JOINT EUROPEAN TORUS JET

JET–P(92)38

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** See Annex*

Preprint of Paper to be submitted for publication in
Europhysics Letters

ABSTRACT.
Test particle transport in two dimensional turbulence over spatial scales much longer than the correlation length has been studied numerically, addressing the dependence of the diffusion features on a convenient dimensionless parameter, the Kubo number. A scaling argument is introduced to discuss the difference between the numerical results and the predictions of current analytic theories.

The study of the motion of tracer particles (test particles) in two-dimensional turbulence is of immediate interest in various branches of applied physics, such as the physics of magnetically confined plasmas, where impurity ions naturally occur, and geophysical fluid dynamics.

So far, most of the analytical as well as numerical investigation has been devoted to the test particle behavior in power-law spectra such as those occuring in the inertial ranges[1-3]. The aim of this class of studies has been to analyze the statistics of particle motion over distances not greater than the correlation length of the background turbulence. The reason of limiting the study to the inertial range lies in the fact that the scale of energy injection of many turbulent systems is of the same order of the system size. However, the correlation length of background turbulence occuring in some important applications, as magnetic plasma confinement, is much smaller than the system size.

Previous numerical work addressing this "large distance regime" employed a model velocity field constructed with very few Fourier components[4-6]. The results of those works are considered rather special, because the spatial structure is not chaotic. The aim of this letter is to present the results of extensive numerical simulations, employing a model of the velocity field which is random in both space and time. These results will be expressed in terms of just one convenient dimensionless control parameter, the Kubo number $K = v\tau_c/\lambda_c$[7-8], which is formed with the size (the r.m.s. value) of the velocity field v and with the correlation length λ_c and the correlation time τ_c as suitable spatial and time scales. In the following, units such that $\lambda_c \sim 1$ and $v \sim 1$ will be used. Then K is a dimensionless measure of the correlation time. The numerical results and the related discussion will mainly concern the statistics of the particle displacements, and especially the average squared displacement $\langle r^2(\tau, K)\rangle$, as a function of K and of the elapsed time τ. Here the average is taken over the ensemble of particles and over the particle history as well.

Whereas little can be said a priori on the functional form of $\langle r^2(\tau, K)\rangle$ for small values of τ, for long enough $\tau \gg \tau_c$, a general argument based on the central limit theorem[9-10] leads to the conclusion that, since the advecting field changes beyond recognition after a correlation time $\tau \sim \tau_c$, ordinary diffusion sets in:

$$r^2(\tau) \approx D(K)\tau \qquad \tau \gg \tau_c \ .$$

Various analytical approximation to the diffusivity $D(K)$ are available. For $K \ll 1$, quasilinear theory gives the proper scaling. In this regime, a given particle experiences a completely changed velocity field after a time of order of the Eulerian correlation time τ_c, before having the time to travel across a characteristic length λ_c. Then the particle correlation time τ_p (Lagrangian correlation time) is of order of the Eulerian correlation time. In the chosen units, the step size of the random walk process is $\delta r \sim \tau_c \sim K$, while the clock is $\tau_p \sim \tau_c \sim K$. Then the quasilinear diffusivity is proportional to K: $D_{\text{quasilinear}} \sim K$.

In the opposite limit $K \gg 1$ the situation is more complicated. Indeed a generic frozen two dimensional incompressible velocity field ($K = \infty$), although spatially disordered, does not lead to diffusion. The reason is that in such a field particles move along the lines of constant stream function, which are closed with probability 1 in the generic case.
When a slow variation in time is introduced, as when $K \gg 1$ but not infinite, the particle motion, which is Hamiltonian in the (x, y) phase space, is still strongly constrained by the existence of adiabatic invariants. Then the actual trajectory of a given particle most of the time departs slowly from a closed trajectory of the frozen system. Quasilinear theory grossly overestimates the actual diffusivity in this limit because it assumes that the particle displacement is $\delta r \sim v\tau_c \gg \lambda_c$[11].

In general, one can define the asymptotic exponent α such that:

$$D(K) \quad \sim \quad K^{-\alpha} \quad , \quad K \gg 1 \ . \tag{1}$$

An upper bound to the actual diffusivity is obtained assuming that the Lagrangian correlation time is of order of the time required to travel across a characteristic length λ_c:

$\tau_p \approx \lambda_c/v \sim 1$. The step-size of the diffusion process is then $\delta r \approx v\tau_p \sim 1$. The resulting diffusivity tends to a constant $D \sim 1$, which is also the result from Markovian closure theory[12]. In a similar way, a lower bound is obtained assuming that the Lagrangian and Eulerian correlation times are of the same order $\tau_p \approx \tau_c \sim K$ and that, since frozen trajectories are closed and typically of size λ_c, the step-size is $\delta r \approx \lambda_c \sim 1$. Then $D \sim 1/K$, and α is constrained in the interval $0 < \alpha < 1$.

The difficulty in estimating α with simple arguments lies in the fact that, whereas most of the particles are constrained to almost closed trajectories of size a of order $a \sim \lambda_c \sim 1$ for long times of order $\tau_c \sim K$, a smaller number is allowed much longer excursions $\delta r \gg 1$ with much shorter correlation times. Over a long time, any given particle would experience long periods of small displacements (effective trapping) and shorter period of long excursions, where most of the contribution to the average diffusivity comes from. This is clearly exhibited in Fig. 1, where a typical trajectory obtained at $K = 10^2$ is shown.

Substantial progress in understanding the statistics of long trajectories has been made in Ref. [14], where an analytic argument suggests $\alpha = 3/10$. The value obtained in the present numerical simulations, $\alpha = 0.2 \pm 0.04$, is quite close to [14] but nevertheless significantly different (of about seven standard deviations). This result and related findings are discussed below.

Numerical results

Our model can be summarized as follows. The position $\vec{x}$ of a test particle in two-dimensional turbulence obeys the equation of motion

$$\frac{d\vec{x}(t)}{dt} = \vec{v}[\vec{x}(t), t] \ , \tag{2}$$

where, assuming incompressible flow, the velocity field $\vec{v}(\vec{x}, t)$ can be derived by a suitable stream function $\Phi(\vec{x}, t)$: $\vec{v}(\vec{x}, t) = \vec{\nabla} \times [\Phi(\vec{x}, t)\,\hat{z}]$. It must be noted that in magnetized plasmas the relevant velocity is the velocity of guiding centers whose effective stream function is a linear combination of the electrostatic potential $\tilde{\phi}$ and the component of the fluctuating vector potential parallel to the magnetic field $\tilde{A}_{\|}$: $\Phi = c\tilde{\phi}/B + v_{\|}\tilde{A}_{\|}/B$, where $v_{\|}$ is the velocity of the particle along the magnetic field lines. In addition, the time dependence of Φ is the one seen by a particle moving along the field line with velocity $v_{\|}$. Thus, a suitable choice of Φ in Eq. 2 allows to treat a large class of physical problems including the motion of particles in stochastic magnetic fields[15].

Here we choose to express the stream function as the sum of a large number $N_w = 64$ of standing waves:

$$\Phi(\vec{x}, t) \quad = \quad \sum_{\vec{k}} a_{\vec{k}}(t) \cos(\vec{k} \cdot \vec{x} + \alpha_{\vec{k}}) \quad ,$$

where the wavevectors $\vec{k}$ and the phases $\alpha_{\vec{k}}$ are randomly chosen with Gaussian probability distribution function of given variance $1/\lambda_c$ and uniform distribution over 2π, respectively. In order to model a spatially random field, no triangularity relations (of the type one would have in periodic domain) are enforced between wavevector triads. Therefore when $N_w \geq 3$ a snapshot of a generic stream function is described by a quasiperiodic surface. By preventing recurrency phenomena over the scale of interest, the large number of waves ensures that the resulting velocity field is effectively disordered. Temporal randomness is implemented by specifying that the amplitudes $a_{\vec{k}}(t)$ are independent, numerically generated, random functions of time with prescribed correlation time τ_c and r.m.s. amplitude A:

$$< a_{\vec{k}}(t) a_{\vec{k}'}(t') > \quad = \quad A^2 \, e^{-\frac{|t-t'|}{\tau_c}} \, \delta_{\vec{k},\vec{k}'} \quad .$$

This constitutes a difference from previous work, where time dependence is mostly prescribed as a deterministic function, usually a given oscillation[16].

Normalizations are chosen such that $v = 1$ and $\lambda_c = 1$, and $\tau_c = K$. Then, the motion of a large number $N_p = 1024$ of particles is followed by integrating Eq.2 with a simplectic algorithm. A systematic study of the diffusion features is carried out, varying the Kubo number between $K = 1. \times 10^{-2}$ and $K = 1. \times 10^{4}$.

Fig. 2 shows a typical plot of the $\langle r^2(\tau) \rangle$ at $K = 1. \times 10^4$ The diffusivity is obtained from the slope of this type of curves for large values of τ. The main result of this work is presented in Fig. 3 where the diffusivity is plotted against the Kubo number, and compared with analytic theories. As expected, quasilinear theory holds only in the limit $K \ll 1$. Closure theory does better, but it still fails to pin down the correct asymptotic exponent of the diffusivity $\alpha = 0.2 \pm 0.04$ for $K \gg 1$.

