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* *See Annex*

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Abstract

Direct numerical simulations have been used for more than a decade to study various reduced models of plasma dynamics. This paper is devoted to a summary of the contribution made by direct numerical simulations of fluid models to the present knowledge of the basic mechanisms of anomalous transport, which have been and still largely remain the main objective of the majority of the investigations. In the first part of the work, the seemingly prevailing claim that numerical results are consistent with quasilinear theory is critically analyzed employing general arguments of turbulence theory. Then the emphasis will be shifted to some emerging physics issues in plasma turbulence which will presumably play a major role in the future numerical investigation. In particular, the problem of the determination of the correlation length, the role of the coherent structures and the question of the subcritical transition to turbulence are addressed. Finally, the subtle role of dissipation is discussed: with the support of recent numerical results and exploiting the analogy of some plasma turbulence models with those employed for ordinary fluids, it is argued that dissipation is likely to enter in a non-trivial way in the global scaling laws for plasma transport.

I – Introduction

One of the great challenges in current fusion research is, needless to say, understanding the mechanisms of plasma transport in confinement devices.

The reason why it is exceeding difficult to make progress in the solution of the transport problem is that one is ultimately faced with the necessity of solving the equations of plasma dynamics. Indeed it was readily recognized that collisional theory alone cannot explain the experimental observations. Collisional theory is based on expansions around *stable* equilibrium states. However it would seem that present day machines operate in such regimes that a stable equilibrium is not achieved. This has been shown by linear theory which predicts that plasma states associated with the experimentally observed profiles of various macroscopic quantities are unstable to various types of perturbations.

When this occurs, macroscopic plasma motion i.e.,— over scales larger than the mean free path, will ensue. Thus the observed profiles (or equilibrium states) can only be regarded as *time averages* around which fluctuations much higher than the thermal fluctuations predicted by collisional theory can occur. That this picture is qualitatively correct was confirmed by the experimental observation of such fluctuations in many devices.

The need to understand the plasma dynamics has motivated over the last decade a lot of work on the derivation of reduced models for the evolution of macroscopic quantities in various instability conditions[1].

Since most of the dynamics relevant to transport are thought to be characterized by frequencies much smaller than the ion cyclotron frequency, the expansion in the small parameter $\omega/\Omega_{ci} \ll 1$ has generally been employed (drift wave expansion). Even so, a reduced model of fluid equations in a pure plasma in a simplified geometry (thereby neglecting trapped particles) would not employ less than six scalar fields with coupled dynamics. The complexity of the problem is self-evident.

In this scenario, numerical simulations of the equations of plasma dynamics has come as a powerful and natural investigation tool. The main focus of this work will be on the conceptual contributions provided by more than a decade of direct simulations of fluid models for the plasma dynamics.

The outline of this work is as follows. In Sec. II the conventional ideas (based on quasilinear theory) about transport induced by microinstabilities are analyzed. Upon employing a simple paradigm model, the forced Hasegawa-Mima equation, the need for high resolution simulations to investigate truly turbulent behavior will be emphasized. This will allow to show that the frequent claim drawn from earlier simulations that "quasilinear theory is qualitatively correct" is not justified.

The subsequent sections are devoted to emerging topics which are attracting interest in the simulations of the recent years. In Sec. III the role of coherent structures in reducing the transport from quasilinear theory is discussed. Sec. IV outlines the problem of subcritical excitations or turbulence selfsustainement. Finally, some issues related to the global simulations will be addressed in Sec. V. In particular the likely role of dissipation in the global scaling law will be emphasized, also employing analogies drawn from the dynamics of normal fluids. Conclusions, given in Sec.VI, will focus on the critical physical issues which will be arguably addressed in the future work.

II – The early simulations and quasilinear theory

Fluid simulations oriented to understand the transport associated to various microinstabilities were carried out for most of the eighties especially by Horton and coworkers [2] and Waltz[3]; see also[4].

The common feature of the early work was the comparison of the numerically found transport coefficients with those analytically derived by means of quasilinear formulas. As a results of the simulations, it was generally stated that quasilinear formulas are substantially valid.

This has given support to the global (indirect) simulations with codes based on diffusion equations and quasilinear formulas for the turbulent transport coefficients. However, once compared with experiments, it turned out that those codes substantially overestimate the actual transport. The most natural explanation of this failure is that quasilinear theory is not adequate to derive the functional form of the turbulent fluxes.

It is indeed the scope of this section to critically review the quasilinear approach to point out the origin of its deficiency in a general way, leaving some explicit examples and other circumstantial evidence against quasilinear theory to the subsequent sections.

The conventional point of view of turbulent transport in confined plasmas is based on few assumptions which were more or less explicitly used in the simulations. First of all, the correlation length λ_c of the type of turbulence under study is generally assumed to be much smaller than the machine size a. Then over lengths l much smaller than the machine size but much bigger than the correlation length, $\lambda_c \ll l \ll a$, one can treat that system as a homogeneous one with prescribed gradients. The latter are ascribed as the "sources" of turbulence as suggested by linear analysis, and kept constant during the simulation. Indeed, modes of interest to transport studies are generally destabilized when a critical gradient is exceeded. Sufficiently far from threshold, the spectrum of unstable modes is generally peaked around a wavenumber $k_\perp$ which scales like the inverse of the ion gyroradius ρ_s: $k_\perp \rho_s \sim 1$. It is then implicitly assumed that the spectrum of the resulting turbulence resembles the spectrum of the unstable modes. This leads to the conclusion that $\lambda_c \sim \rho_s \ll a$.

Second, the role of dissipation was neglected. Dissipation coefficients like viscosity, thermal conductivity etc., were introduced as big as needed to absorb the energy transferred from the unstable modes to the stable ones. This attitude is linked to the low resolution of the early simulation, typically 32×32 in the spectral codes. Such resolution was more or less what was needed to represent all the unstable modes in the system.

It is apparent that the above setup for a numerical simulation does not allow much flexibility. The key feature is that *no small control parameter enters in the system.* Therefore every physical quantity must be simply expressible in terms of the normalization units ρ_s for lengths and a/c_s for times. The consequence is that the diffusivity obtained from these simulations is bound to obey the so-called gyro-reduced Bohm scaling: $D \sim (cT_e/eB)(\rho_s/a)$. In the end one gets what one puts in.

