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Neoclassical Transport in the Presence of Fluctuations

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Abstract

The usual argument for automatic ambipolarity of neoclassical particle fluxes in a tokamak is based on the flux surface averaged toroidal momentum equation. It does not apply when fluctuations are present, because they also contribute to the momentum. In the case of electrostatic fluctuations, their contribution comes from the FLR pressure tensor. The same pressure tensor gives rise to a difference in the drift velocities of ions and electrons, and hence to non-ambipolar anomalous transport. The mean $(\underset{\sim}{j}\times\underline{B})_\varphi$ force introduced by magnetic fluctuations can be much larger, as in the associated non-ambipolar particle flux. Any non-ambipolarity in the anomalous transport affects the ambipolar electric field. Because of their high Z, impurity neoclassical transport is most strongly affected. This can explain the pump-out of impurities by MHD activity.

1. INTRODUCTION

The proof that neoclassical particle transport is automatically ambipolar can be based on either the toroidal or the parallel component of the momentum equation [1-6]. In the absence of fluctuations the toroidal momentum can be averaged over a flux surface in such a way that the divergence of the pressure tensor vanishes. This gives a relation between the mean radial flux and the collisional friction between species. Because collisions conserve momentum, the collisional friction vanishes when summed over all species. This leads to $\Sigma_a e_a \Gamma_a = 0$, i.e. ambipolarity.

When fluctuations are present, as they always are in real plasmas, their first order effects may be separated from first order effects resulting from toroidicity. However, when second order effects in the momentum equation are averaged over a flux surface, the mean forces arising from toroidicity and fluctuations must both be retained. Electrostatic fluctuations introduce a term $\left\langle \left(\nabla \cdot \underline{\underline{\Pi}}_a\right)_\varphi \right\rangle$ into the toroidal momentum balance, where $\underline{\underline{\Pi}}$ is the collisionless anisotropic pressure tensor resulting from finite Larmor radius (FLR) effects. Magnetic fluctuations give rise to a $<\underline{j}\times\underline{B}>_\varphi$ force. Adding the momentum equations for the different species no longer leads to ambipolarity of the neoclassical fluxes.

In Sec. 2, fluctuations are included in the momentum balance equation. Although the effect arising from electrostatic fluctuations is small compared with what can result from magnetic fluctuations, it is considered first because the analysis is particularly straightforward. The adjustment of the ambipolar electric field so that the neoclassical particle flux balances any non-ambipolar anomalous flux can be seen more clearly. The transport due to electrostatic fluctuations must be non-ambipolar, because $\Sigma_a e_a n_a = 0$ while the velocity of the ions differs from that of the electron, due to FLR effects. In the guiding centre description, the averaging of the electric field over the fast gyration of the ion can be expressed by taking the effective electric field acting on the ion to be $\left(1+\left(\rho_a^2/4\right)\nabla_\perp^2\right)\underline{E}$, where ρ_a is the Larmor radius. In the fluid description an equivalent FLR correction to the ion cross-field drift comes from the term - $(\nabla\cdot\underline{\Pi})\times\underline{B}/neB^2$ in the ion perpendicular velocity. Such a radial current must contribute to the moment balance.

The conclusion that the ambipolar condition must include all particle fluxes has important implications for a pure plasma. For example, the electron loss along a stochastic magnetic field can be balanced by an enhanced ion neoclassical flux. However, the effect on the predicted impurity flux is more pronounced, because their larger ionic charge makes them more sensitive to radial electric field. Section 3 describes the equations for particle fluxes in a multi-species plasma originally derived by Connor [3], and how these can be extended to include ion species in different collisional regimes. Section 4 considers the superposition of fluctuations on neoclassical effects, and discusses how the non-ambipolar anomalous flux is balanced by the neoclassical fluxes. Section 5 briefly describes a few of the experimental observations on impurity transport, and how far they can be explained by the foregoing analysis.

2. MOMENTUM BALANCE IN A NEOCLASSICAL PLASMA

Fluctuations will now be included in the derivation of automatic ambipolarity of the neoclassical fluxes, based on the generalised fluid momentum balance equation

$$n_a m_a \frac{d\underline{u}_a}{dt} = n_a e_a(\underline{E} + \underline{u}_a \times \underline{B}) - \nabla \cdot \underline{\underline{P}}_a + \underline{R}_a \tag{1}$$

where subscript a denotes the species, $\underline{\underline{P}}$ is the pressure tensor, and $\underline{R}_a = \Sigma\, R_{ab} = \Sigma\int m_a\, \underline{v}\, C_{ab}(f_a, f_b) d^3v$ is the collisional friction. Although the analysis may be performed for a general toroidally symmetric magnetic configuration [5,6], in the interests of physical clarity the simplest geometry of concentric circular flux surfaces will be used.

a) Inclusion of electrostatic fluctuations

The variation over a flux surface of quantities such as density n and electrostatic potential Φ will be written in the form

$$\begin{aligned} n(r,\theta,\phi) &= \bar{n}(r) + \tilde{n}(r,\theta) + \sum_{m,s} n_{ms}(r)\cos(\psi_{ms} + \alpha_{ms}) \\ \Phi(r,\theta,\phi) &= \overline{\Phi}(r) + \tilde{\Phi}(r,\theta) + \sum_{m} \Phi_{ms}(r)\cos(\psi_{ms} + \beta_{ms}) \end{aligned} \tag{2}$$

Here $\tilde{n}(r,\theta)$ is the neoclassical variation which, to first order in $\varepsilon \equiv r/R_0$, varies as $\tilde{n}_c(r)\cos\theta + \tilde{n}_s(r)\sin\theta$. Superimposed on this is a spectrum of electrostatic waves, with $\psi_{ms} = m\theta + s\phi - \omega t$, representing the density fluctuations which are always present experimentally.

