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Ion Temperature Gradient Driven Impurity Mode

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Ion Temperature Gradient Driven Impurity Modes

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The presence of two (or more) species of ions in an inhomogeneous magnetically confined plasma can present opportunities for a new class of drift-type modes to become unstable. When the thermal speed of a minority ion species (the impurity) is much smaller than that of the primary species, waves with parallel phase velocities intermediate between the two speeds can be driven unstable by a source of free energy (e.g. density or temperature gradient) and Landau damping. Their linear stability theory and quasilinear particle and heat transport are discussed in the context of a shearless slab geometry. The simultaneous appearance of two unstable modes, one propagating in the ion diamagnetic direction and the other in the electron direction, is discussed. When the effective impurity concentration is finite, the threshold for the ion temperature gradient (ITG) instability is raised dramatically.

I Introduction

The control of impurities in magnetically confined toroidal plasmas is fast becoming a crucial problem for present day experiments [1-2], and will likely remain such in the future. Impurity ions that are either resident in the gas that acts as a fuel for the plasma or which are produced via interactions (e.g. sputtering) between the plasma and the wall of the containment vessel can be highly deleterious to the achievement of plasma ignition (namely the "point" in density/temperature parameter space where heating from thermonuclear reaction balances all energy losses). Their two main deleterious effects are dilution of the fuel (D-T or D-D), especially by high-Z impurity ions, and energy loss through radiation. A variety of remedies can be attempted, though it is advisable that the physics of these impurities be known as much as possible before they are implemented.

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The physics of impurities can loosely be grouped into three main areas: production, entry into the main plasma, and (particle and energy) transport consequences. We shall be concerned only with the third area here. We shall assume that an equilibrium exists, with prescribed density and temperature profiles for each species, and study the processes that arise from this situation: unstable spectrum (if any) of low frequency waves and ensuing quasilinear transport of particles and energy. We are interested in processes which arise from the presence of two ion species, which will necessarily fall outside the single fluid description (e.g. ideal or resistive MHD). Since much of current interest is centered around microturbulence due to ion temperature gradient modes (ITG) whose parallel phase velocity is comparable to the thermal velocity of one ion species, a kinetic description is in order.

In the present study we shall concentrate on the transport due to microturbulence that arises from the presence of two ion species in a plasma with a temperature inhomogeneity. The temperature gradient provides the free energy for the instability. The mode becomes unstable, in most situations of interest, through a competition between the (stabilizing) Landau damping of (heavy) impurities and the destabilizing inverse Landau damping of the primary ions. The simplest magnetic configuration, shearless slab, is assumed. Section II deals with the linear theory of impurity modes, their stability properties (e.g. marginal stability, number of unstable modes), and the extent and shape of the unstable spectrum. In Section III we discuss the quasilinear transport of particles and energy, arising from a saturated spectrum of these modes. Section IV contains the conclusions.

II Linear Theory

We consider the simplest equilibrium, a shearless slab: $\vec{B} = \hat{e}_z B$ with a low-β plasma. Hence, only electrostatic perturbations, $\tilde{\vec{E}} = -\nabla\tilde{\phi}$, are of relevance. The equilibrium distribution function in velocity space depends on energy and canonical angular momentum:

$$F(\vec{v},\vec{r}) = \frac{n}{\left(\pi v_T^2\right)^{3/2}} \exp\left(-\frac{v^2}{v_T^2}\right)\left\{1+\frac{1}{L_n}\left(x+\frac{V_y}{\Omega}\right)\left[1-\eta\left(\frac{3}{2}-\frac{v^2}{v_T^2}\right)\right]\right\} \tag{1}$$

where $\Omega = ZeB/mc$ is the cyclotron frequency and $\eta = d\ell nT/d\ell nn$ is the temperature gradient parameter. The thermal speed is $V_T = \sqrt{2T/m}$. The dispersion relation in the local approximation is well known (see, e.g. [3]) and can be written as:

$$O = 1 + \sum_j \frac{Z_j^2 n_j}{n_e}\frac{T_e}{T_j}\left\{1 - \eta_j \frac{\omega\omega_{*j}}{k_\parallel^2 V_{Tj}^2}\Gamma_j + \Gamma_j \frac{\omega}{k_\parallel V_{Tj}} Z\left(\frac{\omega}{k_\parallel V_{Tj}}\right)\right.$$
$$\left.\left[1 - \frac{\omega_{*j}}{\omega} + \eta_j \frac{\omega_{*j}}{\omega}\left(\frac{1}{2} + b_j\delta_j - \frac{\omega^2}{k_\parallel^2 V_{Tj}^2}\right)\right]\right\} \qquad (2)$$