However, small parameters do exist in the original system. Such are for example the ratio of the ion-ion collision frequency to the drift frequency $\nu_{ii}/\omega^\star$, which is a measure of dissipation, and ρ_s/a. Then, a closer inspection would show that the above approach is at the very least rather dubious. Indeed, although the main source of fluctuation may occur at $k_\perp \sim \rho_s$, the nonlinear coupling occuring between modes would eventually transfer the injected energy to other scales until it is ultimately absorbed by some dissipation mechanism. It is the way energy is transferred across wavenumber space and the absorption mechanism which determines the spectral properties and ultimately transport. Mathematically, dissipation operators are often singular perturbations of the dissipationless equations. Therefore the limit of zero dissipation is a delicate one. For example one should not expect analyticity of the turbulent fluxes for small values of the dissipation coefficients. In this limit the fluxes will be independent of dissipation only in special cases.

The situation is schematically illustrated by a model problem. Consider the forced Hasegawa-Mima model for the electric potential ϕ with a passively advected scalar ψ:

$$\partial_t(1-\nabla^2)\phi + \partial_y\phi + \gamma\phi + \vec{v}_E\cdot\vec{\nabla}(-\nabla^2\phi) = -D_{\rm H}\nabla^{2(p_{\rm H}+1)}\phi - D_{\rm L}\nabla^{2(p_{\rm L}+1)}\phi \qquad (1)$$

$$\partial_t\psi \,+\, \vec{v}_E\cdot\vec{\nabla}\psi = 0 \qquad (2)$$

where $\vec{v}_E = \hat{z}\times\vec{\nabla}\phi$ and the usual normalizations for lengths and times have been chosen. The model forcing operator γ is peaked around a forcing wavenumber $k_f \sim \rho_s$ with a growth rate of order $\gamma_{\vec{k}} \sim \gamma_f$ over a bandwidth $\Delta_f \sim \rho_s$. When $\gamma_{\vec{k}}$ falls off to zero fast enough as $k \to 0$, the forcing can be considered localized and the growth rate spectrum can be approximated by a delta function in wavenumber space:

$$\gamma_{\vec{k}} \approx \gamma_f\Delta_f\delta(k-k_f) \quad .$$

The large- and short-scale damping operators are chosen to be hyperviscosities with coefficients $D_{\rm L}$ and $D_{\rm H}$ and indexes $p_{\rm L} < 0$ and $p_{\rm H} > 0$. These dissipation coefficients can be combined in dimensionless, Reynolds-like numbers, which, together with a measure of wave dispersion, constitute the full set of control parameters of the model. In the following, the forcing is assumed to be strong enough that weak turbulence effects can be neglected.

When the Reynolds-like numbers are big enough separation of spatial scales and, in general, of timescales will occur, because the fluctuation energy is absorbed at scales well separated from the injection scale. In this regime, the spectra can be computed with a Kolmogorov-type analysis[5]. In Fig. 1 the difference between the spectra obtained by this method and by quasilinear techniques is sketched.

For the subsequent discussion of passive advection, the main interest is in the cascade behaviour in the large scale range $k \ll k_f$. In this range, the Hasegawa-Mima energy

$E = \sum(1+k^2)|\phi_{\vec{k}}|^2$ is transferred to the large scales a constant rate ϵ, which depends on the actual amount of forcing. The k-space energy density $E(k)$ scales like $E(k) \sim \epsilon^{2/3}k^{-11/3}$, so that the size of the velocity fluctuations behaves like $v_k \sim \epsilon^{1/3}k^{-1/3}$, while the k-dependent energy transfer timescale is $\tau_k \sim \epsilon^{-1/3}k^{-8/3}$.

In order to compare the prediction of quasilinear theory with the one obtained with the Kolmogorov analysis, it is convenient to choose the popular strong plasma turbulence ordering for the forcing: $\gamma_k \sim \omega^\star \sim c_s/a$ over a bandwidth of order $\Delta k \sim \rho_s$. Then, using dimensional variables one has $\tau_k \sim \tau^\star(k\rho_s)^{-8/3}$ and $v_k \sim v^\star(k\rho_s)^{-1/3}$, where $\tau^\star \sim a/c_s$ and $v^\star \sim c_s\rho_s/a$.

Over large enough scales (i.e.,— larger than some suitable Lagrangian correlation length) the (ensemble) averaged passive advection equation Eq. 2 becomes a diffusion equation with an effective diffusivity D_{eff}. This diffusivity depends on the only control parameter of the problem, the Kubo number $K = v\tau_c/\lambda_c$, where v is the r.m.s. velocity fluctuation associated to the potential ϕ, and τ_c and λ_c are the the correlation time and the correlation length.

When the large scale dissipation is small, the turbulent energy is dissipated at a scale $\lambda_0 \ll \rho_s$, which depends on the dissipation operator. Then one can take $\lambda_c \sim \lambda_0$, and using the previous expressions for v_k and τ_k at $k \sim 1/\lambda_0$ as estimates for v and τ_c one gets:

$$K \quad \sim \quad (\lambda_0/\rho_s)^2 \quad \gg \quad 1$$

whereas a quasilinear calculation would give $K \sim 1$. In the large Kubo number regime the effective diffusivity obeys a scaling law $D_{\text{eff}}/(v\lambda_c) \sim K^{-\alpha}$ with $0 < \alpha < 1$[6]. The conclusion is that

$$D_{\text{eff}}/D_{GB} \sim (\lambda_0/\rho_s)^{4/3-2\alpha}$$

instead of the usual quasilinear gyro-Bohm result: $D_{\text{eff}} \sim D_{GB} \sim (cT_e/eB)(\rho_s/a)$.

We conclude this section by summarizing again the main point. Proper turbulence simulations require a large enough resolution to accomodate the wide separation of scales occurring in the original system. When this is not achieved, spurious effects are introduced, the most common deficiency being a too big dissipation. When this occurs, no small parameter exists in the simulated system. The consequence is that any physical quantity automatically results of order one in the normalization units.

For drift waves, this would lead to the wrong conclusion that the gyro-Bohm scaling of the diffusivity is adequate. On the other hand we have seen how a proper analysis on the model (1-2) gives a different answer for the effective diffusivity in the problem of passive advection. In the next section the analysis would be extended to a selfconsistent problem, with more dramatic consequences.