Since the scale length of the neoclassical variation is the plasma radius, finite Larmor radius (FLR) effects are negligible, and the pressure tensor can be treated as diagonal. The measured fluctuations, however, have relatively short wavelengths, and FLR effects are important, while pressure anisotropy is not expected to play a significant role. The pressure tensor will therefore be separated into its two components $\underline{\underline{P}} = \underline{\underline{P}}^N + \underline{\underline{P}}^F$, where the neoclassical part

$$\underline{\underline{P}}^N = p_\perp \underline{\underline{I}} + (p_\parallel - p_\perp)\underline{b}\underline{b}$$

includes the zero order pressure. Here $\underline{\underline{I}}$ is the unit tensor and $\underline{b}$ is a unit vector parallel to the local magnetic field. The fluctuating pressure will be written as

$$\underline{\underline{P}}^F = p^F \underline{\underline{I}} + \underline{\underline{\pi}}$$

where $\underline{\underline{\pi}}$ contains only FLR terms. Its components are given, for instance, by Braginskii [7].

We now examine the φ-component of the pressure tensor.

$$(\nabla \cdot \underline{\underline{P}})_\varphi = \underline{e}_\varphi \cdot \left[\nabla p_\perp + \underline{b}(\underline{b}\cdot\nabla)(p_\parallel - p_\perp) + (p_\parallel - p_\perp)\{(\underline{b}\cdot\nabla)\underline{b} + \underline{b}(\underline{b}\cdot\underline{b})\} + \nabla p^F + \nabla \cdot \underline{\underline{\pi}}\right] \tag{3}$$

$$= \frac{BB_\phi}{r} \Theta \frac{\partial}{\partial\theta}\left[\frac{p_\parallel - p_\perp}{B^2}\right] + \frac{1}{R}\frac{\partial p^F}{\partial\phi} + \left(\nabla \cdot \underline{\underline{\pi}}\right)_\phi \tag{4}$$

where $\Theta = B_\theta/B_\phi$, and the symmetry of the neoclassical pressure in ϕ has been invoked. In the simple geometry considered, and with the tokamak ordering, $B \approx B_\phi \approx B_o/h(\theta)$ where $h(\theta) = 1 + \varepsilon \cos\theta$, and Θ is independent of θ. To eliminate $p_\parallel - p_\perp$, multiply the ϕ-component of Eq. (1) by h^2 and integrate over θ and ϕ, giving

$$\frac{m_a}{e_a}\left\langle n_a \frac{du_{a\phi}}{dt} h^2 \right\rangle = \Gamma_a B_{\theta o} + \left\langle \bar{n}_a E^A h^2 \right\rangle + \frac{1}{2R}\sum s n_{ams} \Phi_{ms} \sin(\beta_{ms} - \alpha_{ms})$$
$$- \left\langle \left(\nabla \cdot \underline{\underline{\pi}}_a\right)_\phi \right\rangle + \left\langle R_{a\phi} h^2 \right\rangle / e_a \tag{5}$$

where $\langle A \rangle = \oint d\phi \oint d\theta \, A/4\pi^2$, and $\Gamma_a = \langle n_a u_{ar} h \rangle$ is the mean particle flux across a magnetic surface. $E^A \underline{e}_\phi$ is the externally induced electric field. Because $\nabla \cdot \underline{\underline{\pi}}$ consists of terms of the general form [7]

$$\frac{\partial}{\partial x}\left[\frac{nT}{\Omega}\frac{\partial u_z}{\partial y}\right]$$

where ∂x and ∂y may be $r\partial\theta$ or $R\partial\phi$, and u_z any velocity component, it is already a nonlinear term. It generally gives a non-vanishing mean when integrated over a flux surface.

We will now separate off the classical particle flux (such as occurs in a plasma slab) and the anomalous flux due to the electrostatic fluctuations. What is left we will call the neoclassical flux. We write

$$\Gamma_a = \left[\frac{\left\langle h^2 \underline{R}_a \right\rangle \times \underline{B}_o}{e_a B_o^2}\right]_r + \bar{n}_a \left[\frac{\left\langle h^2 \underline{E}^A \right\rangle \times \underline{B}_o}{B_o^2}\right]_r$$
$$+ \sum_{ms} \left(\frac{\left\langle n_{ams} \underline{E}_{ms} \times \underline{B} \right\rangle}{B^2}\right)_r - \left(\frac{\left\langle \nabla \cdot \underline{\underline{\pi}}_a \times \underline{B} \right\rangle}{e_a B^2}\right)_r + \Gamma^N \tag{6}$$

The classical flux (the first two terms) and the anomalous flux (the third and fourth terms) can be obtained by vector multiplying Eq. (1) by $\underline{B}$, and integrating over a magnetic surface. The variation in B does not affect this integral, since it is not in phase with the fluctuations.

Substituting Eq. (6) for Γ_a in Eq. (5) gives

$$\frac{m_a}{e_a}\left\langle n_a \frac{d\underline{u}_{a\phi}}{dt} h^2 \right\rangle = \Gamma_a^N B_{\theta o} + \left\langle \left(\bar{n}_a E_\parallel^A + R_{a\parallel}/e_a \right) h^2 \right\rangle$$
$$+ \frac{1}{2B}\sum_{m,s} \left(\underline{k}_{ms} \cdot \underline{B} \right) n_{ams} \Phi_{ms} \sin(\beta_{ms} - \alpha_{ms}) - \frac{1}{e_a}\left\langle \left(\nabla \cdot \underline{\underline{\pi}}_a \right)_\parallel \right\rangle \quad (7)$$

where $\underline{E}^A = E_\parallel{}^A \underline{b} - \Theta E^A \underline{e}_\perp$, $R_{a\phi} = R_{a\parallel} - \Theta R_{a\perp}$, and $\underline{e}_\perp$ is a unit vector in the flux surface, perpendicular to $\underline{B}$. $\underline{k}_{ms}$ is the wave number of the fluctuation, so $\underline{k}_{ms} \cdot \underline{B} = mB_\theta/r + sB_\phi/R$.

b. Automatic ambipolarity in the absence of fluctuations

Assuming a steady state plasma with no fluctuations, and neglecting convective inertia, Eq. (7) reduces to

$$e_a \Gamma^N B_{\theta o} = -\left\langle \left(\bar{n}_a e_a E_\parallel^A + R_{a\parallel} \right) h^2 \right\rangle \quad (8)$$

Summing Eq. (8) over all species, including electrons, $\Sigma R_{a\phi} = 0$ because total momentum is conserved in collisions, and hence $\Sigma e_a \Gamma^N = 0$. Thus the particle fluxes are ambipolar, without having to impose it as a condition [1-6].