where the summation is over ion species (j=i,I) and the electron response has been taken to be adiabatic, $\tilde{n}_e = n_e e\tilde{\phi}/T_e$ since $|\omega| << k_\parallel V_{Te}$. Perturbed quantities are Fourier analyzed in two directions $\tilde{\phi}(\vec{r},t) = \hat{\phi}\exp(-i\omega t + ik_\parallel z + ik_\perp y)$. The diamagnetic frequency is

$$\omega_{*j} = k_\perp c T_j / Z_j e B L_{nj} \qquad (3)$$

while the finite Larmor radius (FLR) parameter is $b_j = k_\perp^2 T_j/m_j\Omega_j^2$ with $\Gamma_j = \exp(-b_j)\, I_0(b_j)$ and $\delta_j = 1 - I_1(b_j)/I_0(b_j)$. The quality $Z(\omega/k_\parallel V_{Tj})$ is the standard dispersion function [4]. The collisionless limit, $\nu < (\omega_{*j}, k_\parallel V_T)$ is considered to be of relevance, as we are interested in modelling keV-level plasmas that occur in the main body of experiments. For instance, for JET, we may take $T \geq 0.5$ keV, $n \sim 3\cdot 10^{13}\text{cm}^{-3}$, $B \sim 4$ Tesla, $R \sim 300$cm, $a \sim 100$cm, $q \sim 5/2$ and $L_n \sim 100$cm, and obtain: $\nu_i \leq 2.5\cdot 10^3\text{sec}^{-1}$, $k_\parallel V_{Ti} \geq V_{Ti}/qR \sim 4\cdot 10^4\text{sec}^{-1}$, ω_{*i} ($k_\perp \equiv 2n^o q/a) \geq 10^4\text{sec}^{-1}$ (for $n^o \equiv$ toroidal mode number = 20).

A) Marginal stability - fluid impurities

It has been known for a long time [5] that the presence of a second, massive species of ions can give rise to a new instability: the impurity drift mode. In the absence of temperature gradients, this mode is unstable provided that the impurity density gradient is directed in the direction opposite to that of the primary ion (and is strong enough) and that the impurity thermal speed is finite (thereby providing a reactive instability through coupling to the impurity sound mode or dissipation through inverse Landau damping). Subsequent work [6-8] has considered the effect of primary ion temperature gradient, under the assumption of a fluid response by the impurities to the perturbation. We begin the linear analysis by considering the limit $T_I \to 0, \eta_I \to 0$. Noticing that $n_i = n_e - Z_I n_I$, the dispersion relation becomes:

$$O = 1 + \frac{Z_I n_I}{n_e}\left(b_i\mu_I + \sigma_I\frac{\omega_{*i}}{\omega} - \frac{1}{2\mu_I}\frac{k_\parallel^2 V_{Ti}^2}{\omega^2}\right) + \left(1 - \frac{Z_I n_I}{n_e}\right)$$
$$\left\{1 - \eta_i\frac{\omega\omega_{*i}}{k_\parallel^2 V_{Ti}^2}\Gamma_i + \Gamma_i\frac{\omega}{k_\parallel V_{Ti}}Z\left(\frac{\omega}{k_\parallel V_{Ti}}\right)\left[1 - \frac{\omega_{*i}}{\omega} + \eta_i\frac{\omega_{*i}}{\omega}\left(\frac{1}{2} + b_i\delta_i - \frac{\omega^2}{k_\parallel^2 V_{Ti}^2}\right)\right]\right\} \quad (4)$$

where we define $\mu_I \equiv (1/Z_I)(m_I/m_i) \cong 2$ (for a plasma with hydrogen as a main species) and $\sigma_I = d\ell n\, n_I/d\ell n\, n_i$. Marginal stability occurs when the following pair of equations is simultaneously verified:

$$O = 1 + \frac{Z_I n_I}{n_e}\left(b_i\mu_I + \sigma_I\frac{\omega_{*i}}{\omega} - \frac{1}{2\mu_I}\frac{k_\parallel^2 V_{Ti}^2}{\omega^2}\right) + \left(1 - \frac{Z_I n_I}{n_e}\right)\left(1 - \eta_i\frac{\omega\omega_{*i}}{k_\parallel^2 V_{Ti}^2}\Gamma_i\right) \quad (5a)$$

$$O = 1 - \frac{\omega_{*i}}{\omega} + \eta_i\frac{\omega_{*i}}{\omega}\left(\frac{1}{2} + b_i\delta_i - \frac{\omega^2}{k_\parallel^2 V_{Ti}^2}\right) \quad (5b)$$