III – Coherent Structures

The word "coherent structure" (CS) has become increasingly common in the simulation literature over the last few years. Since some confusion has arisen about what to call CS, the following definition will be employed in this work:

For a given set of model equations, coherent structure is defined to be any solution of the inviscid (dissipationless) equations which is stationary in a suitable reference frame.

Various coherent-type solutions to the equations of reduced models for plasma turbulence have been found in the recent years, particularly by Horton and collaborators[7].

It must be noted that the above definition is not yet satisfactory from the experimental point of view, because it refers to a specific set of equation. However, in practise, CS's are characterized by time independent functional relations between the fields describing the system. Then the observation of slowly decaying correlations between those fields would be the sign that some sort of coherent behaviour is occuring in the system.

Furthermore, it must be pointed out that strictly speaking, global coherent structure solutions of the model problems do not exist because of the presence of dissipation. However, from the practical point of view, one is interested in systems whose behaviour is *approximated* by the one of CS's in some spatial subdomain. Then one will be faced with systems that exhibit "coherent" behaviour in some spatial region and more chaotic behaviour in others. The emerging picture is one of a system where islands of coherent or vortex-like behavior where dissipation is negligible are separated by "turbulent" boundary layers where most of the dissipation occurs. See Fig. 4.

There is presently no systematic approach to the problem of CS's in the presence of dissipation. However, a certain amount of numerical results is now available.

Coherent structures were first observed in numerical simulation by McWilliams[8] in a model of Rossby-wave turbulence. Subsequently, very high resolution simulations (up to 1024 × 1024 employing spectral codes) of the two-dimensional Navier-Stokes where performed by other groups[9]. The important discovery of this line of work was that CS's in the form of long living vortices dominate in the enstrophy inertial range. These vortices are quasi-stationary solutions of the 2-D Euler equation:

$$[\phi, \nabla^2\phi] \approx 0$$

where ϕ is the stream function. Vorticity then becomes functionally dependent on the stream function: $\nabla^2\phi = f(\phi)$[9-10]. The consequence is that enstrophy transfer across wavenumber space is strongly inhibited and energy power spectra steeper than the k^{-3} law[11] predicted by a Kolmogorov-type argument occur.

The role of CS's in plasma transport were first observed in high resolution spectral simulations of a model of η_i-turbulence in Ref. [12]. In order to illustrate the main points, Eq. 1-2 are modified by introducing linear coupling terms between the two equations. The new model, which is a simplification of the one discussed in Ref. [12], is:

$$\partial_t(1-\nabla^2)\phi + \partial_y\phi - \epsilon\partial_y p + \vec{v}_E \cdot \vec{\nabla}(-\nabla^2\phi) = -D_1\nabla^4\phi \tag{3}$$

$$\partial_t p + (1+\eta_i)\partial_y\phi + \vec{v}_E \cdot \vec{\nabla} p = D_2\nabla^2 p \tag{4}$$

One can easily verify that Eq. 3-4 admit a class of inviscid solutions in the form of coherent structures travelling with speed u, $\phi(x,y,t) = \phi(x, y-ut)$, $p(x,y,t) = p(x,y-ut)$, and characterized by the functional relations:

$$\begin{aligned} p &= \alpha\phi \\ \nabla^2\phi &= \beta\phi \\ -\alpha u + 1 + \eta_i &= 0 \\ -u(1-\beta) + 1 - \epsilon\alpha &= 0 \end{aligned} \tag{5}$$

The relation of functional dependence between p and ϕ has a deep consequence on transport. Indeed one can immediately verify that, in the regions where such relation holds, the heat flux $\langle p\vec{v}_E \rangle$ is identically zero in any subdomain bounded by a closed equipotential line (which then encircles a maximum or a minimum of ϕ). In Fourier space, the functional relation between p and ϕ implies that the phase difference between the Fourier components of the two fields is zero. On the other hand, quasilinear theory, which *treats fluctuations linearly* predicts a finite phase difference (and hence a nonzero flux) which depends on the growth rate of the given Fourier component.

Coherent structures characterized by an approximate linear functional relation between p and ϕ were indeed reported in Ref. [12]. In Fig. 2 the contour plot of the potential is shown for two stages of the simulation: the early, essentially linear stage, which would be used in the quasilinear estimates and a later stage corresponding to the saturated turbulence. Large scale structures are present in the saturated states. These structures present a good degree of coherence as shown in Fig. 3, where the pressure is plotted against the potential for each grid point in order to emphasize the linear relationship. The date are scattered around a straight line because the coherence is only approximate. Although dissipation prevents the development of exact inviscid solutions to the model equations, the observed overall behaviour of the system is in line with the previously drawn picture. Fig. 4 shows a sketch of an array of coherent structures separated by turbulent boundary layers with steep gradients.

Because the CS's contribute negligibly to the heat flux, the observed overall transport was substantially less than the one expected from analytic estimates of the quasilinear type. A more important observation was that the actual flux scales with some (small) power of dissipation. A dependence of the flux on the small dissipation parameter $D \sim \nu_{ii}/\omega^\star$ of the form $F \sim D^\alpha$ with $\alpha = .3 \div .5$ was reported (Fig. 5). This dependence cannot be predicted by quasilinear theory because the dissipation does not enter in the growth rate of the most unstable modes in a significant way. Thus quasilinear theory fails not only in the magnitude of the estimated transport, but, more important, in the prediction of the *correct scaling law* of the heat flux.

It must be stressed that the observation of coherent structures would not have been possible with lower resolution than the one employed in Ref. [12] (up to 256×256). Indeed a too small resolution requires a too big dissipation to allow the formation of almost inviscid solutions of the model equations. Likewise, the study of the scaling of transport with small parameters like D required high resolution.

We conclude this section by commenting on some open questions on the issue of CS's.

The first point is that one needs a criterion for the formation of CS's for a given set of model equations. Some help could come from the stability analysis of ideal CS's. Indeed, although the actual structures are only dissipative approximations of the ideal ones, a good starting point would seem to assume that the observable structures are close to the *ideally stable* inviscid solutions of the model equations. However, to our knowledge, little is known about the stability properties of the various families of ideal CS's occurring in the literature.