A more restrictive form of neoclassical automatic ambipolarity has recently been derived, known as the principle of detailed balance [4-6]. For each species in a multi-ion plasma, some lower energy particles fall within the Pfirsch-Schlüter collisional range, while higher energy particles satisfy the banana/plateau collisionality condition. Thus $\Gamma_a{}^N = \Gamma_a{}^{PS} + \Gamma_a{}^{BP}$. In [4-6] it is stated that each contribution can be written in a form similar to Eq. (8), i.e.

$$\Gamma_a^{BP} = -\frac{1}{e_a B_{\theta o}} \left\langle \bar{n}_a e_a E_\parallel^A + R_{a\parallel} \right\rangle, \quad \Gamma^{PS} = -\frac{1}{e_a B_{\theta o}} \left\langle \left(\bar{n}_a e_a E_\parallel^A + R_{a\parallel} \right) \left(h^2 - 1 \right) \right\rangle$$

Summing over species now leads to $\Sigma e_a \Gamma_a{}^{BP} = 0$ and $\Sigma e_a \Gamma_a{}^{PS} = 0$, i.e. the Pfirsch-Schlüter and banana/plateau components of the particle fluxes must be separately ambipolar.

Even in the absence of fluctuations, the automatic ambipolarity of the neoclassical fluxes can be questioned, because of the neglect of convective inertia, $\bar{n}_a m_a \langle \tilde{u}_a \cdot \nabla \tilde{u}_{a\varphi} \rangle$. Evaluating this for the Pfirsch-Schlüter regime, for example, using the expressions in Ref. 8, gives

$$m_a \bar{n}_a \frac{\tilde{u}_{a\theta}}{r} \frac{\partial \tilde{u}_{a\parallel}}{\partial \theta} \sim 0 \left[\frac{\rho_{a\theta}}{r_n} \right]^2 e_a B_\theta \Gamma_a^N$$

where $\rho_{a\theta}$ is the Larmor radius in the poloidal magnetic field, and r_n is the density scale length. Although $\rho_{a\theta}/r_n$ is assumed small, when convective inertia is included momentum balance does not produce exact ambipolarity. It will be neglected in the following analysis, however, because fluctuations produce a larger effect.

c. Non-ambipolarity of neoclassical fluxes with fluctuations

When fluctuations are present, multiplying Eq. (7) by e_a and summing over species, neglecting convective inertia, gives,

$$B_{\theta o}\sum_a e_a \Gamma_a^N = \sum_a \left\langle \left(\nabla \cdot \underline{\underline{\pi}}_a\right)_{\parallel} \right\rangle \tag{9}$$

Thus the neoclassical fluxes are no longer ambipolar.

The above equations may be interpreted in a different way. Multiplying Eq. (5) by $e_a R_o$ and summing over species, the result can be written in the form

$$\sum_a m_a \left\langle R n_a \frac{du_{a\phi}}{dt} h \right\rangle = \left\langle R B_\theta j_r h \right\rangle - \sum_a \left\langle R\left(\nabla \cdot \underline{\underline{\pi}}_a\right)_\phi h \right\rangle \tag{10}$$

$$= R_o B_{\theta o} \sum_a e_a \Gamma_a^N - \sum_a \left\langle R\left(\nabla \cdot \underline{\underline{\pi}}_a\right)_{\parallel} \right\rangle \tag{11}$$

But Eq. (10) is just the angular momentum equation about the major axis, integrated over the annulus between two flux surfaces. (In the simple geometry used, the spacing between the flux surfaces is constant). In the absence of fluctuations, there is no $\nabla \cdot \underline{\underline{\pi}}$ term. If the convective inertia is neglected, it then follows that the net $R(\underline{j} \times \underline{B})_\phi$ force must vanish. This happens to coincide with $\Sigma e_a \Gamma_a = 0$. Since the classical fluxes are automatically ambipolar, it follows that $\Sigma e_a \Gamma_a^N = 0$. This does not result from any special property of neoclassical transport (the loss mechanism in Eq. (10) need not even be specified). It merely states that the angular momentum is constant only if the net moment vanishes. If there is no other ϕ-directed force, this requires $\langle R j_r B_\theta h \rangle = R_o B_{\theta o}$ $\Sigma e_a \Gamma_\alpha = 0$.

When electrostatic fluctuations are present, then from Eq. (6) ambipolarity requires

$$\sum_a e_a \Gamma_a^N - \frac{1}{B}\sum_a \left\langle \nabla \cdot \pi_{a\perp} \right\rangle = 0 \tag{12}$$

where $\perp$ denotes the component in the flux surface perpendicular to $\underline{B}$. This does not generally ensure that the right side of Eq. (11) vanishes, i.e. the condition for ambipolarity is not generally the same as for vanishing angular momentum about the major axis. Momentum balance will be discussed in a later paper.

d. The FLR viscous tensor

Since this plays such an important role, we will briefly discuss its physical origin. The most important terms, arising from the general form given earlier for $\nabla \cdot \underline{\underline{\pi}}$, combine to give

$$\frac{nT}{\Omega_i} \nabla_\perp^2 \underline{u} = \frac{1}{2} n_a e_a B \rho_a^2 \nabla_\perp^2 \underline{u}$$

This arises because an ion responds to the electric field averaged over its very fast gyration motion. In the guiding centre description the same effect gives rise to the FLR correction to the electric drift, i.e.

$$\left[1 + \left(\rho_a^2/4\right)\nabla_\perp^2\right] \frac{\underline{E} \times \underline{B}}{B^2}$$

It is apparent that electrostatic fluctuation must produce non-ambipolar fluxes, since quasi-neutrality requires that $\Sigma Z_a n_a = 0$, while the ion drift velocity is less than that of the electrons. The measured fluctuations typically have $k_\perp \rho_a \sim 0.1$-0.3, consistent with prediction, so the non-ambipolar flux must be of order 1% of the total. This would produce a very rapid build-up in space charge if no compensating non-ambipolar loss is present.