In order to assess the validity of the analytic predictions, special care has been taken in the evaluation of the error on the asymptotic exponents, which is due to the finite statistics (finite number of particles as well as sampling errors in the generation of the velocity field). The difficulty lies in the fact that the error on the value of $\langle r^2(\tau) \rangle$ is correlated with the error on the value of $\langle r^2(\tau') \rangle$. Thus, although standard error analysis can be generalized to deal with correlated errors, the very fact that the statistics of such errors is not known prevents one from a reliable estimate of the propagated error on the diffusivity if the latter is obtained by an arbitrary linear regression of the $\langle r^2(\tau) \rangle$ curve. This difficulty is overcome by estimating the diffusivity from a value of the squared displacement measured at a large enough elapsed time τ_G that the statistics can be considered Gaussian.
Then the standard deviation of the squared displacement is
$\delta r^2(\tau_G) = (\tau_G/T)^{1/2}(1/N_p)^{1/2} \langle r^2(\tau_G) \rangle$, where T is the total elapsed time (the length of the simulation).

The deviation from Gaussianity is measured by a kurtosis-like quantity
$\kappa = \langle r^4(\tau) \rangle / \langle r^2(\tau) \rangle^2 - 2$. The general behavior for $K \gg 1$ is such that κ peaks a certain time $\tau_{\mathbf{peak}} \ll K$, whereas it is already approaching zero at a much later time $\tau_G \approx K$. This behavior is exhibited in Fig. 4, where κ is plotted for a run at $K = 10^2$. As shown later, $\tau_{\mathbf{peak}}$ is af order of the correlation time of long trajectories, while a complete loss of memory of the past history occurs only after that a much longer Eulerian decorrelation time K has elapsed.

In Fig. 5 the probability distribution function (p.d.f.) of the (radial) displacements from the initial position is shown at different times. The displacement is normalized to the actual value of $\langle r^2(\tau)\rangle$. At $\tau_{\rm peak}$ the p.d.f. has a long tail, while at later times $\tau \sim K$ it approaches the normalized Gaussian distribution $P(r) = 2re^{-r^2}$.

Discussion; scaling argument

The features of particle dynamics leading to Eq.1 are clarified by introducing a scaling argument. Consider for the moment a frozen field. In the generic case, trajectories coincides with the lines of constant stream function and are closed with probability 1 in such a configuration. Then one can classify the various trajectories according to their size a, which is defined as the diameter of the smallest circle enclosing the trajectory.
For long enough trajectories, $a \gg 1$, the absence of a characteristic length suggests a power law for the probability distribution function of orbits of size between a and $a + da$: $p(a) \sim a^{-\mu}$. Moreover, though obviously differentiable on the small scale, such long orbits are effectively fractal[14] when measured with a ruler of intermediate length l such that $1 \ll l \ll a$. Then the trajectory length scales as $L(a) \sim a^d$, where $d < 2$ is the fractal dimension. The time necessary to complete a given orbit is also $\tau_{\rm comp} \sim a^d$. On a shorter timescale, the particle displacement from the starting point scales like $\delta r \sim t^{1/d}$.

When the velocity field is allowed to fluctuate on a long time scale $K \gg 1$, the state of a particle at a given time can still be labelled according to the size of the orbit the particle would follow if the field where frozen at that time. However, fluctuations allow transitions between orbits of different size. For long trajectories, the orbit lifetime, which is the typical time a given particle spends on an orbit of size a, is assumed to scale like $\tau_p \sim Ka^{-1/\nu}$.

The various regimes of particle motion are sketched in Fig. 6. Of special importance are the crossover times and scales $\tau_M \sim K^{-d/(d+1/\nu)}$ and $\lambda_M \sim K^{-1/(d+1/\nu)}$. Particles on small size orbits $a < \lambda_M$ are able to complete their orbit before decorrelation intervenes. Larger orbits are decorrelated before their completion time.

For long enough times, $\tau > K$, the diffusivity can be estimated by summing over the contributions of the various trajectories of different sizes. For $a < \lambda_M$, the step size of the diffusive process is just the orbit size, $\delta r(a) \sim a$, while the clock of the process is the orbit lifetime $\delta t(a) \sim K a^{-1/\nu}$. Then the contribution to the diffusivity from orbits of size a is $D(a) \sim \delta r^2(a)/\delta t(a) \sim K^{-1} a^{2+1/\nu}$.
Similarly, when $a > \lambda_M$, $\delta r(a) \sim \tau_p^{1/d}$ and $\delta t(a) \sim \tau_p$ yield $D(a) \sim K^{2/d-1} a^{-(2/d-1)/\nu}$. Then the dominant contribution to the diffusivity comes from orbits of size λ_M if the probability distribution function of orbit sizes is not too steep: $D(a) \sim (\lambda_M^2/\tau_M) p(\lambda_M) \lambda_M$. This leads to the scaling exponent of the diffusivity:

$$\alpha = \frac{\mu + d - 3}{d + 1/\nu} \quad . \tag{3}$$

For times smaller than K, nondiffusive contributions come from particles living on short orbits with long decorrelation times: $a < (\tau/K)^{-\nu}$. One can recognize that, when $\tau_M < \tau < K$, the dominant contribution to the squared displacement is still linear in τ with scaling exponent of the diffusivity given in Eq.(3).
However, for even shorter times $1 < \tau < \tau_M$, the dominant contribution is nondiffusive and comes from a fraction $p(a)a$ of particles with $a \sim \tau^{1/d}$. This gives:

$$\langle r^2(\tau) \rangle \sim \tau^\gamma \quad , \quad \gamma = \frac{3-\mu}{d} \quad , \quad 1 < \tau < \tau_M \quad .$$

The anomalous portion of the displacement law is apparent in Fig. 2b.

Similarly one can compute the contributions to the Kurtosis κ. Since the fourth moment of the displacement grows initially like $\tau^{\gamma'}$, with $\gamma' = (5-\mu)/d$, κ grows like τ^σ , $\sigma = (\mu - 1)/d$ until $\tau_{\text{peak}} \sim \tau_M$ when it reaches its peak value $\kappa_{\text{peak}} \sim K^\delta$, with $\delta = (\mu - 1)/(d + 1/\nu)$. Then it decreases like $\tau^{-\sigma'}$ with exponent $\sigma' = \nu(\mu - 1)$.

To summarize, four regimes of the displacement law are found for $K \gg 1$: a ballistic regime for $\tau < 1$, a subdiffusive regime for $1 < \tau < \tau_M$, a Fick-type regime with non-Gaussian statistics for $\tau_M < \tau < K$, and a proper diffusive regime with Gaussian statistics for $\tau > K$.

Ref. [14] employes results from percolation theory[17] to obtain $d = 7/4$, $\mu = 2$ and $\nu = 4/3$. In order to assess the validity of these results, d and μ are evaluated numerically by following the particle motion in a frozen velocity field ($K = \infty$). The distribution function of the orbit size is shown in Fig. 7. The measured exponent $\mu = 1.97$ is obtained by fitting the data of the distribution function of the orbit size with $a > 10$. Similarly the fractal dimension $d = 1.88$ is estimated from the longest trajectories in the sample. These results for the "static" exponents are in good agreement with Ref. [14] and one can attribute the difference to statistical errors.

Although the discrepancy in the value of α between Ref. [14] and our numerical results could be attributed to (subdominant) corrections to the asymptotic scaling (1), there is a body of evidence that the difference must be traced to the estimate of the "dynamic" exponent ν of orbits lifetimes. Indeed a value of ν around 0.5 is needed to match the observed scaling law of the diffusivity. Unfortunately, there is no easy way to measure ν directly. An indirect estimate must necessarily rely on one of the above scaling relations. A possibility would be to use the exponent of the Kurtosis peaking time, which is a measure of the crossover time τ_M, as given in Fig. 8. In Tab.1 the equivalent choice is made to take the numerical value of α as reference and to assume the values of d and μ from Ref. [14] as exact this yields $\nu = 1/2$. The comparison between the "expected" exponents obtained in this way and those obtained from Ref. [14] on the basis of the scaling argument shows that the first choice gives a better agreement with numerical findings for all the measured exponents.

As a final remark we note that the result of closure theory ($\alpha = 0$) is recovered from Eq. 3 in the limit $\nu \to 0$. One can then observe from Fig. 6 that the Lagrangian decorrelation time becomes $\tau_p \sim 1$. In this approximation, the diffusion process is a random walk with unit step-size and correlation time. The statistics of the displacement becomes Gaussian already at $\tau \sim 1$ and the intermediate power law range of the displacement law is not obtained.

Acknowledgements

The author would like to thank Dr. M. Isichenko for enlightening discussions and prof. D. Düchs, Dr. M. Hugon, Dr. P. H. Rebut and Dr. W. Kerner for useful comments.

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[13] More details on this and other points will be given in a longer paper (M. Ottaviani, in preparation).

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Source	Exponent	(a)	(b)	(c)
diffusivity $D \sim K^{-\alpha}$	$\alpha = (d + \mu - 3)/(d + 1/\nu)$	0.2	0.2	0.3
$\tau_{\text{peak}} \sim \tau_M \sim K^{\beta}$	$\beta = d/(d + 1/\nu)$	0.54	0.47	0.70
$\langle r^2(\tau) \rangle \sim \tau^{\gamma}$, $\tau < \tau_M$	$\gamma = (3 - \mu)/d$	0.74	0.57	0.57
$\langle r^4(\tau) \rangle \sim \tau^{\gamma'}$, $\tau < \tau_M$	$\gamma' = (5 - \mu)/d$	1.85	1.71	1.71
kurtosis $\kappa \sim \tau^{\sigma}$, $\tau < \tau_M$	$\sigma = (\mu - 1)/d$	0.36	0.57	0.57
kurtosis $\kappa \sim \tau^{-\sigma'}$, $\tau > \tau_M$	$\sigma' = \nu(\mu - 1)$	0.54	0.50	1.33
kurtosis peak $\kappa_{\text{peak}} \sim K^{\delta}$	$\delta = (\mu - 1)/(d + 1/\nu)$	0.23	0.27	0.40

Tab. 1 Summary of the scaling exponents. Numerical results (a). Scaling argument employing $\mu = 2$, $d = 1.75$, $\nu = 0.5$ (b). Same with $\nu = 4/3$ as given in Ref. [14] (c).