Solving the first equation in the limit of very small impurity concentration, we obtain:

$$\omega \cong \frac{2}{\eta_i\Gamma_i}\frac{k_\parallel^2 V_{Ti}^2}{\omega_{*i}}\left\{1 + \frac{1}{2}\frac{Z_I n_I}{n_e}\left[1 + b_i\mu_I + \frac{1}{2}\sigma_I\eta_i\Gamma_i\left(\frac{\omega_{*i}}{k_\parallel V_{Ti}}\right)^2 - \frac{1}{8\mu_I}\left(\eta_i\Gamma_i\frac{\omega_{*i}}{k_\parallel V_{Ti}}\right)^2\right]\right\} \quad (6a)$$

$$\omega \cong \pm\frac{1}{2}Z_I k_\parallel V_{Ti}\left(\frac{m_i}{m_I}\frac{n_I}{n_e}\right)^{1/2} \quad (6b)$$

The first root is a frequency shifted (upshift for $\sigma_I > 0$) ion temperature gradient mode (ITG). It is well known that impurities with $\sigma_I > 0$ tend to stabilize it (see, e.g. [7] and [9]). This is by far the most common situation for toroidal plasmas (the exception being a transient state in impurity injection experiments, cf. [10]). Both roots indicated in Eq. (6b) have low frequency $|\omega| << k_\parallel V_{Ti}$, hence they correspond to the same (positive value) of η_i at threshold: $\eta_{ic} \cong 2/(1+2b_i\delta_i)$, which is the critical η_i for the ITG mode in the absence of impurities, in the long parallel wavelength regime ($k_\parallel L_{ni} << k_\perp \rho_i$, cf. [3]). Note that three roots (ITG and two impurity drift modes) can be simultaneously unstable according to this analysis. This can be verified using Nyquist diagram techniques and is also a feature of the full equation (i.e. Eq. (2)

when all species are kinetic). The properties of the two impurity drift waves (propagating in opposite directions) are similar, as opposed to those of the ITG mode. Thus, for simplicity we only follow the one with $\omega \cong \omega_{*e} Z_I n_I / n_e$ in detail.

For larger concentrations of impurities, the dominant terms in Eq. (5a) are the adiabatic responses of the electrons and primary ions ($\cong 2$) and the $\vec{E} \times \vec{B}$ drift of the impurities ($\sigma_I \omega_{*i}/\omega$) so that the relevant solution becomes:

$$\omega \cong -\frac{\sigma_I}{2 - Z_I n_I / n_e} \frac{Z_I n_I}{n_e} \omega_{*i} \tag{7a}$$

namely a low frequency mode that propagates in the electron diamagnetic direction. We have assumed that $b_i \mu_I < 1$, for simplicity. This solution is found in regimes where one finds $\sigma_I > (1/2\ \mu_I)(k_{\|} V_{Ti}/\omega_{*i})^2$ and $(1 - Z_I n_I/n_e)\ \eta_i \Gamma_i\ (\omega_{*i}/k_{\|} V_{Ti})^2 < 1$. The corresponding value of η_i at marginal stability is

$$\eta_i = 2\frac{1 + \sigma_I \hat{n}_I}{1 + 2b_i\delta_i - (\sigma_I \hat{n}_I)^2 (\omega_{*i} / k_{\|} V_{Ti})^2} \tag{7b}$$

where $\hat{n}_I \equiv (Z_I n_I/n_e)/[2 - (Z_I n_I/n_e)]$. Note that for sufficiently large $\sigma_I \hat{n}_I$ the impurity drift mode is unstable at any positive value of the ion temperature gradient parameter. This clearly establishes the existence of an instability threshold in impurity concentration, for fixed ion temperature gradient.