Other terms in $\nabla \cdot \underline{\underline{\pi}}$ have the form

$$\frac{\partial}{\partial x}\left(\frac{p}{\Omega}\right)\frac{\partial u}{\partial y}$$

These terms arise because the full equation for the flux includes the convective inertial term $(n_a/\Omega_a)(\underline{u} \cdot \nabla \underline{u}) \times \underline{b}$ (neglected in Eq. (6)). The above terms cancel the diamagnetic part of $\underline{u}$, leaving the guiding centre convective inertial term. It is physically reasonable that the true convective inertial term should contain the guiding centre, and not the diamagnetic velocity. This term, however, is not considered here.

e. Inclusion of magnetic fluctuations

Since this follows similar lines to the inclusion of electrostatic fluctuations, the analysis is abbreviated. The local current carried by the a^{th} species, associated with the fluctuations, is denoted by $\underline{j}_a = n_a e_a \underline{u}_a$. The first order fluctuations in magnetic field and charge flux are denoted by $\tilde{\underline{B}}$ and $\tilde{\underline{j}}_a$. Since the effect of the FLR pressure tensor and electrostatic fluctuations have already been analysed, they will now be omitted.

The equation corresponding to Eq. (5) is

$$m_a\left\langle n_a \frac{du_{a\varphi}}{dt} h^2\right\rangle = e_a B_{\theta o}\Gamma_a + e_a\left\langle \bar{n}_a E^A h^2\right\rangle + \left\langle \tilde{j}_{ar}\tilde{B}_\theta - \tilde{j}_{a\theta}\tilde{B}_r\right\rangle + \left\langle R_{a\varphi}h^2\right\rangle \tag{14}$$

As before, the anomalous transport due to flow along the perturbed magnetic field and the classical flux are separated off, leaving the neoclassical flux

$$\Gamma_a = \left[\frac{\left\langle h^2 \underline{R}_a\right\rangle \times \underline{B}_o}{e_a B_o^2}\right]_r + \bar{n}_a\left[\frac{\left\langle h^2 \underline{E}^A\right\rangle \times \underline{B}_o}{B_o^2}\right]_r + \frac{\left\langle \tilde{j}_{a\parallel}\tilde{B}_r\right\rangle}{e_a B_o} + \Gamma_a^N \tag{15}$$

Substituting this into Eq. (14) gives

$$m_a\left\langle n_a \frac{du_{a\varphi}}{dt} h^2\right\rangle = e_a B_{\theta o}\Gamma_a^N + \left\langle\left(\bar{n}_a e_a E^A + R_{a\parallel}\right)h^2\right\rangle + \left\langle \tilde{j}_{ar}\tilde{B}_\theta - \tilde{j}_{a\perp}\tilde{B}_r\right\rangle \tag{16}$$

where $\tilde{j}_{a\perp} = \tilde{j}_{a\theta} - \Theta\, \tilde{j}_{a\parallel}$ is the fluctuating charge flux lying in the magnetic surface and perpendicular to the unperturbed field. summing Eq. (16) over species gives

$$\sum m_a\left\langle n_a \frac{du_{a\varphi}}{dt} h^2\right\rangle = B_{\theta o}\sum_a e_a\Gamma_a^N + \left\langle \tilde{j}_r\tilde{B}_\theta - \tilde{j}_\perp B_r\right\rangle \tag{17}$$

where $\tilde{\underline{j}} = \Sigma_a \tilde{\underline{j}}_a$.

As with electrostatic fluctuations, conservation of momentum in collisions does not imply ambipolarity of the neoclassical fluxes when magnetic fluctuations are present, even when convective inertial is ignored. Ambipolarity must be imposed as a separate condition

$$\sum e_a\Gamma_a^N + \left\langle \tilde{j}_{a\parallel}\tilde{B}_r\right\rangle = 0 \tag{18}$$

As will be discussed in Sec. 4, this allows electron loss along an ergodised magnetic field to be balanced by neoclassical ion flux.

3. CONNOR'S SOLUTION FOR AN IMPURE PLASMA

To study the implications of Eq. (18), particularly for impurity fluxes, we need specific expressions for the neoclassical fluxes in an impure plasma. Such expressions were first derived by Connor [3], who considered a multi-ion plasma in which all the species are predominantly in the banana collisionality regime. In practice, impurity ions are more likely to be in the plateau or collision-dominated regimes. Such a situation has been considered by several authors [6,9,10], but the Connor analysis will be used for its analytic simplicity. Its generalisation to other regimes will be discussed later in this section.