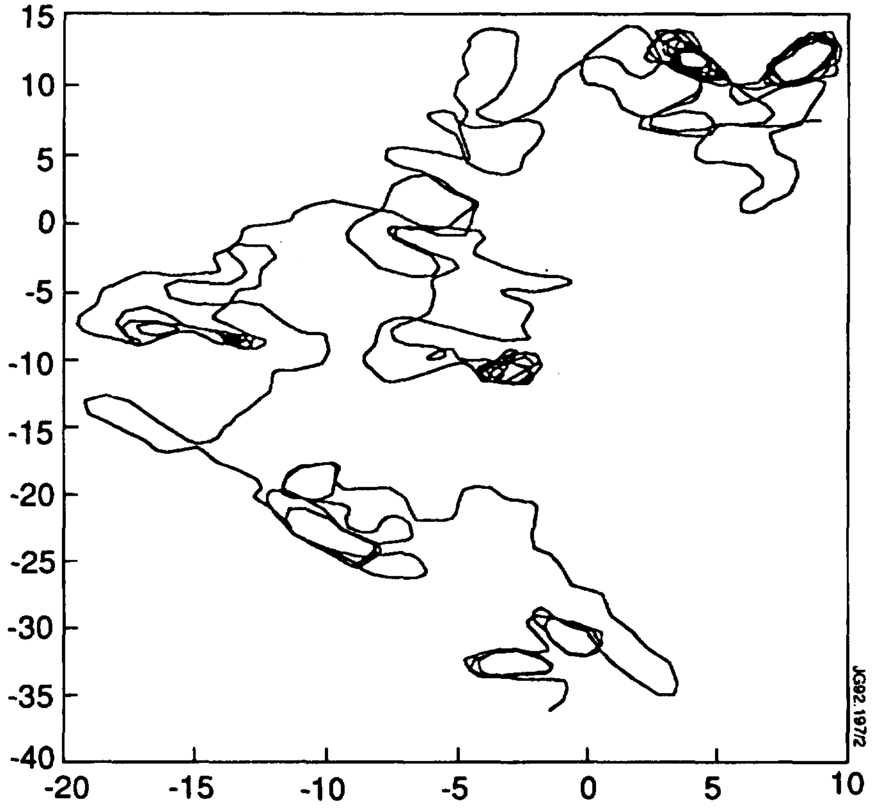


Fig. 1 Example of particle trajectory at high Kubo number $K = 10^2$.

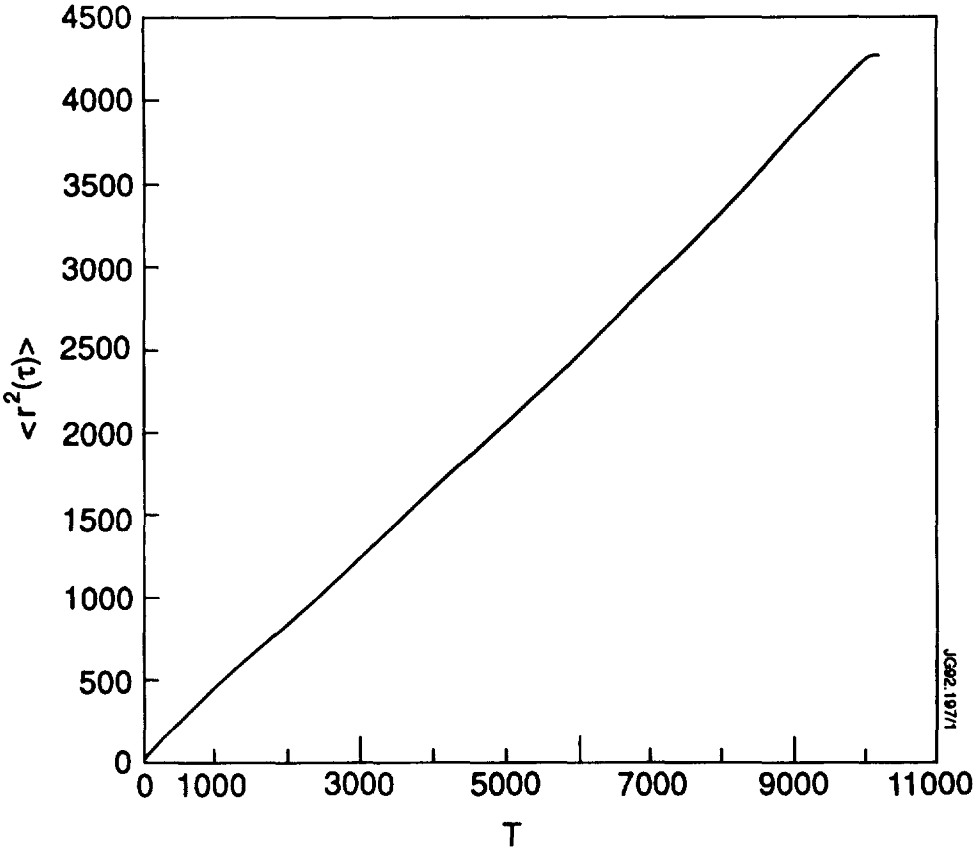


Fig. 2a Squared displacement vs τ for a run at $K = 10^4$, linear scale. The diffusivity is obtained from the slope of this curve at large enough τ.

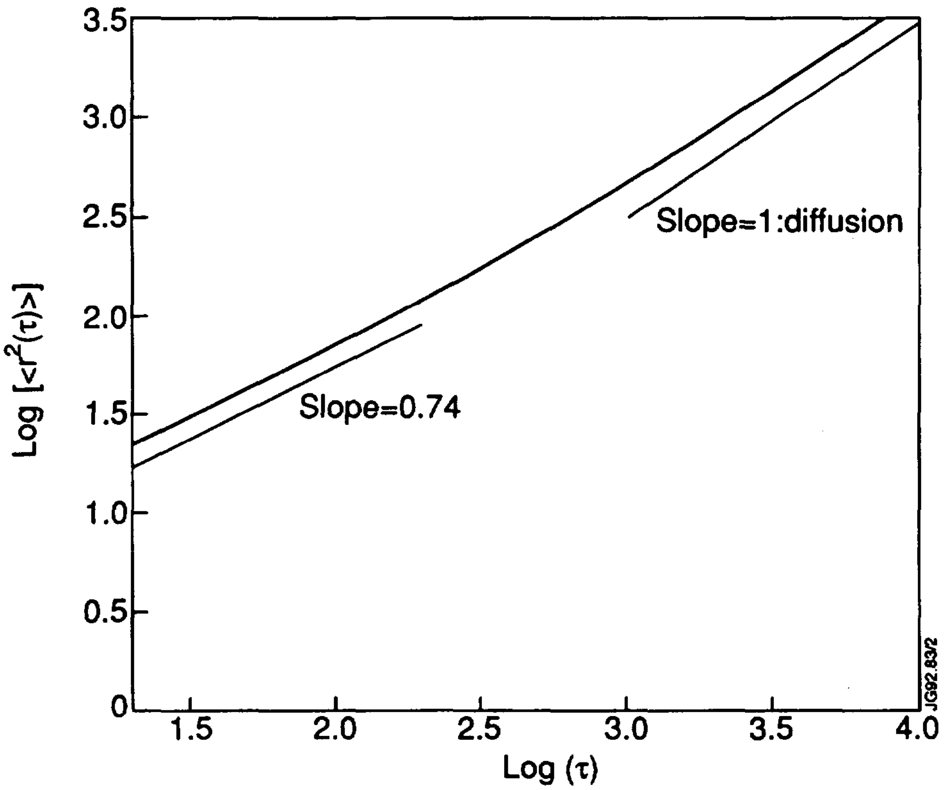


Fig. 2b Squared displacement vs τ for a run at $K = 10^4$, logarithmic scale. The short time subdiffusive regime is evidentiated.

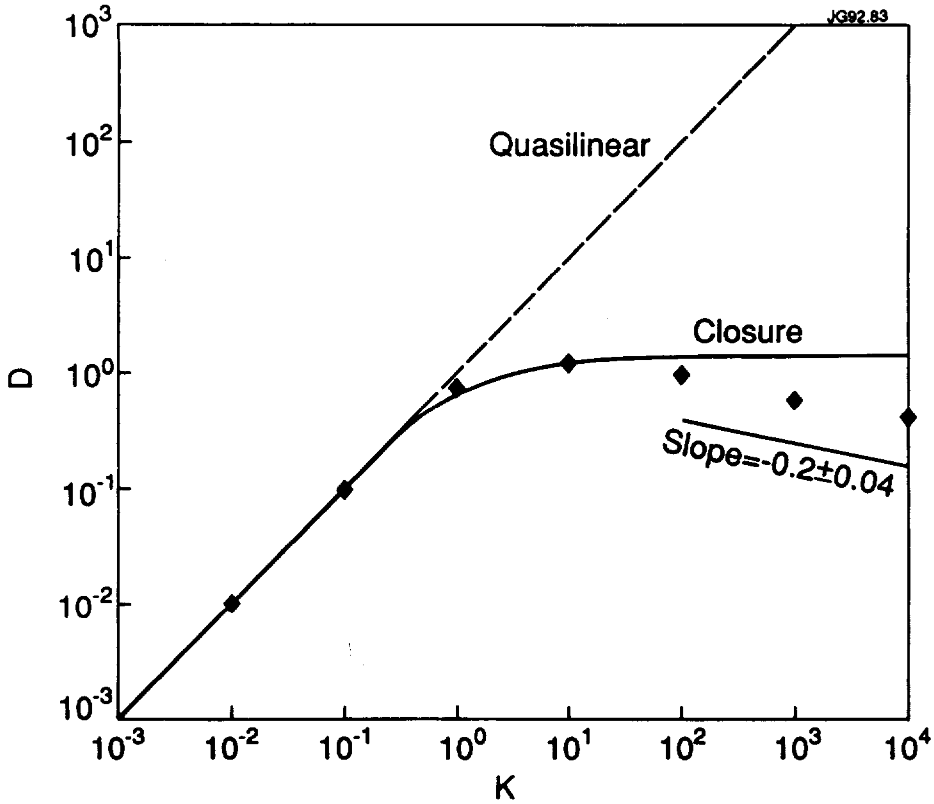


Fig. 3 Diffusivity as a function of the Kubo number. Marks: numerical result. Solid line: closure theory. Dashed line: quasilinear theory.