An interesting criterion has been introduced by Leith[13] to explain the formation of CS's in the two-dimensional Navier-Stokes equation. This criterion, which can be seen as a version of the selective decay hypothesis[14], states that CS's are regions of the fluid where enstrophy is minimized for a given energy. The rationale beneath this hypothesis is that in turbulence decay experiments enstrophy is dissipated faster than energy in the limit of high Reynolds number. Then states of minimum enstrophy would seem natural states even for a forced system. However, whereas the vorticity of Leith-type CS's is proportional to the stream function, a detailed analysis in Ref. [10] suggests a more complicated functional relation. In any case it would seem that the Leith approach is worth pursuing also for other systems. In the case of plasmas the only example so far available is Taylor's relaxation theory. Indeed Taylor states are inviscid stationary solutions of the MHD equations (MHD coherent structures).

The second comment is about the role of dissipation observed in the transport scaling law when CS's are present. Although there is presently no analytic argument that predicts the observed exponent of the scaling law, it must be pointed out that some dependence of transport on dissipation is expected. Indeed, because of the zero flux property of the CS's, the only significant contribution to transport comes from the the boundary layer region between the structures. Then, it is natural to expect that the width of such region decreases as the dissipation decreases, thereby decreasing the average flux.

It is must be noted that the above results have been obtained in homogeneous systems.

The question of the role of dissipation in the transport scaling laws will be reproposed again in a later section when discussing inhomogeneous systems of the type found in global simulations.

IV – Subcritical excitation and selfsustainment

In this section the conventional quasilinear wisdom on plasma turbulence will the object of further criticism from a different direction.

Quasilinear theory treats the fluctuations linearly. In order to produce a non-trivial, non-zero result, quasilinear theory requires a set of unstable modes. Instability occurs when some control parameter, say R, becomes greater then a critical value $R > R_{\text{crit}}$.

However, for the turbulence to be excited, one does not need a linearly unstable equilibrium. Even when $R < R_{\text{crit}}$ one may have a stationary turbulent behavior provided that some conditions are met.

The first condition is that the system possesses more than just one basin of attraction for its dynamics. One of these basins would be some neighborhood of the stable equilibrium. However, if a second condition is met, that the initial state of the system is far enough from the stable equilibrium, the system will evolve into a final state which is different from the reference equilibrium. Then, depending on the problem, this final state may exhibit turbulent features.

A possible bifurcation diagram illustrating subcritical excitations is shown in Fig. 6.

The possibility of subcritical excitations is well known in the fluid dynamics literature. For plasmas, the first example of a system showing subcritical behaviour is probably reported in a paper by Biskamp and Walter[15], for a model problem of drift waves.

More recently, detailed numerical simulations by Scott on collisional electron drift-waves[16] has brought back the issue to the general attention. Collisional drift-waves are especially appealing because of the universal nature. However, linear theory predicts shear damping stabilization for such modes.

Using a two-dimensional slab code with a single rational surface but introducing all the detailed electron dynamics in the singular layer, it was shown that subcritical behavior indeed occurs. A noteworthy result is that full selfsustainement occurs when the electron temperature fluctuations are included in the model. No additional energy source, beside the intrinsic "free energy" associated to the gradients is necessary to achieve a stationary state with a finite amount of energy in the fluctuations.

Closing this section, It is worth noting that the above results has again required a very accurate numerical treatment of the singular layer around the rational surface. This is the reason why only two-dimensional simulations have been so far performed. Thus we can see again that high resolution has been a necessary technical ingredient in the new finding.

V – Towards high resolution global simulations

Plasmas in confinement machines are inhomogeneous systems. Therefore plasma turbulence relevant to transport is also in principle inhomogeneous.

In real systems, inhomogeneity can occur because of two reasons. The first obvious reason is that the background equilibrium fields are not constant. Then, whichever model problem one may consider, the outcome would in principle depend on the whole profile of the equilibrium fields. The second one, not less important, is that the system is bounded. Then the final result would also depend on the chosen boundary conditions for the fluctuating fields.

The large majority of the simulations performed so far are at least local if not even homogeneous. This is certainly the case when spectral methods have been employed. Indeed the applicability of this method requires periodic boundary conditions and the system is homogeneous with all the equilibrium fields kept constant.

Even when the most obvious source of inhomogeneity, magnetic shear, is taken into account, the simulation have been set up in a local fashion[17-18]. The simulation domain is generally a three-dimensional slab with periodic boundary conditions in the two homogeneous directions, say y and z. In the inhomogeneous direction x, the gradients of the equilibrium quantities are kept fixed, while the fluctuating fields are set to zero at the slab boundary. A noteworthy consequence of this choice is that the radial velocity is zero at the boundary.

The rationale behind this choice is that it is expected that the shear has a localizing effect on turbulence. When the slab width is substantially larger than the correlation length, it is argued that the boundary conditions are irrelevant, because there is no correlation between the fluctuations occurring in proximity of the two slab edges. The system is then assumed as representative of a small plasma volume where gradients can be taken constant. "Local" turbulent transport coefficients are then obtained as a function of the local parameters, which include the local gradients of the equilibrium fields such as the magnetic shear parameter.

This argument, however, is not convincing. Indeed, since the particular choice of boundary conditions discussed before set to zero the component of the velocity perpendicular to the boundary, the turbulent flux is also zero at the boundary. Then all the transport occurs through the collisional channel, with the heat flux being given by expressions of the type $F = \chi(\vec{\nabla}T)_{\mathbf{boundary}}$. The consequence of this modeling is that as dissipation decreases thermal boundary layers with steep temperature gradients would form at the slab boundary. Indeed, as soon as the total transport exceeds the collisional value, the heat flux must scale with χ with a power smaller than one in order to be bigger than the collisional value: $F \sim \chi^{\alpha}$ with $\alpha < 1$. Then $\vec{\nabla}T \sim \chi^{(\alpha-1)} \rightarrow \infty$.

Now the key question is whether this boundary effect has any influence on the transport in the slab interior. The previous argument shows only that the boundary gradients are big. However, since the average gradient is kept constant, the gradient in the slab interior may itself be affected if the temperature jump across the boundary layer grows bigger as the dissipation decreases. When this happens the interior gradient is reduced and the local turbulence would depend on this new, smaller gradient rather than the one on started from.