Connor assumed a stationary toroidal plasma without fluctuations. Starting from the kinetic equation with Fokker-Planck collision operator, he found the diffusion flux of the a^{th} species to be

$$\Gamma_a^N = -1.46\varepsilon^{1/2} n_a \left[\frac{m_a \bar{\nu}_a}{e_a B_\theta^2} \left\{ \frac{T_a}{e_a} \left(\frac{n_a'}{n_a} - \gamma_a \frac{T_a'}{T_a} \right) - E_r + \frac{B_\theta}{\bar{\nu}_a} \sum_b \bar{\nu}_{ab} U_b^a \right\} + \frac{E_\parallel^A}{B_\theta} \right] \tag{19}$$

where

$$\bar{\nu}_a = \sum_b \bar{\nu}_{ab}, \quad \nu_{aa} = 0.53\nu_{ao}, \quad \tilde{\nu}_{aa} = 0.71\nu_{ao}, \quad \gamma_a = \frac{3}{2} - \sum_b \tilde{\nu}_{ab} / \bar{\nu}_a$$

$$\bar{\nu}_{ab} = 0.4\tilde{\nu}_{ab} = \nu_{ao} \frac{n_b e_b^2}{n_a e_a^2} \left[\frac{m_b T_a^3}{n_a T_b^3} \right]^{1/2} \text{ when } m_b < m_a$$

$$\bar{\nu}_{ab} = \tilde{\nu}_{ab} = \nu_{ao} \frac{n_b e_b^2}{n_a e_a^2} \text{ when } m_b > m_a$$

$$\nu_{ao} = \frac{4}{3}(2\pi)^{1/2} \frac{n_a e_a^4}{m_a^{1/2} T_a^{3/2}} \ell n\, \Lambda$$

In the derivation of Eq. (19), the radial electric field, E_r, is treated as an arbitrary parameter. U_b^a, which is approximately the parallel mass flow of the b^{th} species, is defined more precisely in [3]. For brevity we will denote the term containing it by its approximate value, $B_\theta u_{a\parallel}$, but this can be replaced by its exact value if greater accuracy is desired. Since the Ware pinch, the last term in Eq. (19), is automatically ambipolar, it plays no part in the following discussion and will be dropped.

The remainder of this section will summarise the impurity behaviour predicted by Eq. (19) when neoclassical transport is the only loss mechanism. Connor [3] derived the radial electric field from the ambipolar condition, $\Sigma e_a\Gamma_a = 0$, and then eliminated it from Eq. (19). Since the coefficients of all terms in Γ_e are 0 $(m_e/m_i)^{1/2}$ smaller than comparable terms in the ion diffusion, the ion fluxes must cancel to this order. For simplicity he discussed the case of a light main ion, and one dominant heavy impurity, denoted by subscripts 1 and 2 respectively. After eliminating E_r, determined by ambipolarity, the impurity flux becomes

$$\Gamma_2 = \frac{1.46\varepsilon^{1/2}}{e_2^2 B_\theta^2}\left(\frac{n_1 m_1 \bar{v}_1 \quad n_2 m_2 \bar{v}_2}{n_1 m_1 \bar{v}_1 + n_2 m_2 \bar{v}_2}\right)\left[T_1 \frac{e_2}{e_1}\left(\frac{n_1'}{n_1} - \gamma_1 \frac{T_1'}{T_1}\right) - T_2\left(\frac{n_2'}{n_2} - \gamma_2 \frac{T_2'}{T_2}\right) + e_2 B_\theta (u_{1\parallel} - u_{2\parallel})\right] \quad (20)$$

Γ_1 may be obtained by interchanging subscripts 1 and 2, or from $\Gamma_1 = - Z_2\Gamma_2/Z_1$.

Although Eq. (20) is the more suitable form for evaluation, the origin of the rather complex variation in neoclassical flux may be seen more clearly when E_r is retained, as in Eq. (19). The neoclassical particle flux in all collisional regimes may be written in the general form

$$\Gamma_a^N = n_a D_a\left[-\frac{n_a'}{n_a} + \gamma_a \frac{T_a'}{T_a} + \frac{e_a}{T_a}\left(E_r - B_\theta u_{a\parallel}\right)\right] \quad (21)$$

where the expressions for D_a and γ_a vary as the collisionality changes from the banana, through the plateau, to the Pfirsch-Schlüter regime. When the ion species are in different collisionality regimes, an approximation to the more exact analysis [6], [9-10] may be obtained by using in the ambipolar condition the expression appropriate to each component species. This gives the ambipolar value of $E_r - B_\theta u_\parallel$, which can then be eliminated to obtain the flux for each species. This approach is described in detail in [11] and [12].

As an example, a possible variation of $Z_a^2 D_a$ for the two-ion species example is sketched in Fig. 1. This shows the main ion species just reaching the plateau regime, while the impurity, because its higher atomic charge increases its collisionality, reaches the Pfirsch-Schlüter regime. Whichever species has the larger value of Z^2D dominates the ambipolar condition. E_r must then adjust itself so that the bracket in Eq. (21) nearly vanishes for this species. This reduces its flux so that it is equal, but opposite, to that of the slower diffusing species.

As illustrated in Fig. 1, the most common situation is where the main ions dominate the ambipolarity equation, because of their higher number density. An inward directed radial electric field is then set up to reduce the outward ion flux. The inward pull of this field on the impurity

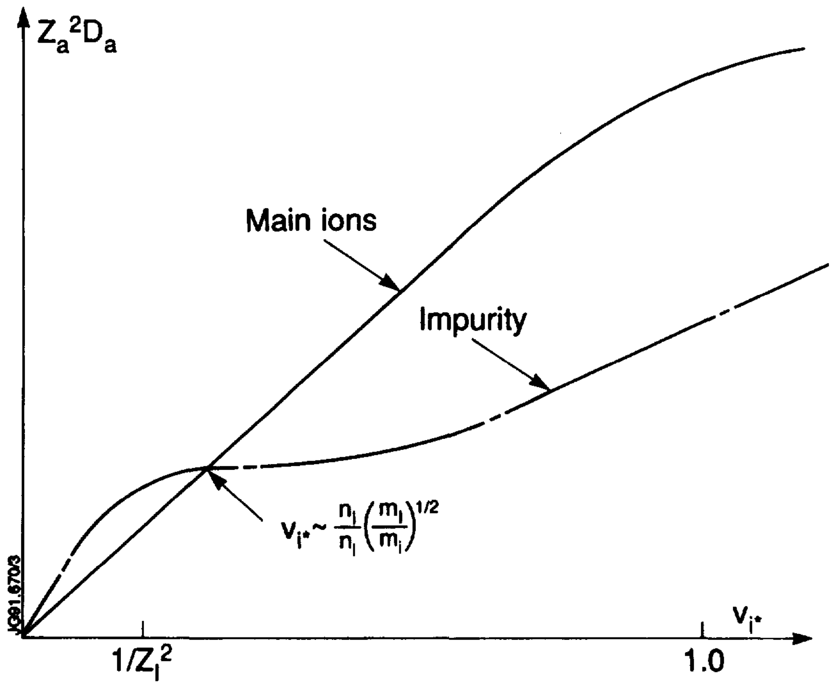

Fig. 1. Variation of the diffusion coefficient with collision frequency for main plasma and impurity ions