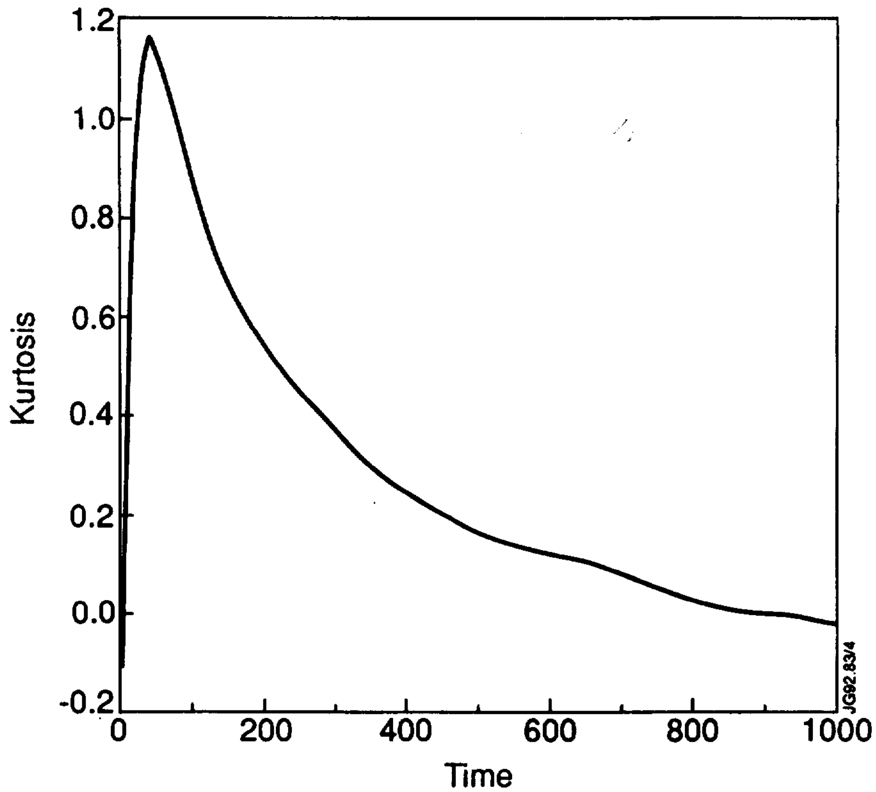


Fig. 4 Kurtosis vs time. $K = 10^2$.

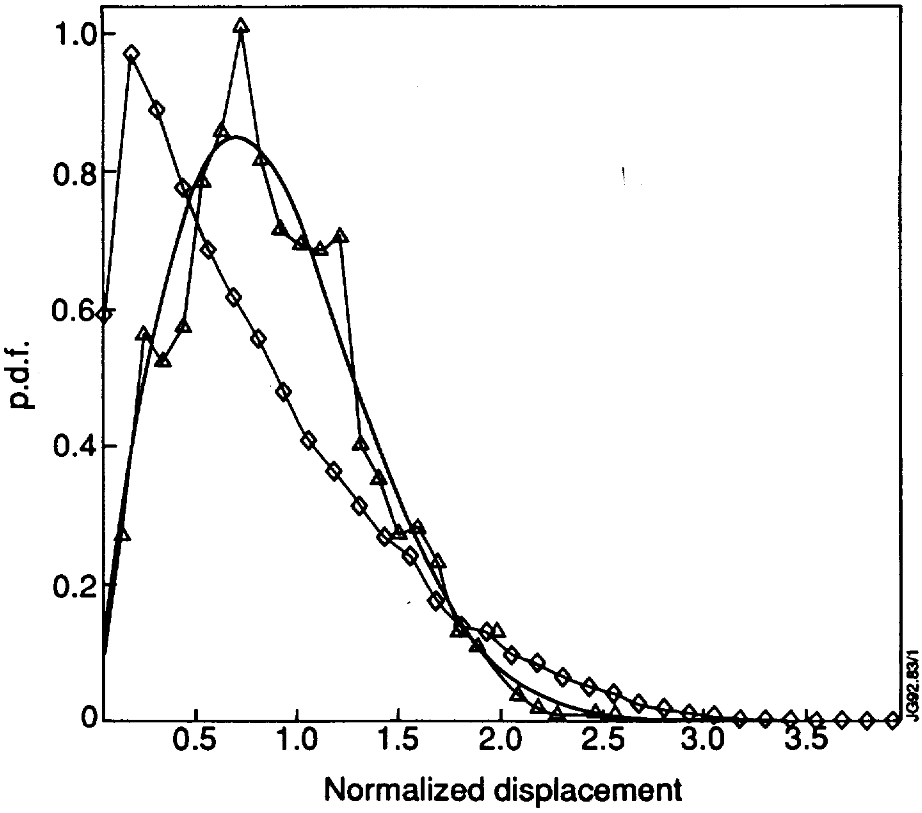


Fig. 5 Normalized p.d.f. of the displacements at the kurtosis peak $\tau = \tau_{\mathbf{peak}} = 32.$ ($\Diamond$) and in the Gaussian regime $\tau = 1024.$ ($\triangle$). Solid line: Gaussian p.d.f.

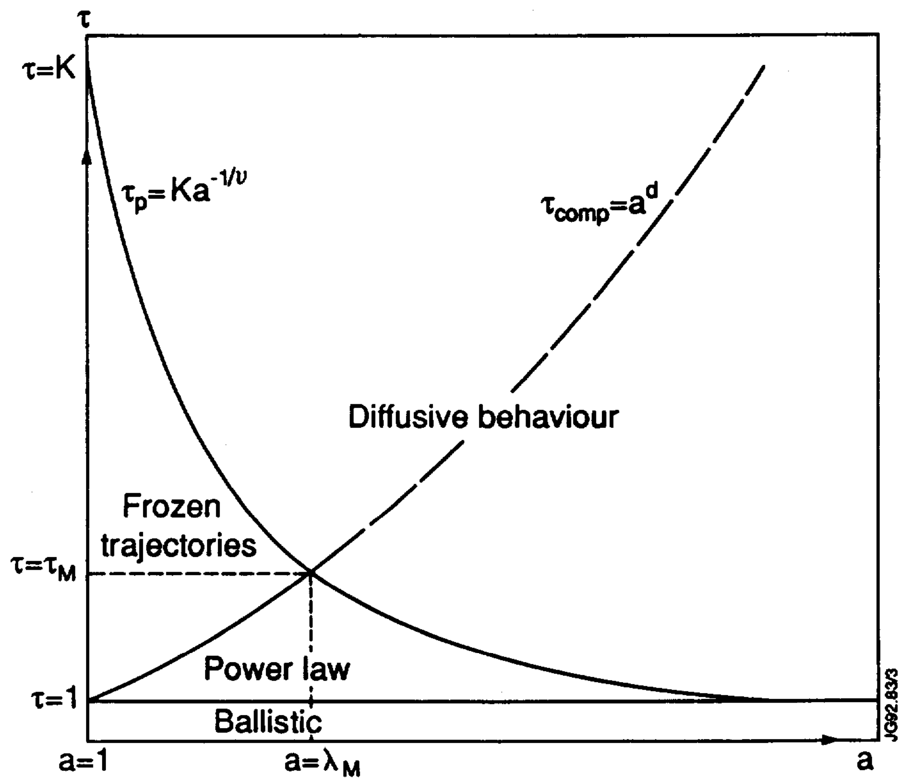


Fig. 6 Different regimes of particle motion as a function of the orbit size a and of the elapsed time τ.

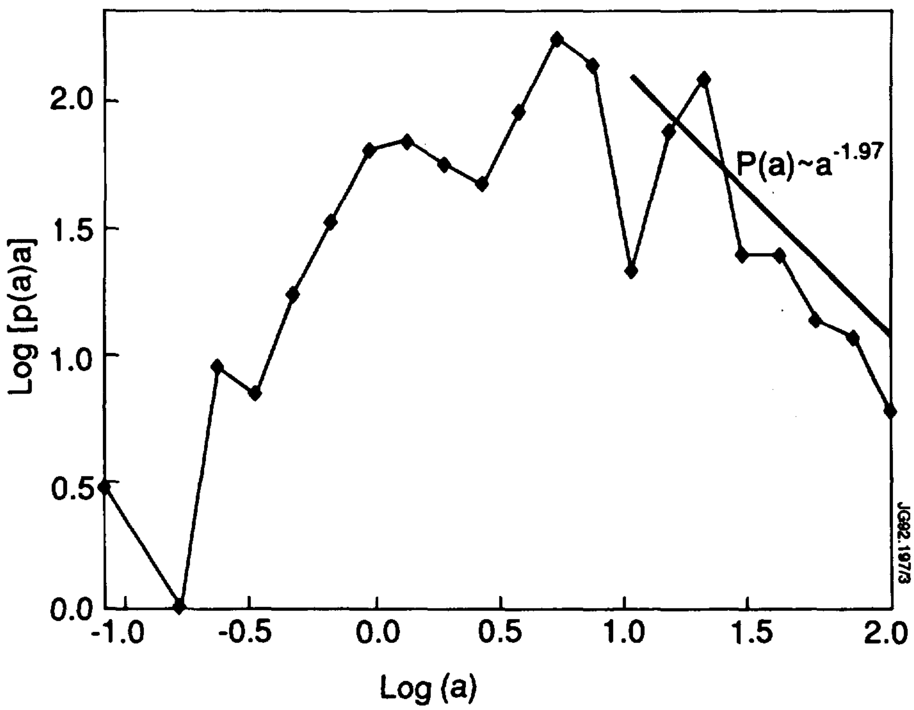


Fig. 7 Probability distribution function of the orbit size.

