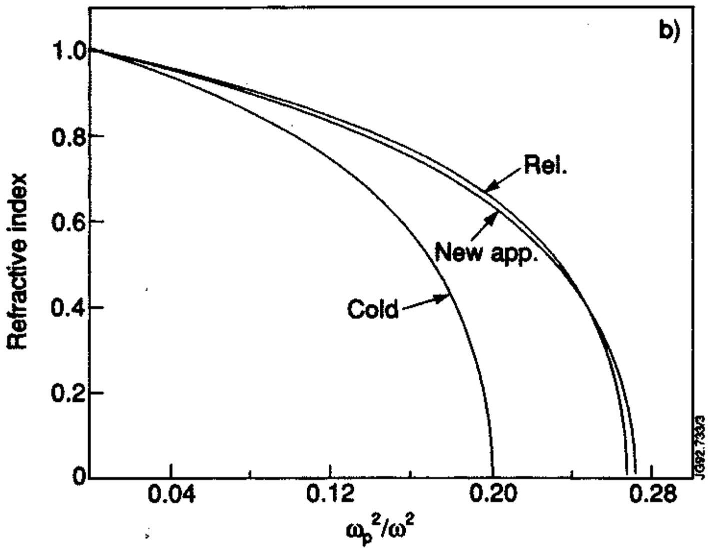


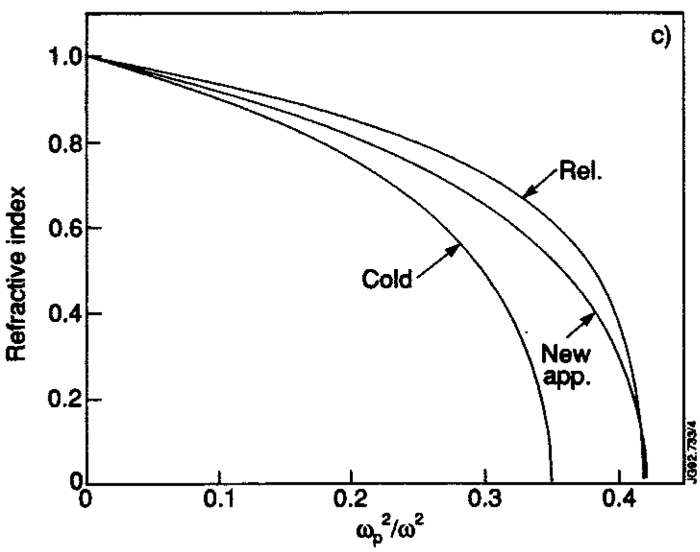
Figure 1: Relativistic density increase of R–cutoff predicted with the new approximation [1] divided by the density increase predicted with the fully relativistic expression [6] as a function of ω_c/ω.



a)
Refractive index
1.0
0.8
0.6
0.4
0.2
0
0.02
0.04
0.06
0.08
0.10
0.12
$\omega_p{}^2/\omega^2$
Cold
Rel.
New app.



b)
Refractive index
1.0
0.8
0.6
0.4
0.2
0
0.04
0.12
0.20
0.28
$\omega_p{}^2/\omega^2$
Rel.
New app.
Cold



c)
Refractive index
1.0
0.8
0.6
0.4
0.2
0
0.1
0.2
0.3
0.4
$\omega_p{}^2/\omega^2$
Rel.
Cold
New app.