Examples of marginal stability curves for the ITG and impurity drift modes are shown in Fig. 1 for oxygen and carbon impurities, solving Eq. (5) numerically. One clearly sees that the threshold values of η_i for instability of the ion temperature gradient mode are raised dramatically above the values found for pure plasmas, $Z_{eff} = 1$ as was previously noted in [7] for the slab branch of the ITG and in [9] for the toroidal branch. The threshold for the impurity drift mode is consistently lower. There are no qualitative differences between fully ionized impurity species for which $\mu_I = 2$: from Eq. (5), the only nontrivial species dependence is in Z_{eff}. The markedly increased threshold values of η_i, for the ITG instability, are likely to be of relevance to recent observations [11-13], that experiments can operate at varying values of the temperature gradient parameter $\eta_i > 1$ (the nominal threshold value for pure, $Z_{eff} = 1$, plasmas) with no appreciable change in transport properties.

As indicated in Fig. 1, each mode becomes unstable once η_i crosses a threshold value. It is important to realise that once both threshold values are exceeded, both modes are simultaneously unstable. Thus, there exist regimes in which the unstable spectrum is predicted to have two peaks, one with positive $\omega/k_\perp$ (by our convention this would be the ITG mode, propagating in the ω_{*i} direction) and the other with negative $\omega/k_\perp$ (the impurity drift mode). This is in agreement with what is observed, e.g. in the TEXT experiments [14] which first identified an ion feature in the spectrum of microturbulence, where one has $k_\perp\rho_i \cong 0.4$, $k_\| V_{Ti}/\omega_{*i} \cong 0.2$ and $\eta_i \cong 2.5$. From Fig. 1, we see that $Z_I n_I/n_e \geq 0.1$ is sufficient to predict a two peaked spectrum (e.g. a concentration of a few per cent of impurities with $Z = 4$). An alternative explanation, used previously, for this kind of spectrum involves modes driven unstable by two different processes, e.g. an ITG mode (due to $\eta_i > \eta_{ic}$) and a trapped electron mode (due to $\eta_e > 0$ and collisional detrapping).

The preceding remarks are meant as proof-of-principle arguments in favour of an important role of impurity drift mode in plasmas. So far, all calculations have assumed that the impurities are "cold" in the sense that their thermal speed is much lower than the parallel phase velocity of the mode. Note, from Eqs. (2) and (4) that the temperature, T_I, is not the small parameter. Indeed, it cancels out from the impurity drift terms, leaving the FLR, sound term, and $\vec{E} \times \vec{B}$ terms (the first two include the factor of μ_I) in Eq. (4). In realistic cases, the ratio of impurity thermal speed to mode phase velocity is finite and the impurities contribute a finite dissipative term (i.e., Landau damping) to the linear dispersion relation. As we shall see in the next section, this will considerably restrict the part of the wave spectrum, in ($k_\perp\rho_i$ - $k_\| L_n$) space, where these impurity drift modes are unstable. It will also bring in a strong dependence on the ion mass ratio, m_i/m_I [8], and on the charge state, Z_I.

B) Unstable impurity modes - fully kinetic ions

In this section we solve the dispersion relation, Eq. (2), with full kinetic responses for all ion species. We will fix the density gradient scale lengths, $\sigma_I = d\ell n\, n_I/d\ell n\, n_i = Z_{eff} = 1 + (Z_I n_I/n_e)(Z_I - 1)$, to simulate a central peaking of impurities (we have consistently found that $\sigma_I > 1$ is needed for instability of the impurity drift mode). Such a peaking is expected to occur as a result of ion parallel friction, as predicted by fluid theories (e.g. Braginskii equations). We shall also take $T_I = T_i = T_e$, for simplicity, and to model high temperature plasmas. This has the unavoidable disadvantage of bringing the two thermal speeds, V_{TI} and V_{Ti}, closer together, thereby weakening the instability. We find that, given an impurity species (m_I/m_i, Z_I), the instability exists in a rather restricted region of multi-dimensional parameter space (η_i, n_I/n_e, $k_{\|}L_{ni}$, $b_i = k_\perp^2\rho_i^2/2$). This is due to the delicate balance between the mode phase velocity (determined mostly by the parameter $\omega_{*I}Z_I^2n_I/n_e = \sigma_I\omega_{*i}\, Z_In_I/n_e$) and the two thermal speeds. The result of this balance is that the competition between effective inverse Landau damping by the primary ion ("effective" because of the η_i factors multiplying it) and the (stabilizing) Landau damping due to the impurity, determines the overall sign of the mode "growth rate".