This type of phenomenon has been indeed observed in several simulations and is commonly given the improper name of quasilinear relaxation of the equilibrium profiles. This phenomenon is sketched in Fig. 7.

Two dimensional shearless model like Eq. 3-4 are likely to exhibit this kind of behavior. Indeed, in this case, correlation length will be set by the large scale dissipation mechanism. If the latter is not existent or simply ineffective, large scale structures would form that occupy all the available volume, as a consequence of the inverse energy cascade.

In order to prevent the flattening of the profiles in the slab interior, gradients are generally "frozen" by artificially removing the components of the fluctuations which survive spatial averaging along the homogeneous directions.

This procedure, unfortunately, leads to additional problems. Freezing the profiles is equivalent to add unphysical heat or particle sources and sinks that act as an instantaneous feedbeck on the average profile. Since the turbulent flux at the boundary is zero because of the above boundary conditions, any statistically stationary state of the system would not have a constant turbulent flux across the slab. Then one is faced with a system with constant gradients but space varying fluxes. This effect would make impossible to obtain a one to one correspondence between gradients and fluxes, unless additional conditions are met.

There is currently no criterion to predict whether thermal boundary layers with big jumps would form in a given slab model. In order to obtain local transport coefficients, the only way to proceed is to run a simulation and verify a posteriori whether the boundary layer can be effectively separated from the core. This would require that a larger and larger core region of constant gradient develops as the separation beween the slab boundary increases.

Magnetic shear has been sometimes invoked as a mechanism capable of preventing the occurence of boundary effects. Unfortunately, this is not the case. Hamaguchi and Horton[18] have observed the formation of steep boundary layers in their three-dimensional simulations of η_i-turbulence with magnetic shear. It turns out that although magnetic shear is effective in controlling the correlation length of the observed turbulence, the temperature drop in the slab core is substantial.

Thus, it would seem that there is a fundamental difficulty in investigating turbulent transport issues with the above set-up. A possible alternative that still retains the feature of having constant gradients, could simply be to employ different boundary conditions. A free boundary approach, for example, would seem more appropriate, but, to our knowledge, it has not yet been attempted.

A better choice, of course, is to operate with global simulations. Before discussing this option, it is instructive to digress on the problem of turbulent thermal convection in ordinary fluids.

First, one must note the analogy between plasma models like Eq. 3-4 and the equations employed in thermal convection between plates in ordinary fluids. This latter system is generally treated in the so-called Boussinesq approximation. One can then recognize that Eq. 3-4 reduces to the two-dimensional Boussinesq equation when the drift frequency is suppressed. When drift terms are retained the Eq. 3-4 are homologous of equations used to model convection in rotating fluids[19].

Now, it is known that thermal convection is characterized by the build-up of boundary layers which account for most of the temperature drop across the plates. These boundary layers are known to be the key elements of the transport process. Indeed, according to an old hypothesis due to Priestley[20], turbulent convection operates with marginally stable boundary layers. Then the Raleigh number of the boundary layer is approximately equal to the critical value for the onset of convection: $Ra \sim ((\Delta T)_{\mathrm{boundary}} d^3)/(\nu\chi) \approx (Ra)_{\mathrm{crit}}$, where ν is the viscosity. Assuming that $(\Delta T)_{\mathrm{boundary}}$ is of order of the total temperature drop ΔT across the slab of thickness L, the width of the boundary layer d is determined as a function of the Raleigh number of the system: $d/L \approx (Ra/Ra_{\mathrm{crit}})^{-1/3}$. Then the heat flux across the boundary layer, which is also the average flux across the slab, scales like

$$F \approx (\chi \Delta T/L)(Ra/Ra_{\mathrm{crit}})^{1/3} \sim \chi^{2/3}\nu^{-1/3}(\Delta T)^{4/3} \quad .$$

This scaling law has been verified experimentally and numerically[21]. Most recently, the well-known experiment by the Chicago group[22] has observed a transition to a different regime with a weaker dependence on R_a as R_a exceeds a critical value of order 10^7.

It is useful to compare the experimental findings with the prediction of quasilinear theory. A simple calculation on the homogeneous Boussinesq system gives the dissipation-independent result

$$F \approx (\Delta T \chi/L)(R_a P_r)^{1/2}$$

where $P_r = \nu/\chi$ is the Prandtl number. The quasilinear scaling is then stronger (at fixed Prandtl number) than the experimental one. One finds once more that quasilinear theory is not adequate, in the sense that not only overestimates the observed flux, but also predicts the wrong scaling law.

The lesson is again that, contrary to what happens in quasilinear theory where dissipation does not plays any role, in the real system dissipation is crucial in the formation of the boundary layers that control the overall transport in certain conditions. Therefore the above analysis is sufficient to strongly motivate the investigation of the possible role played by dissipation in the global scaling laws of plasma transport.

In the following and final part of this section the problem of global simulations will be discussed. By global simulation we mean a simulation such that the equations of whichever model one considers are solved in a domain with the same topology as the confinement machine of interest. In practice such a domain would be a true torus or a cylinder with periodicity enforced along its axis (straightened torus).

Consider now the question of the boundary conditions. In global simulations, boundary conditions will be imposed essentially at the plasma edge. Therefore much depends on the edge model one considers. In particular, if one imposes the usual stress-free boundary conditions, the radial velocity at the boundary is zero whereas the poloidal edge velocity is determined by the evolution. This choice would still produce thermal boundary layers as discussed before to compensate the drop in the convective component of the flux with the enhanced collisional flux due to the steep gradient.

However, in the context of global simulations this edge effect is not necessarily a drawback and could even be related to a genuine physical phenomenon. It is worth recalling here that steep gradients are observed to form spontaneously in certain types of discharges (H-modes).

The formation of boundary layers poses again the previous question whether the profile should be allowed to relax or should be frozen. Whereas the common practice of the few global simulations so far performed[23] has been to freeze the profile, we here advocate a different choice that we consider closer to the experimental situation. Real experiments are characterized by energy and particle sources or sinks. Thus the energy equation to be employed in the simulation should take the following schematic form:

$$\partial_t p + \vec{v}_E \cdot \vec{\nabla} p = \chi \nabla^2 p + S$$

Here the form of the heating function S would generally embody both additive terms like those associated with additional heating (whose deposition profiles are in principle controllable) and nonlinear, selfconsistently adjustable terms like the Ohmic heating function $\sigma(T)E^2$. Therefore the system is not forced by an internal "free energy source" associated with a frozen gradient but by an external energy source. To our knowledge, this possiblity of forcing a turbulent plasma model has been so far employed only in two-dimensional simulations[12,24].