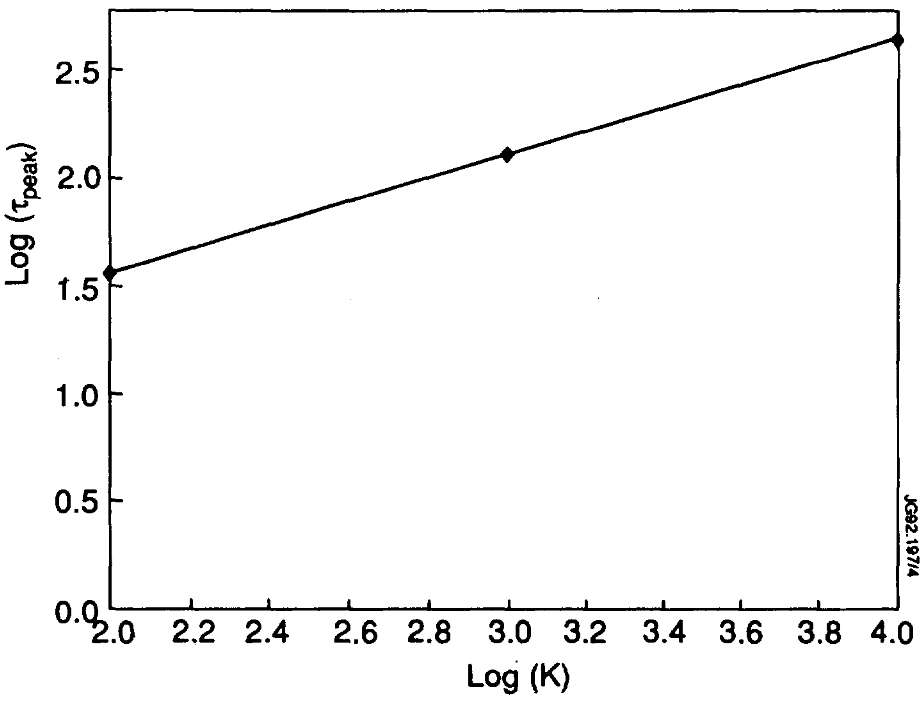


Fig. 8 Peaking time of the kurtosis as a function of K.

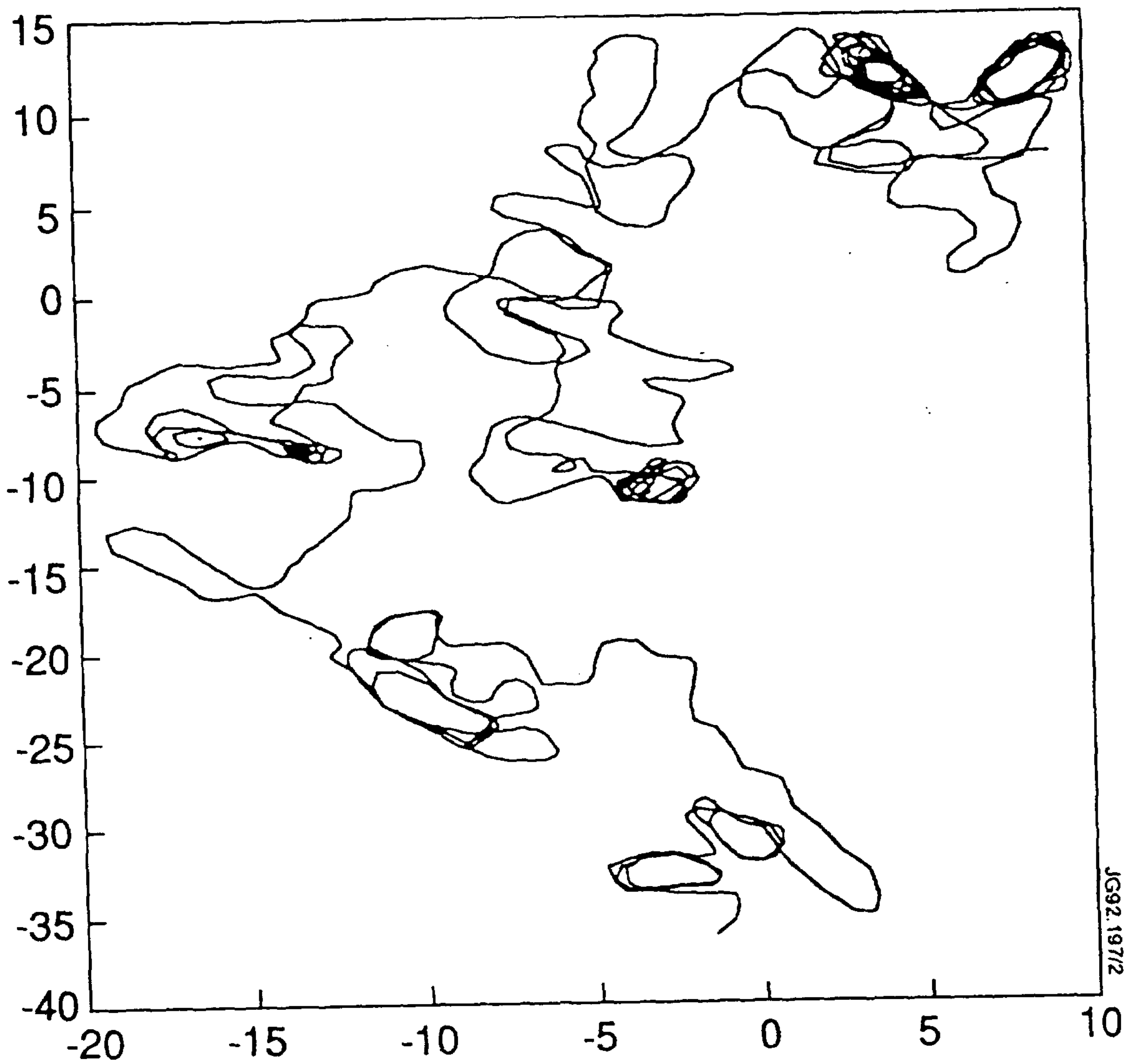


Fig. 1

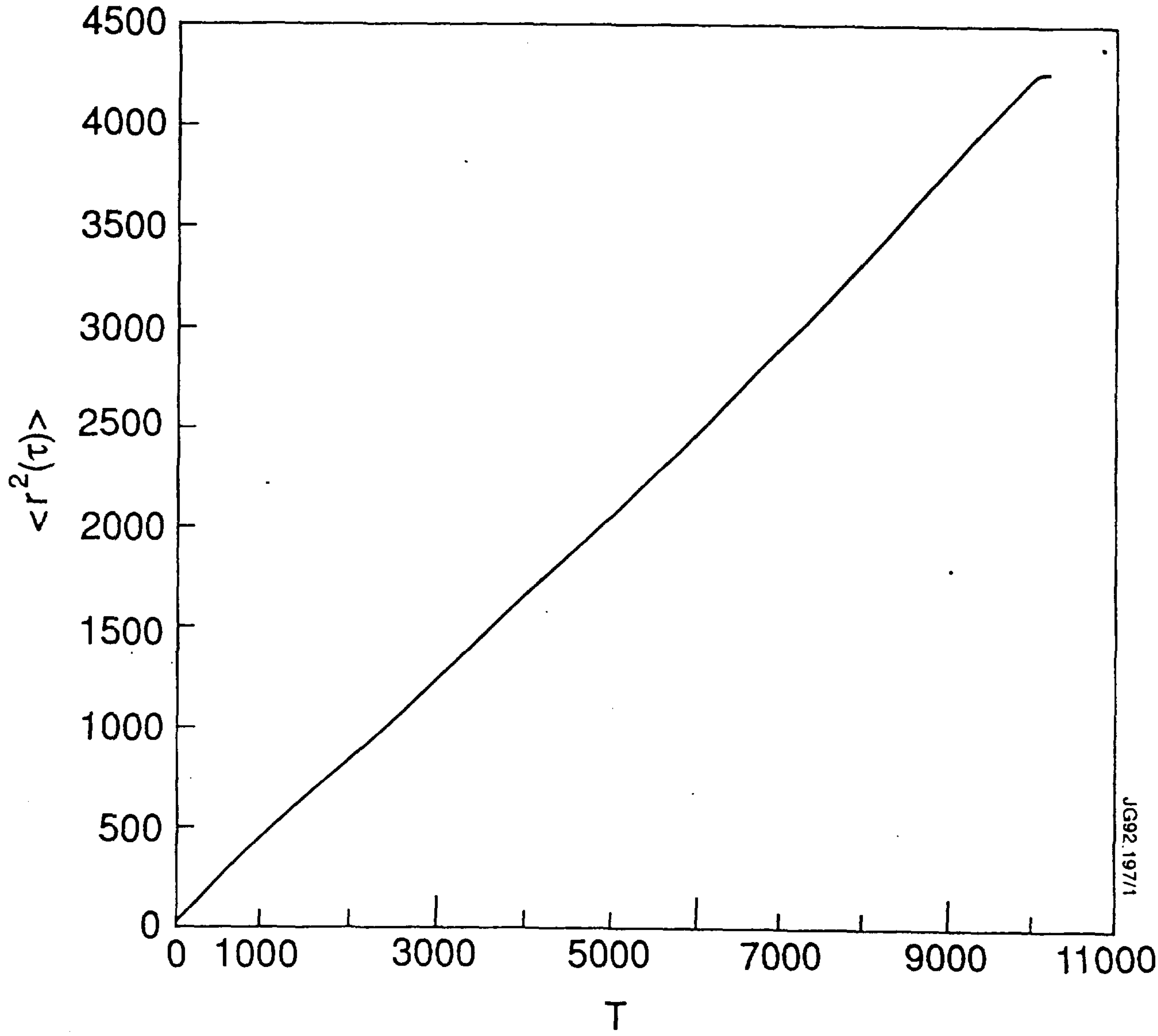

4500
4000
3500
3000
2500
2000
1500
1000
500
0
$<r^2(\tau)>$
0
1000
3000
5000
7000
9000
11000
T
JG92.197/1