As can be expected from these considerations, the unstable spectrum is broader with larger mass ratio, m_I/m_i, and the maximum growth rate is larger (cf. Fig. 2). In this figure, we have taken $\eta_i = 2$ and $k_{\|}L_{ni} \sim 0.1$, which are values appropriate for the middle of the plasma ($r/a \cong 1/2$) in JET ohmic and so-called low-T_i L-mode discharges. We only show the growth of the impurity drift mode here; the standard ITG is also found to be unstable over part of this $k_\perp\rho_i$ spectrum. We present the case of fully ionized oxygen as a case relevant to JET; the curves for Z = 8 chlorine are shown to provide a comparison with a heavier species (i.e. more instability). The unstable spectrum, when it exists, is consistently characterised by modes with parallel phase velocities that are comparable (though somewhat lower) than the thermal speed of the primary ion: $\omega/k_{\|}V_{Ti} \cong -0.4 \to -0.6$ (cf. Fig. 3). This is reminiscent (apart from the minus sign) of the values found for the ITG mode and is not far from the value for which the (inverse) Landau damping of primary ions is maximum $(\omega/k_{\|}V_{Ti})^2 \cong (1/\eta)(2\eta - 1/2 - b_i\delta_i)$ for $\omega_{*i}/k_{\|}V_{Ti} >> 1$. This helps to partially explain why the mode turns off over a large region of parameter space: the primary ions try to drive it unstable at the aforementioned frequency, while the impurities attempt to force an oscillation at their diamagnetic frequency, $\sigma_I\omega_{*i}$. A sufficient mismatch between the two precludes growth. Thus, for example, having set $Z_In_I/n_e = 0.2$, we find that $Z_I = 6$-8 impurities are unstable (for oxygen, $Z_I = 7$ maximises growth) in a hydrogen plasma, but carbon ($m_I/m_i = 12$, $Z_I \leq 6$) is stable. By analyzing the relative effect of terms within the dispersion relation, we have determined that, at each end of the $k_\perp\rho_i$ spectrum of Fig. 2, stabilization occurs due to an increase in Landau damping by the impurity species, accompanied by a minor decrease in inverse Landau damping from the primary. The shift in mode frequency (from the last unstable point) is too small to play a role, when stabilization occurs. Note, cf. Figs. 2-3 that for one case (chlorine, $k_{\|}L_{ni} = 0.05$) an upshift (in the negative direction) of the mode frequency causes a temporary "rebound" in the growth rate. The upshift results as a combination of $\sigma_I \omega_{*i}/k_{\|}V_{Ti}$ which increases as $b_i^{1/2} = k_{\perp}\rho_i/\sqrt{2}$ and a decrease of the primary ion term owing to the decay of the Bessel functions.

Just like the unstable spectrum is finite in $k_{\perp}\rho_i$-space, it also occurs over a finite portion of $k_{\|}L_{ni}$-space ($k_{\|}L_{ni} \lesssim 0.2$ for oxygen) and for a finite range of impurity density concentration. Although given a choice of impurity species and charge state, one can always find a set of plasma and parameters for which the impurity drift mode is unstable, the slab branch of this instability is "fragile": it exists only for a restricted set of parameters. This is due to its inherently kinetic nature (for $T_I \cong T_i$): finite parallel compression of both ion fluids is required for instability (which causes the perturbed pressure to be out of phase with the perturbed electric field, leading to reinforcement of the initial charge separation and thus growth). This parallel compression also engenders the two competing Landau damping contributions and, even more importantly, couples the mode to the ion sound waves ($k_{\|}V_{Ti}$ and $k_{\|}V_{TI}$). When the mode frequency controlled by the $\vec{E} \times \vec{B}$ drift of the impurities, becomes much larger than an ion sound inverse transit time ($k_{\|}V_{Ti}$), the two waves decouple and the pressure perturbation of the primary ion is no longer able to maintain the instability (the same thing happens to the slab branch of the ITG mode when η_i is increased to arbitrary large values [15]: the growth rate is a non-monotonic function of η_i). In this sense, the toroidal branch of this instability, which arises because of perpendicular compression of the ion fluid, $\nabla \cdot \vec{V}_E \neq 0$, will likely be more robust, in the relatively short perpendicular wavelength region, $k_{\perp}\rho_i > k_{\|}L_{ni}$.