It is worth pointing out that the simulations with prescribed energy source can be viewed as the inverse problem of the simulations with frozen gradients. In the former, the total amount of energy losses are given, while the profiles are derived. However in this case the profiles must be considered as time averages of the instantaneous profiles which are in principle fluctuating quantities. Thus the two approaches are not in general equivalent.

VI – Conclusions

After more than a decade of local simulations of plasma dynamics with fluid models, the challenge is now posed by high resolution global simulations. These type of studies will allow to address key questions that simpler investigations have not been able to tackle properly.

Among them the question of what determines the correlation length of turbulence is a central one. Is the shear the important quantity or is large scale dissipation the relevant one? This crucial point has been investigated only marginally in some three-dimensional local simulations. An answer to this question is essential in order to assess to what extent one can employ a diffusive model of large scale transport.

Another important point to investigate is the role of dissipation and whether effects like coherent structures and boundary layers play a role in the global scaling law.

A more specific question is whether global simulation of a given model would exhibit transitions between different transport regimes, like the L-H transition observed in real experiments. To this extent, it is crucial to operate with a code that takes into account the strong variation of the collisional transport coefficients with density and temperature experienced at the plasma edge.

High resolution is required to investigate the role of small parameters without preconceptions. From the experience already accumulated one evaluates that a resolution of at least 500 radial points with 256 poloidal harmonics and 64 toroidal harmonics may be needed to obtain meaningful results. For a typical model problem of interest for ion transport, such as the 3-d cylindrical η_i model, these resolution poses a memory requirement which is close to the maximum capability of present day supercomputers. Numerical simulations of the 3-d Navier-Stokes equation have already been performed with similar resolution. However, for plasma transport studies aimed to extract information on the global scaling laws, one would probably need longer runs, lasting several *"longest turnover times"*, in order to achieve good statistics.

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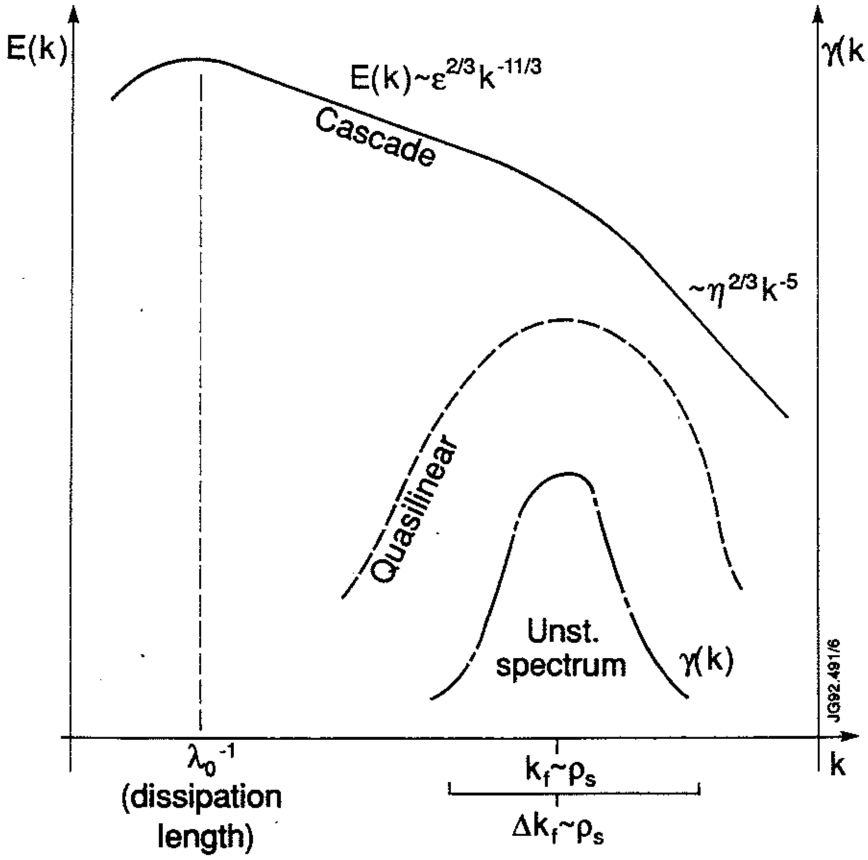


Fig. 1: Sketchy comparison between the spectrum predicted with the Kolmogorov method (cascade, continuous line) and the one expected on the basis of quasilinear theory (broken line). Also shown is the shape of the spectrum of the unstable modes (dashed line).