Fig 2a

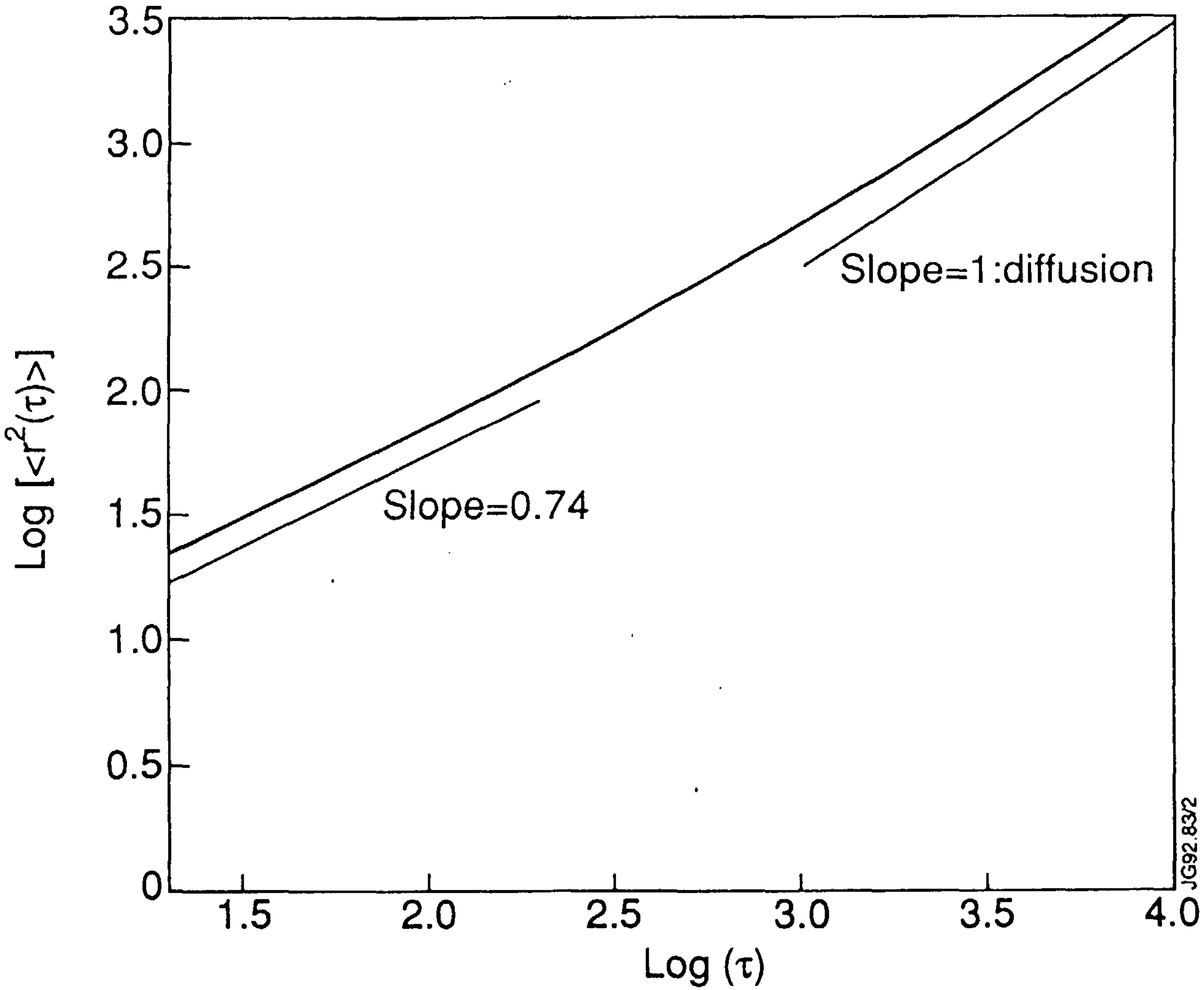

Log [<r²(τ)>]
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
1.5
2.0
2.5
3.0
3.5
4.0
Log (τ)
Slope=0.74
Slope=1:diffusion
JG92.83/2