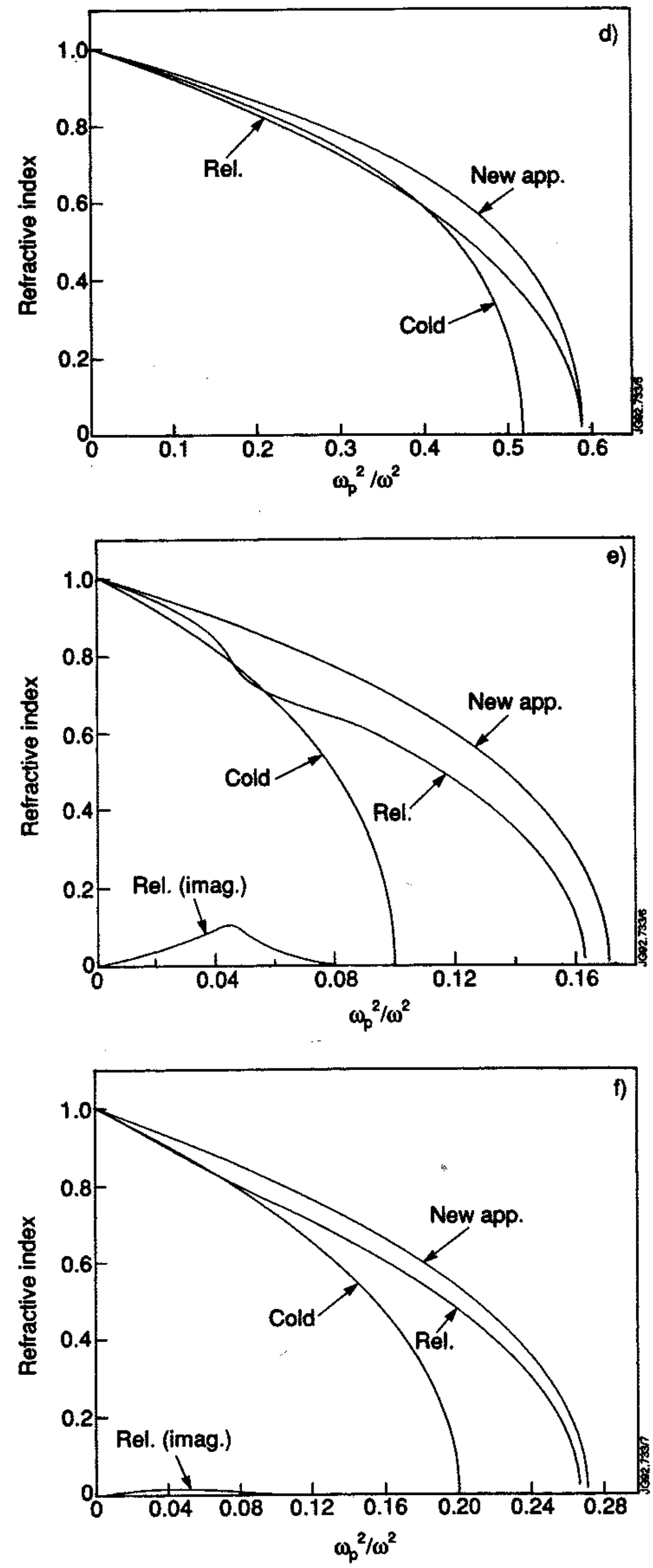


Figure 2: Refractive indices for X–mode radiation predicted by the cold plasma model, the new approximation [1] and the relativistic model as functions of ω_p^2/ω^2 for (a) $\omega_c/\omega = 0.95$, $\theta = 90°$, (b) $\omega_c/\omega = 0.80$, $\theta = 90°$, (c) $\omega_c/\omega = 0.65$, $\theta = 90°$, (d) $\omega_c/\omega = 0.48$, $\theta = 90°$, (e) $\omega_c/\omega = 0.90$, $\theta = 45°$, (f) $\omega_c/\omega = 0.80$, $\theta = 30°$. Under most of the conditions presented here the imaginary part of the relativistic refractive index is negligible. Where this is not the case the imaginary part is also plotted.

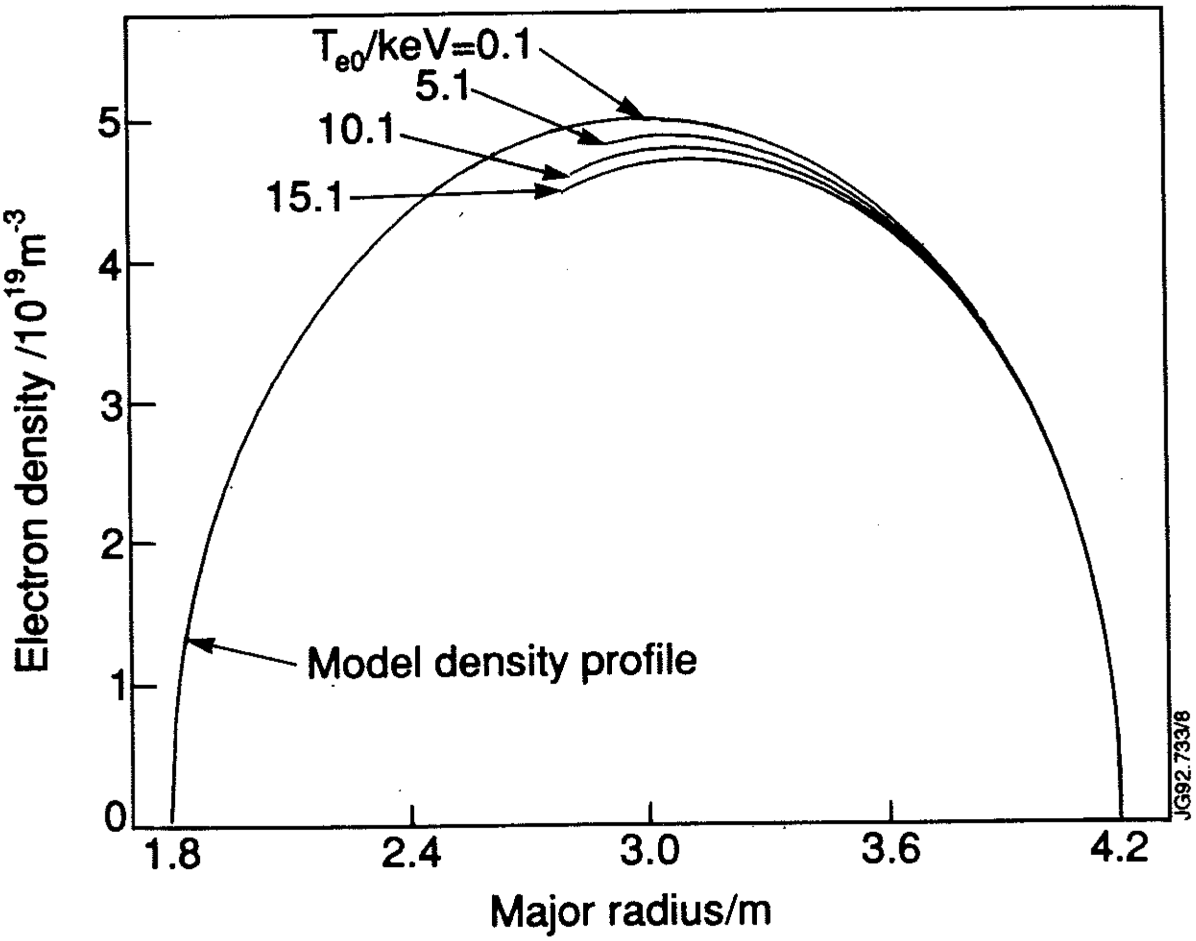


Figure 3: Model density profile and reconstructed density profiles derived from simulated reflectometry data using the new approximation to the dielectric properties of the plasma. Parameters are given in the text. They are identical to those used for the simulation presented in Figure 6 b of [4].

Appendix I

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