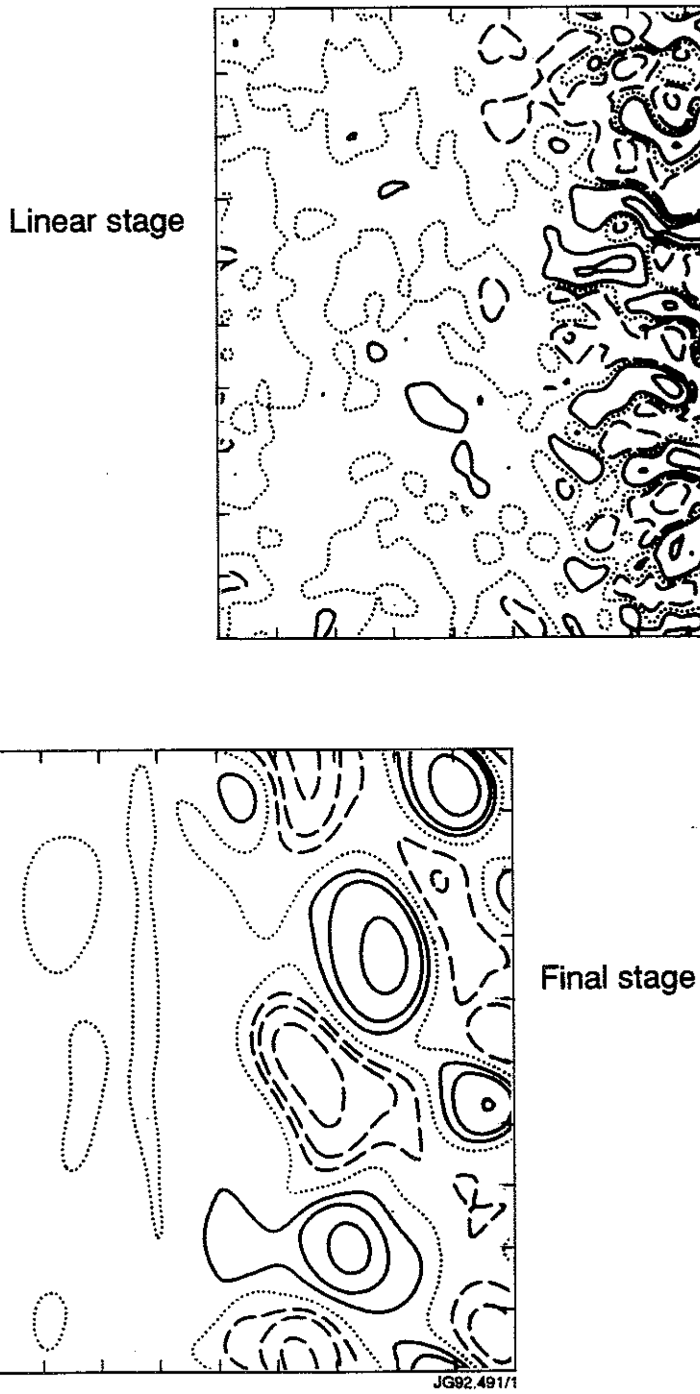
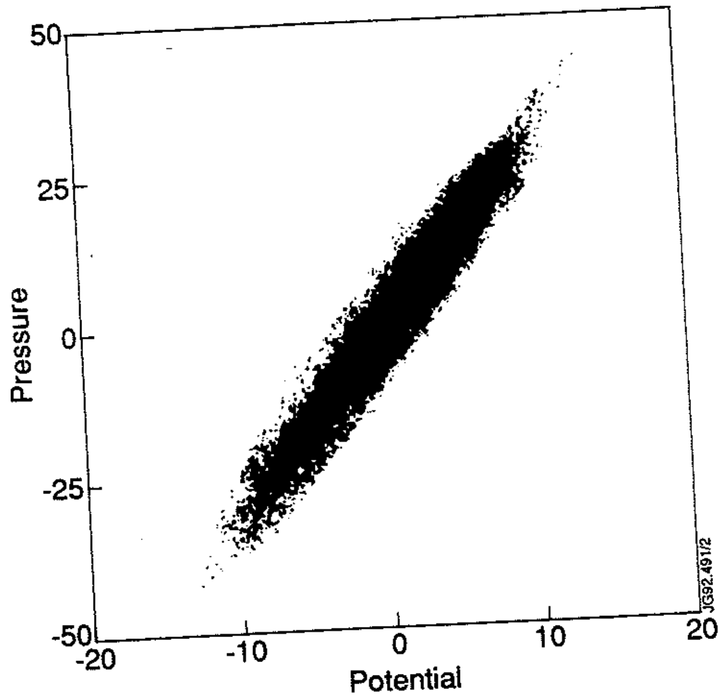
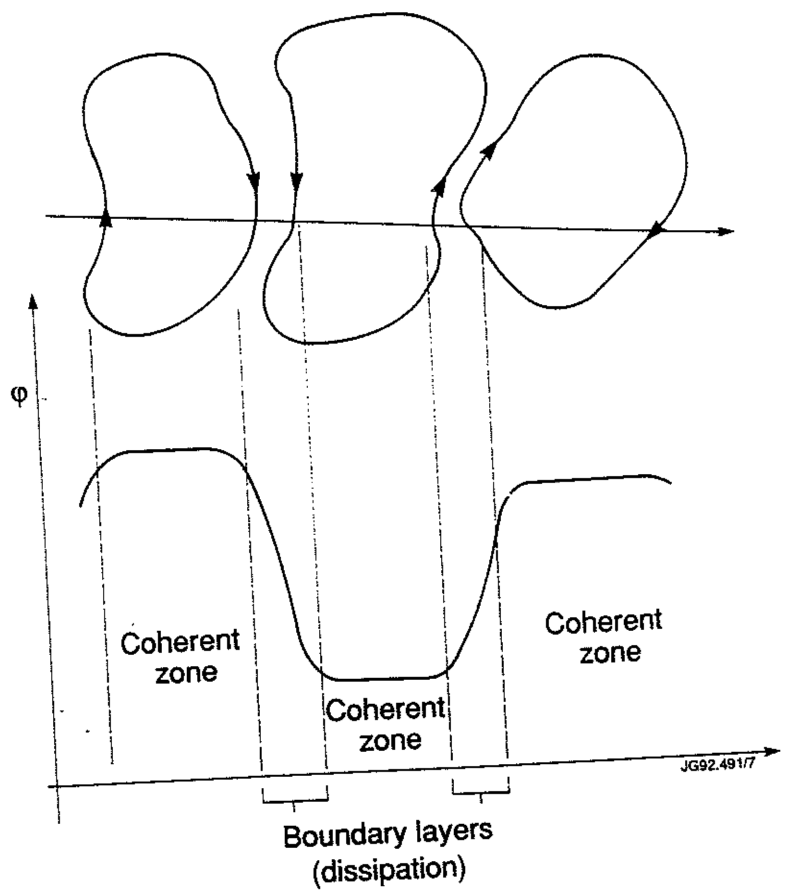