Fig 24

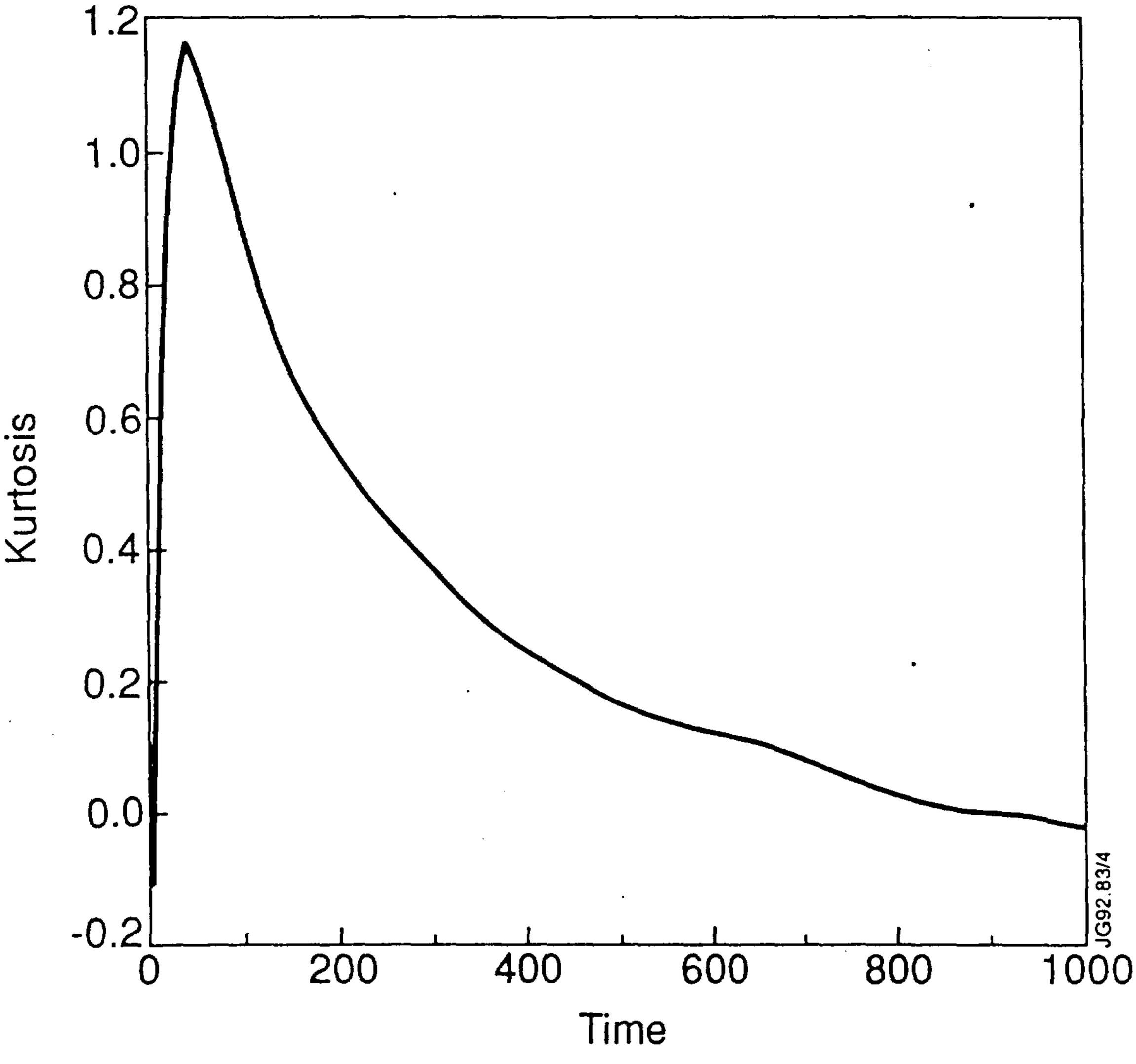
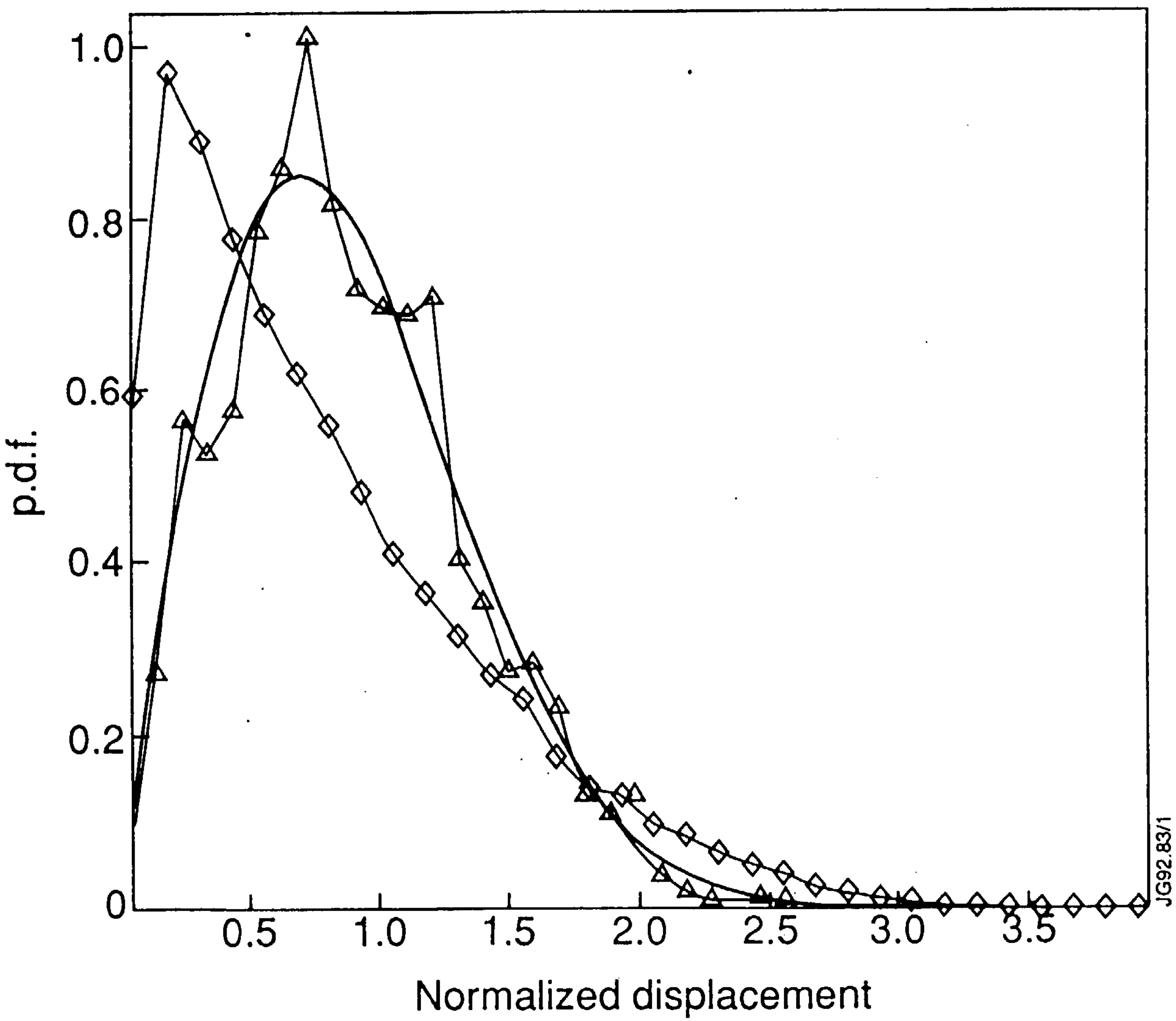
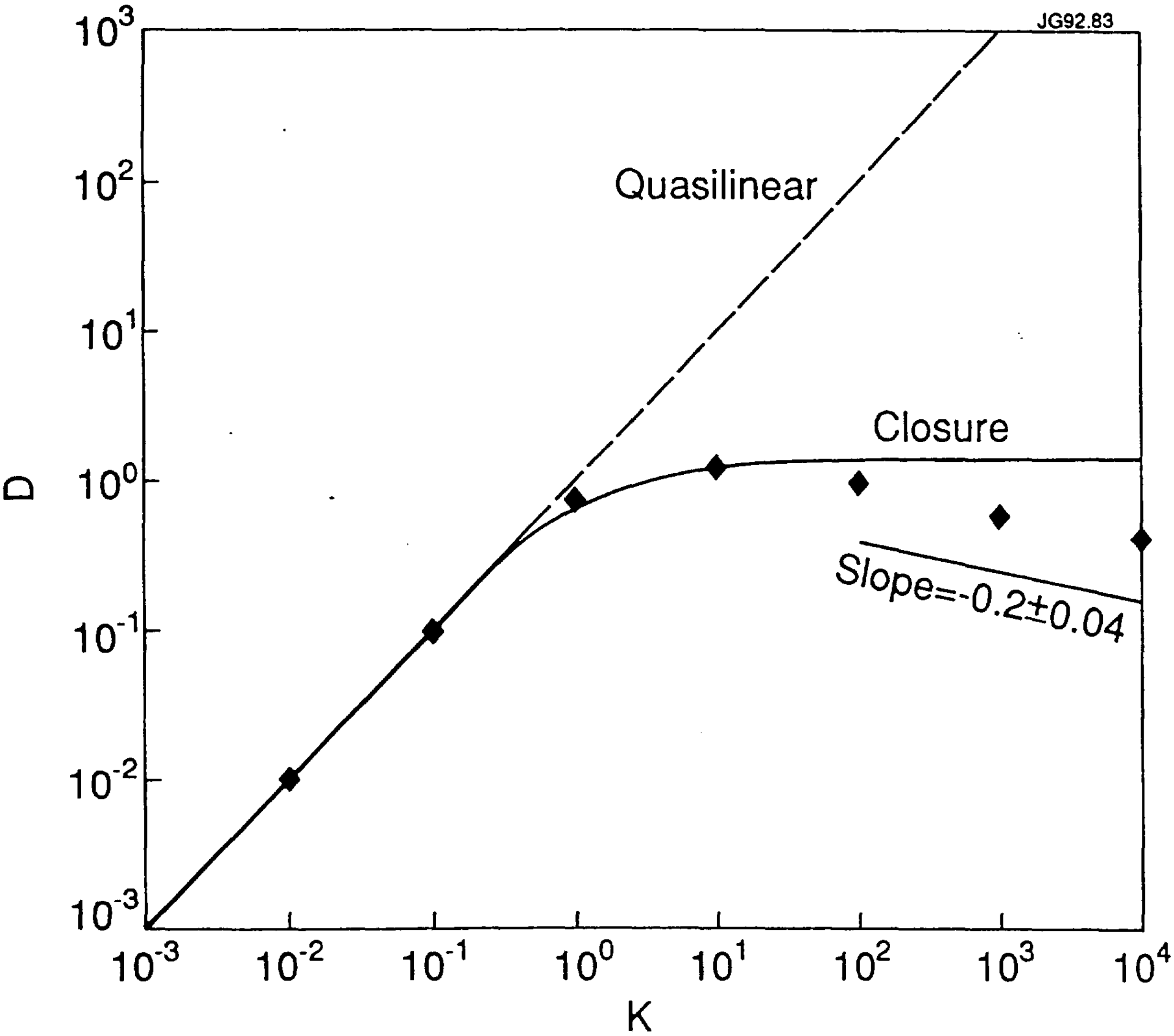

1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
Kurtosis
0
200
400
600
800
1000
Time
JG92.83/4

Fig. 3



p.d.f.
1.0
0.8
0.6
0.4
0.2
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
Normalized displacement
JG92.83/1

Fig. 4



JG92.83
Quasilinear
Closure
Slope=-0.2±0.04
D
K
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-3} 10^{-2} 10^{-1} 10^0 10^1 10^2 10^3 10^4