ions is enhanced by their larger charge, convecting them inwards even when their profile becomes more peaked on axis than the main ions. The impurities continue to accumulate on axis until a steady state profile is reached when

$$\frac{n_2(r)}{n_2(0)}\cdot\left[\frac{T_2(r)}{T_2(0)}\right]^{-\gamma_2} = \left[\frac{n_1(r)}{n_1(0)}\right]^{k}\cdot\left[\frac{T_1(r)}{T_1(0)}\right]^{-k\gamma_1} \tag{22}$$

where $k = Z_2T_1/Z_1T_2$. Thereafter, both main and impurity ions have a slow outward flux, their sum equalling that of the electrons.

4. NEOCLASSICAL TRANSPORT IN THE PRESENCE OF FLUCTUATIONS

Now consider how the Connor analysis is modified by the presence of electrostatic fluctuations. In the kinetic equation we substitute Eq. (2) for the electrostatic potential, and the analogous expression

$$f(r,\theta,w,\mu) = \bar{f}(r,w,\mu) + \tilde{f}(r,\theta,w,\mu) + \sum_{m,s} f_{m,s}(r,w,\mu)\cos(\psi_{ms} + \gamma_{ms}) \tag{23}$$

for the velocity distribution function, where W and μ are the particle kinetic energy and magnetic moment, and $\tilde{f} = 0(\varepsilon\bar{f})$ is the neoclassical variation. Treating ε and $f_{ms}/\bar{f}$ as small quantities, the kinetic equation may be linearised and solved for $\tilde{f}$ and f_{ms}. When the sum of the linearised solutions is substituted into the quasilinear particle flux $\int d^3v v_r f$, and then integrated over a flux surface, only products of neoclassical terms or of fluctuation terms survive. A similar separation of the first order variation can be made when magnetic fluctuations are superimposed on a neoclassical plasma.

Because the wave frequencies are expected to be larger than the ion bounce frequency (their ratio for a drift wave is $qk_y\rho_a\varepsilon^{-3/2}$), the $\underline{E} \times \underline{B}$ drift displacement of a trapped ion should be small. Thus nonlinear interaction between the two effects should be negligible. The neoclassical analysis is also valid in the presence of magnetic fluctuations. A trapped particle is not aware that the field line wanders radially, provided the displacement from a magnetic surface is small over one bounce length. The escaping electrons have largest $v_{||}$, while neoclassical electron diffusion results from those with small $v_{||}$. Hence the interaction should again be small, and the Connor derivation of the neoclassical flux is still valid when fluctuations are present.

The two effects first interact when the ambipolar condition is imposed. The fluctuation driven fluxes generally result in a residual non-ambipolar charge flux $e\Gamma_c$. In the case of electrostatic fluctuations, Eq. (6) gives

$$e\Gamma_c = \sum_a e_a \Gamma_a^{an} = -\frac{1}{B^2} \sum_a \left\langle \nabla \cdot \underline{\underline{\pi}}_a \times \underline{B} \right\rangle_r \tag{24}$$

In an ergodised magnetic field the electron loss is a function of the strength of the magnetic fluctuation, the ratio of the mean free paths to the fluctuation correlation length, and the radial electric field. Ambipolarity requires

$$\sum_a e_a \Gamma_a^N (E_r) + e\Gamma_c = 0 \tag{25}$$

Because the coefficient of the neoclassical electron flux in Eq. (19) is smaller than that for the ion flux by a factor $(m_e/m_a)^{1/2}$, in a pure plasma the ambipolar $E_r - B_\theta U_{||}$ must reduce the ion flux to balance that of the electron. Thus $E_r - B_\theta U_{||}$ almost cancels the other terms in the bracketed factor for the ion flux. When electrostatic fluctuations are present, a small change in this ambipolar value produces the increased ion neoclassical flux required to balance the non-ambipolar part of the fluctuation-driven flux.

Now consider an ergodised magnetic field such that, in the absence of any radial electric field, the electron loss rate is $\Gamma_e^{an}(0)$. This loss rate has been estimated for different collisional regimes by many authors, e.g. [13-15]. If $\Gamma_i^N(0) >> \Gamma_e^{an}(0)$, the ambipolar $E_r - B_\theta U_\parallel$ reduces the ion neoclassical flux to a value of order $\Gamma_e^{an}(0)$. If, on the other hand, $\Gamma_e^{an}(0) >> \Gamma_i^N(0)$, the radial electric field must restrain the electron ergodic loss to the order $\Gamma_i^N(0)$.