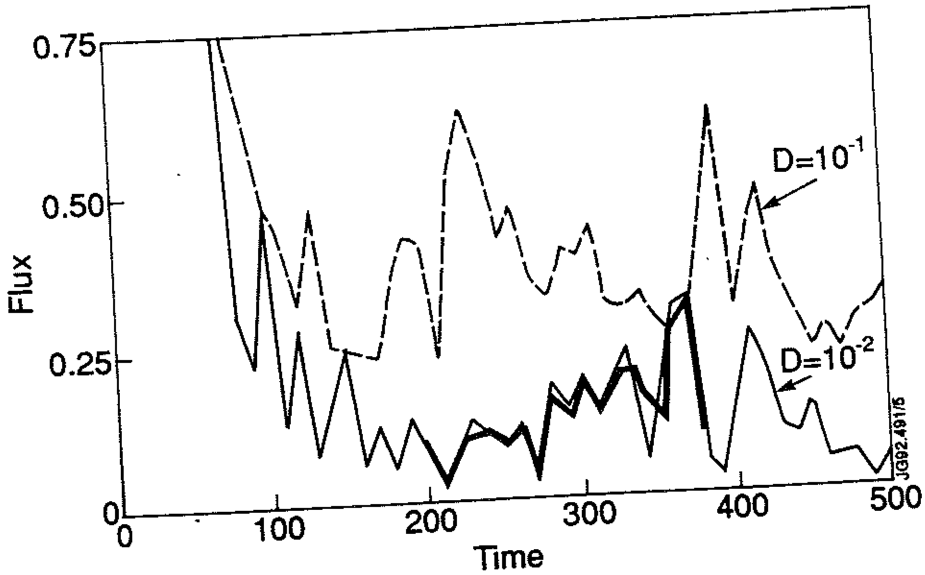
Fig. 2: Contour plots of potential fluctuations at two different stages of the simulation; **upper plot**: linear stage; **lower plot**: final saturated state (From Ref. [12]).



: Plot of potential vs pressure showing the approximate functional dependence (From Ref. [12]).



g. 4: Sketch of the potential profile across the coherent structures showing the boundary layer with steep gradients.



'ig. 5: Turbulent heat flux F for different values of the dissipation parameter $D = \nu_{ii}/\omega^{\star}$. Quasilinear theory predicts $F \approx 2$. (From Ref. [12]).

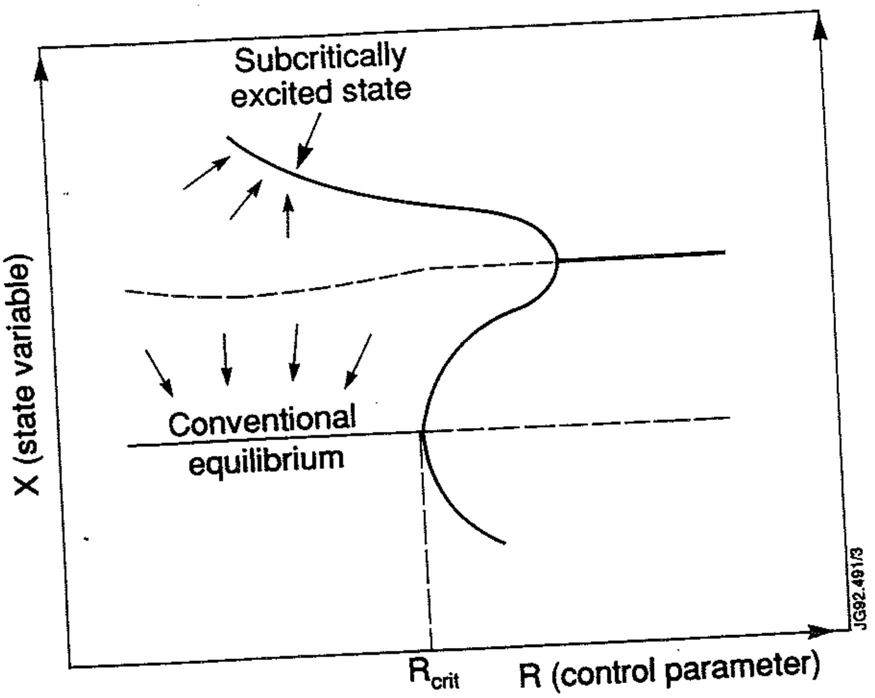


Fig. 6: Picture of a subcritical bifurcation. **Continuous line:** stable bifurcating equilibria; **broken line:** unstable equilibria.

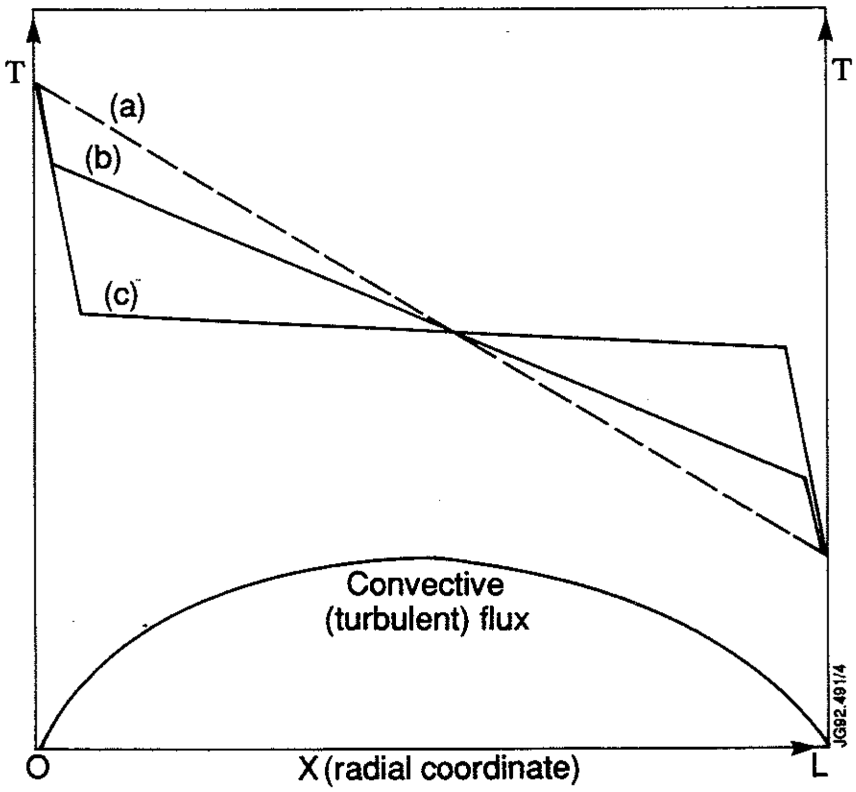


Fig. 7: Sketch of the formation of thermal boundary layers. **(a)** (dashed line): reference constant-gradient profile. **(b)**: a profile almost unaffected by the boundary layer. **(c)**: a profile with a flattened inner gradienty layer; the boundary layer accounts for most of the temperature drop across the slab.

Appendix I

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