Fig. 5

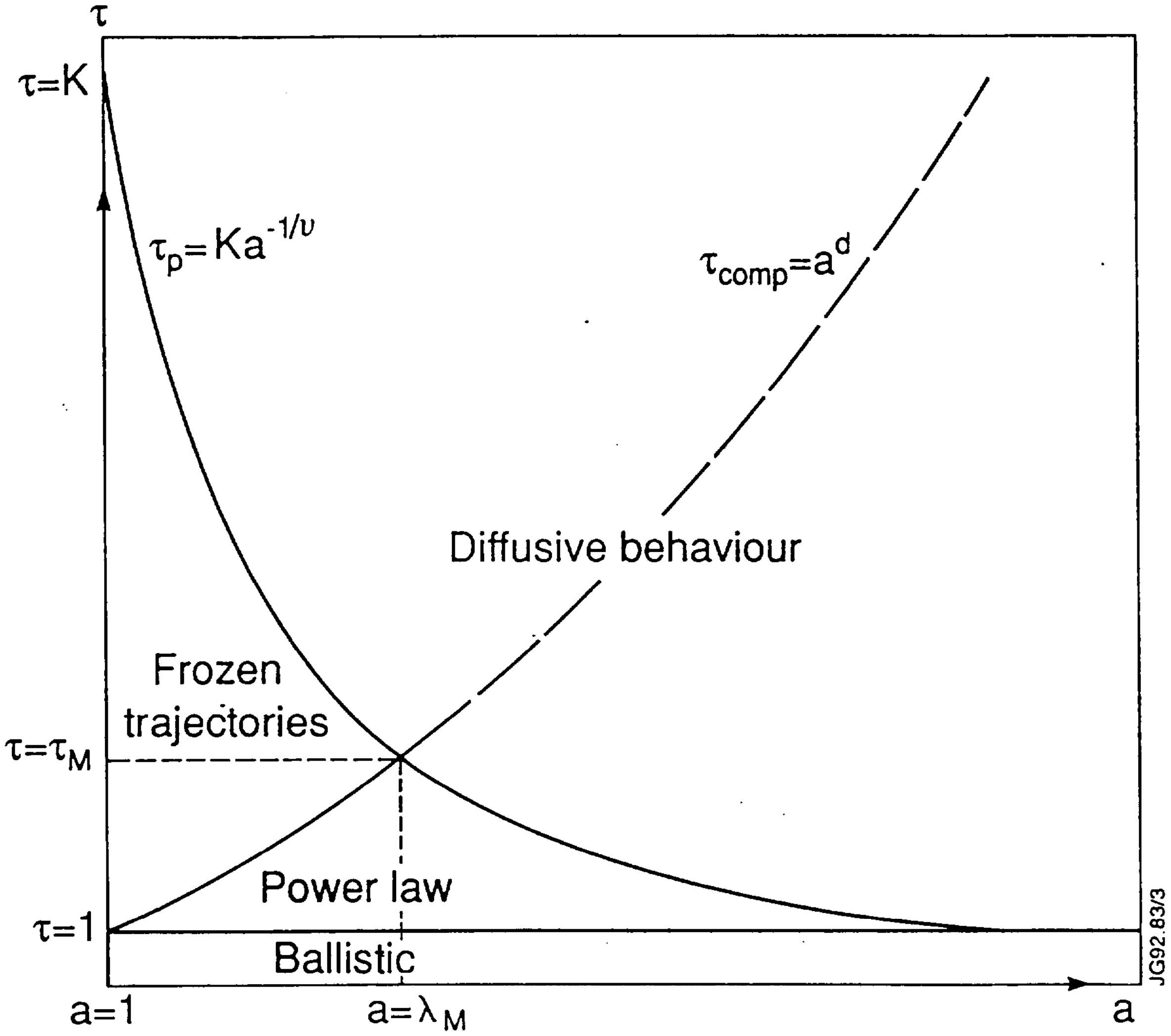

τ
τ=K
τp=Ka-1/υ
τcomp=ad
Diffusive behaviour
Frozen
trajectories
τ=τM
Power law
τ=1
Ballistic
a=1
a=λM
a
JG92.83/3

Fig 6

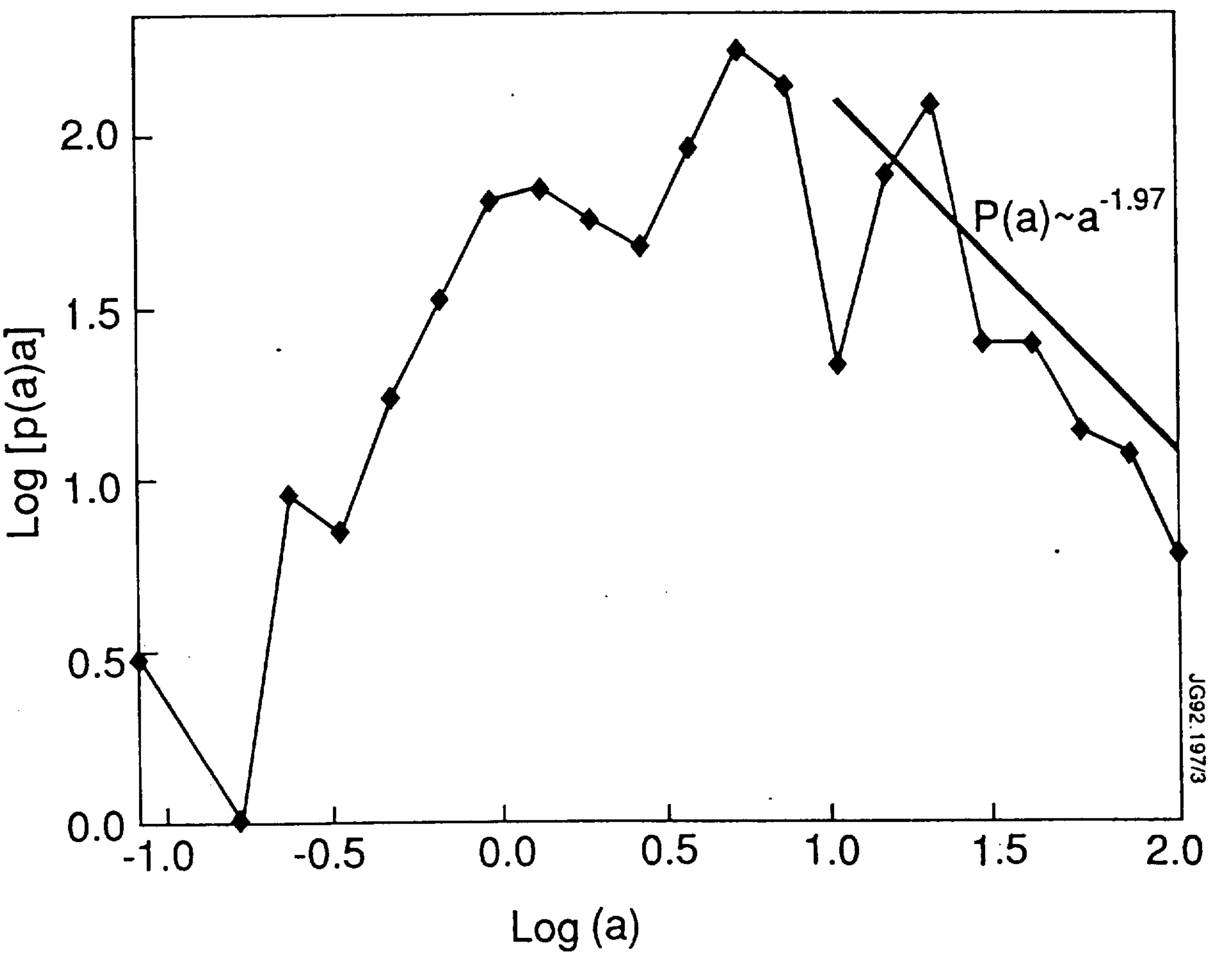


Fig. 7

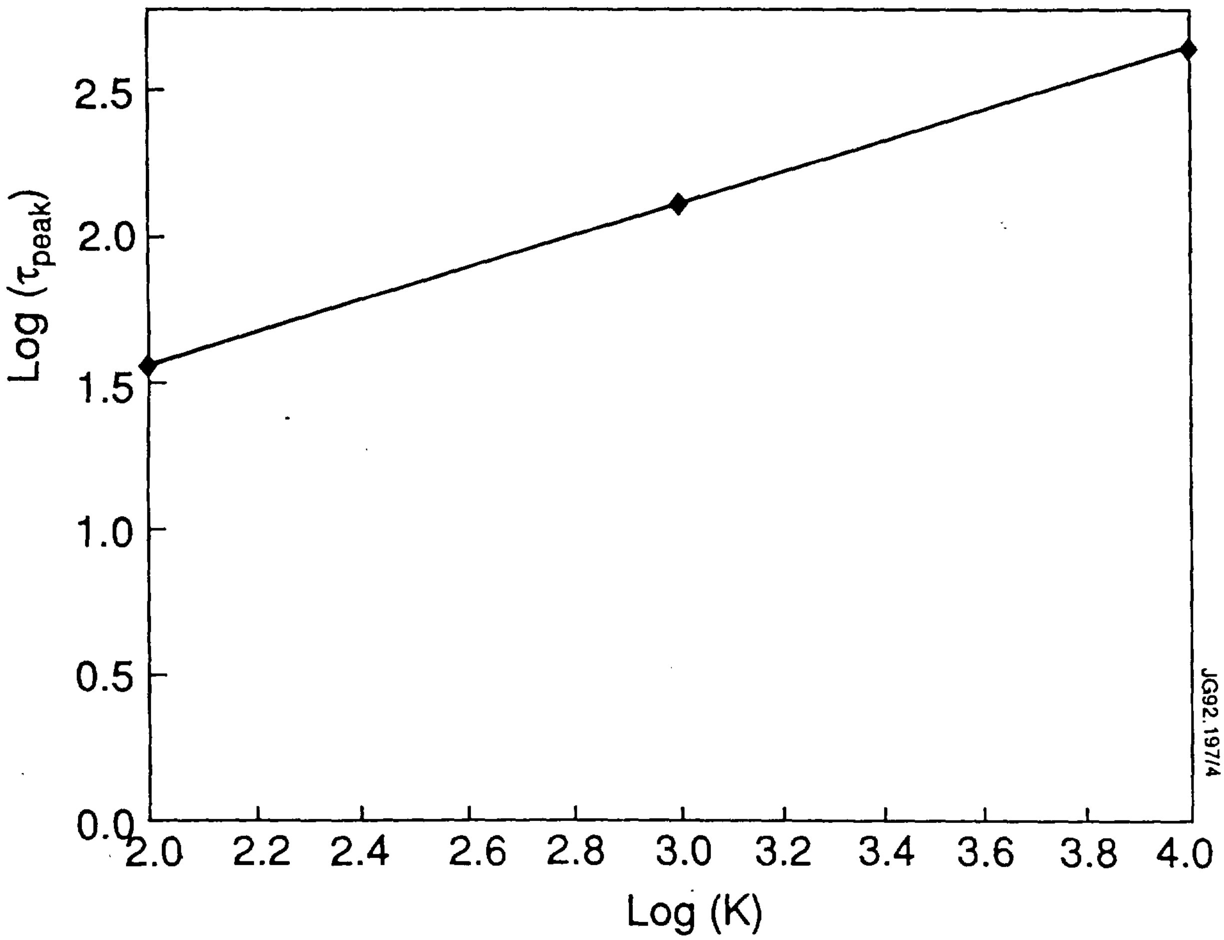

Log (τ_{peak})
2.5
2.0
1.5
1.0
0.5
0.0
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
3.8
4.0
Log (K)
JG92.197/4

Fig. 8

ANNEX

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