Anomalous loss can have a more pronounced effect in impurity fluxes, because their large Z makes them more sensitive to changes in the radial electric field. For the example of a single dominant impurity, the ambipolar value of $E_r - B_\theta U_\parallel$ is given by

$$\left[1+\frac{n_2 m_2 \bar{v}_2}{n_1 m_1 \bar{v}_1}\right]\left(E_r - B_\theta U_\parallel\right) = \frac{T_1}{e_1}\left[\frac{n_1'}{n_1} - \gamma_1 \frac{T_1'}{T_1} + \frac{T_2}{T_1}\frac{Z_1}{Z_2}\frac{n_2 m_2 \bar{v}_2}{n_1 m_1 \bar{v}_1}\left\{\frac{n_2'}{n_2} - \gamma_2 \frac{T_2'}{T_2}\right\}\right]$$
$$-0.685\frac{eB_\theta^2}{\epsilon^{1/2} n_1 m_1 \bar{v}_1}\Gamma_c \qquad (26)$$

As can be seen from Eq. (26), stochastic electron loss ($\Gamma_c = -\Gamma_e^{an}$) reduces the inward directed value of $E_r - B_\theta U_\parallel$. From Eq. (19), the effect of any change in $E_r - B_\theta U_\parallel$ on the impurity fluxes increases in proportion to their ionic charge. Thus moderate changes in $E_r - B_\theta U_\parallel$, which have only a modest effect on the main ion flux, may reverse the direction of the impurity neoclassical flux from inwards to outwards. If $\Gamma_e^{an}(0) > \Gamma_i^N(0)$, as is expected for strong ergodicity, the sign of E_r may be reversed, producing a significant increase in the main ion outward flux, and a rapid pump-out of impurities.

5. EXPERIMENTAL EVIDENCE

We now consider the experimental evidence for the behaviour predicted in the foregoing sections. Removing the ambipolarity condition on the neoclassical fluxes allows a large increase in the ion neoclassical flux when it can be balanced by stochastic electron loss. However, it is difficult to distinguish neoclassical flux from that driven by electrostatic fluctuations. The most readily recognised effect is the enhanced response of impurities to electron stochastic loss. Relevant experimental impurity behaviour will now be briefly reviewed.

Early tokamak plasmas usually showed no evidence of impurity accumulation on axis, with Z_{eff} approximately constant over the cross-section. In some, however, impurities did accumulate near the centre, but this accumulation relaxed during periods of enhanced MHD activity. For example, during stable discharges in T-4 the central impurity density increased monotonically [16]. However, when a kink instability occurred, the impurity content dropped abruptly, rising again after the magnetic oscillations died out. The accompanying changes in electron density and temperature were small. When a disruption instability occurred, the central impurity density dropped by an order of magnitude, while the central electron density fell by only 5% [16].

Later a bimodal behaviour was observed in several tokamaks, in which nominally identical discharges showed major differences in their impurity behaviour. For example, in D-III [17] the more normal discharge (type-S) developed sawtoothing early and no impurity peaking occurred thereafter. In type-O discharges, however, the impurity density peaked sharply inside $r/a = 0.25$, implying a large inward convection, as predicted neoclassically. In these discharges there was no MHD central activity. Later in type-O discharges a large $m = 1$, $n = 1$ oscillation built up rapidly, and the central impurity density decreased. Thereafter it behaved as a type-S. Which mode developed was determined by differences in the plasma-wall interaction during the initial phase. The difference in impurity behaviour was attributed to the difference in central MHD activity [17].

Impurity pump out during a sawtooth crash was studied in Alcator C by Segum et al [18]. They found the impurity to accumulate on axis between sawteeth, while during the crash it flattens inside the inversion radius. This prevents any steady build-up of impurity on axis.

The above experimental behaviour is qualitatively consistent with analytic predictions based on Sec. 4. The absence of impurity accumulation in early tokamaks could have been due to significant electron loss along stochastic magnetic field, due to the higher MHD activity and to the larger magnetic field errors. This would wholly or partly short out the radial electric field which causes impurity accumulation in the purely neoclassical plasma. The impurity accumulation during quiescent periods in T-4 could be due to the absence of magnetic ergodicity. Kink oscillations could provide sufficient ergodicity to allow the electrons to short out the radial electric field. During a disruption the ergodicity could be so strong that an outward electric field is necessary to restrain the parallel electron loss. Because of their high Z, this would cause a very rapid pump out of impurities.

The absence of sawteeth in Type-O discharges in D-III can be attributed to the early impurity influx from wall interaction [17]. Radiation cooling lowers the central temperature, so that current does not peak on axis, keeping q(o) above unity. The impurity behaviour is qualitatively consistent with neoclassical transport inside $r/a < 0.25$, which in type-S discharges is opposed by impurity pump-out during sawtooth crashes.

Evidence that, in the absence of fluctuations, the impurity transport is neoclassical has emerged from a recent work on JET [19,20]. Here the sawtooth period is long enough to evaluate the transport during the recovery phase between sawtooth crashes. This is best done by studying the evolution of the density profile of injected impurities. The parameters in the impurity flux $\Gamma_Z = - D_Z \, dn_Z/dr - nV_Z$, in a transport simulation code, were adjusted to obtain the best fit to the soft X-ray and spectroscopic measurements.

The local impurity transport in JET is found to be different in a central core and in the outer plasma, as illustrated in Fig. 2, taken from Ref. [20]. For $\rho \equiv r/a$ less than about 0.25, the diffusivity and convection in ohmic and L-mode plasmas are both consistent with neoclassical prediction. The diffusivity in the outer plasma, typically $\rho > 0.4$, is more than an order of magnitude higher, being 30-60 times neoclassical prediction. Over the intermediate transition zone, the diffusivity increases exponentially between these two values. The variation in a H-mode plasma is qualitatively similar, but the good confinement and transition zones are wider [20].

This behaviour implies a quiescent central region in JET, where only neoclassical transport occurs between sawtooth crashes. Outside $\rho/a > 0.25$ anomalous transport is so large as to completely dominate the neoclassical flux. The rapid drop in central impurity content, observed after the crash, is consistent with the predicted effect of a brief period of ergodic electron loss.

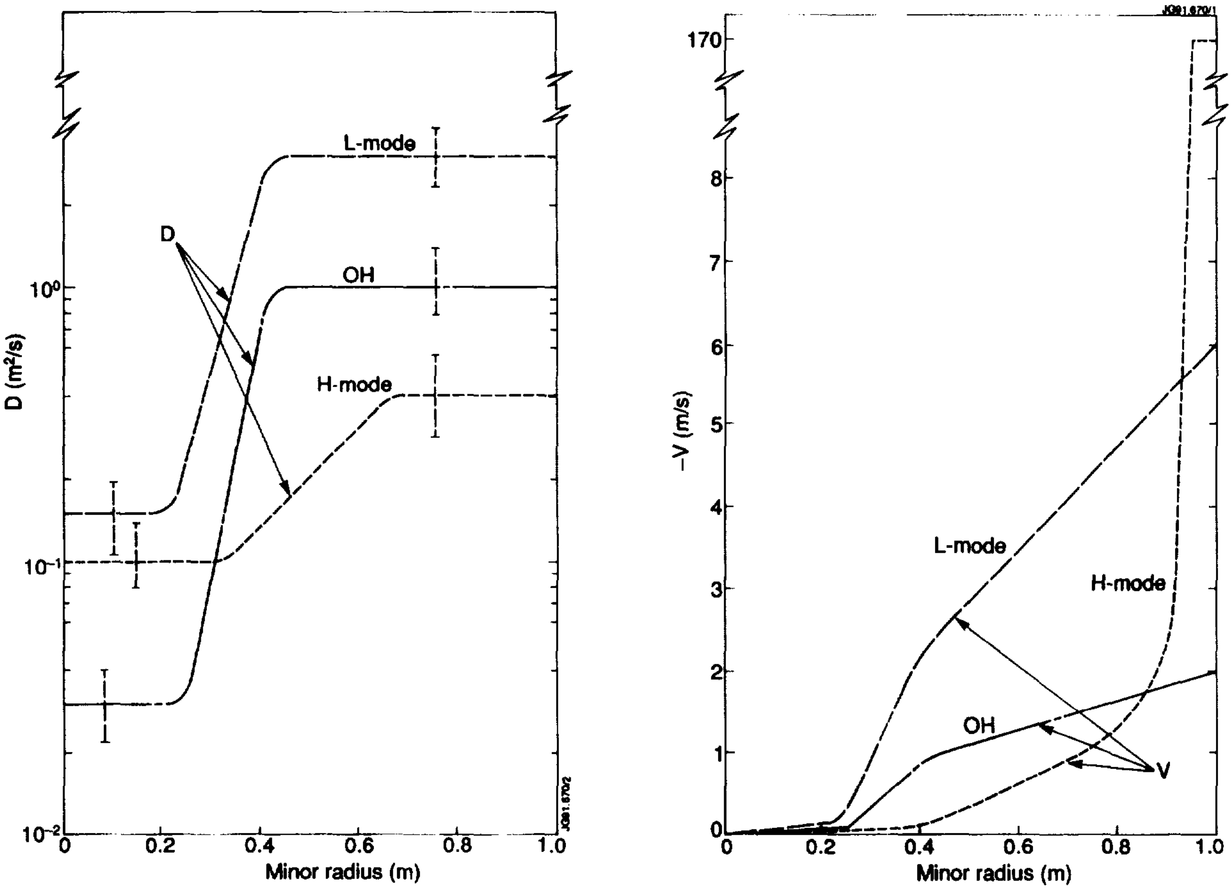

Fig. 2 Radial Variation of D(r) and V(r) for OH, L and H-modes in JET

6. CONCLUSIONS

1. Neoclassical particle transport is not automatically ambipolar when fluctuations are present, as they always are in real plasmas. The usual argument for ambipolarity is based on the φ-component of momentum balance, suitably averaged over a flux surface. The fluctuations also contribute a mean force in this direction. For electrostatic fluctuations the contribution comes from the FLR pressure tensor $<(\nabla\cdot\pi_a)_\varphi>$. Magnetic fluctuations contribute a $<\underline{j} \times \underline{B}>_\varphi$ force. This upsets the simple relation between particle flux and collisional friction which led to automatic ambipolarity.

2. Anomalous transport is generally non-ambipolar. In the case of electrostatic fluctuations, FLR effects introduce a difference between the cross-field drift of ions and electrons. Although this difference is relatively small, if not balanced by some other loss mechanism it would lead to a

very rapid build-up of electric field and plasma rotation. Magnetic ergodicity can produce much stronger non-ambipolarity.

3. The electron, ion, and impurity neoclassical fluxes are all functions of the radial electric field, as shown in Connor's [3] original analysis. In the presence of non-ambipolar anomalous particle transport, the radial electric field adjusts itself so that this non-ambipolar flux is balanced by the neoclassical fluxes.

4. In a purely neoclassical plasma, ions have the capacity for more rapid diffusion, but are restrained by the more slowly diffusing electrons. Additional electron loss, such as by stochastic diffusion, may be balanced by increased ion neoclassical flux.

5. Because of their higher Z, the impurity neoclassical flux is more strongly affected by a change in radial electric field. The small change in electric field necessary to balance the anomalous transport driven by electrostatic fluctuations will produce only a modest effect. However, the magnetic ergodicity resulting from a sawtooth crash or a minor disruption could permit a large electron parallel flow, shorting out the radial electric field. This would produce a rapid pump-out of impurities, with a relatively smaller effect on the main plasma.

6. Outside a central region the experimental impurity diffusivity is generally much larger than neoclassical prediction, by at least an order of magnitude. Presumably this is because the neoclassical diffusion is dominated by the anomalous. Inside a central region, typically $r/a < 0.25$, the diffusivity is much smaller and is consistent with neoclassical [19-21].

7. The observed pump-out of impurities during periods of high MHD activity, such as sawteeth or disruptions, can be explained by the foregoing analysis. An outward ambipolar electric field may be necessary to restrain the electron loss along the stochastic magnetic field. Because of their large Z, impurities are then strongly ejected from the centre, while the effect on the main plasma is much less. The less pronounced change in impurity content in certain

transitions, such as from H to L-mode, may possibly be due to a more modest change in MHD activity.

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Appendix I

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