III Quasilinear Transport

Given our results from linear theory, namely a weak ($\gamma < |\omega_r|$) kinetic instability, we can expect the unstable spectrum to saturate at moderate amplitudes via a relatively gentle quasilinear process: relaxation of the temperature profile accompanied by a small frequency shift (cf. [15] for weakly unstable ITG modes in a slab geometry). Then, particle and energy will be carried radially by quasilinear fluxes. We follow Manheimer's methodology [16] in deriving these fluxes. From the quasilinear form of Vlasov's equation (the so-called slow-time-scale portion) one takes the first and second moments. Considering the y component (i.e. poloidal) of the first moment equation (i.e. momentum conservation) one quickly finds:

$$\Gamma_j \equiv n_j V_{jx} = \frac{c}{B} < \tilde{n}_j \tilde{E}_y > \tag{8}$$

where x is the radial-like slab coordinate, the bracket indicates an ensemble average over the phases of the linear mode spectrum, and we have assumed that the equilibrium state has no y or z dependence. This is equivalent to saying that particle transport occurs via the radial $\vec{E} \times \vec{B}$ drift induced by the perturbation. When no degeneracy in ($k_\parallel$,$k_\perp$) occurs (i.e. there exists only one linearly unstable mode for a given wave number), one can replace the ensemble average by an average over the ignorable coordinates (y and z in our case). This provides a more physical description of the quasilinear flux as the flux-averaged over a closed surface. Evaluating this expression for electrons ($|\omega| << k_\parallel V_{Te}$) and impurity ions ($b_I < 1$, $\omega_{*I} > |\omega| > k_\parallel V_{TI}$) one obtains:

$$\Gamma_e = -2\sqrt{\pi}\left(\frac{dn_e}{dx} - \frac{n}{2T_e}\frac{dT_e}{dx}\right)\sum_{k>0}\left|\frac{e\tilde{\phi}_k}{T_e}\right|^2\left(\frac{k_y c T_e}{eB}\right)^2\frac{1}{|k_\parallel| V_{Te}} \tag{9a}$$

$$\Gamma_I = -2\sqrt{\pi}\sum_{k>0}\left|\frac{e\tilde{\phi}_k}{T_e}\right|^2\left(\frac{k_y c T_e}{eB}\right)^2\exp\left(-\zeta_{kI}{}^2\right)\frac{1}{|k_\parallel| V_{TI}}\left(\frac{dn_I}{dx} + \frac{n_I}{T_I}\frac{dT_I}{dx}\zeta_{kI}{}^2\right) \tag{9b}$$

where $\zeta_{kI} \equiv \omega_k / k_\parallel V_{TI}$. Obviously, the impurity drift modes result in a particle outflow of the impurities (a self-limiting process for the instability). Note that we do not give an explicit expression for the primary ion flux; that is not necessary. As these perturbations maintain plasma neutrality, $\tilde{n}_i = \tilde{n}_e - Z_I \tilde{n}_I$, then $\Gamma_i = \Gamma_e - Z_I \Gamma_I$ results automatically to verify ambipolarity of the flow. In particular, as $k_\parallel V_{Te} \to \infty$, the electron flux disappears and thus $\Gamma_i = - Z_I \Gamma_I$: primary ions flow inward as the impurities flow out. This is the "mixing mode" process first proposed by Coppi and Spight [17], and later elaborated in [3], using the ITG (ion temperature gradient mode) to control the accumulation of impurities. Note that the slight "inconsistency" in taking an adiabatic electron response in the linear theory and a non-adiabatic response in the expression for quasilinear fluxes arises because all in-phase terms cancel out of Γ_j (and Q_j, the energy flux), thus non-adiabatic electron terms contribute to zeroth order in the quasilinear calculation but represent only small corrections to the linear eigenvalue problem. For cases relevant to JET, $\eta_e \gtrsim 2$, we predict (from Eq. (9a)) an inward flux of electrons, and hence of primary ions. In fact, the inward flow of these ions (the fuel for thermonuclear reactions) would be stronger, as it scales with Z_I.

We compute an effective diffusion coefficient for the impurities, $D_I \equiv -\Gamma_I/(dn_I/dx)$, as follows. We use a mixing length estimate for the saturated mode amplitude: when the mode $\vec{E} \times \vec{B}$ drift in the y-direction exceeds the speed characteristic of the source of free energy (in our case $V_{*T} = V_{Ti}\rho_i/2L_{Ti}$), the growth is stopped. Hence, $|e\tilde{\phi}/T_e| \sim (Z_I n_I/n_e)(1/k_x L_{Ti})$. The impurity concentration multiplier is inserted because the mode growth depends (at least for small concentrations) linearly on this parameter. Since the modes under consideration have relatively short wavelengths, $k_y\rho_i \sim 1$, isotropy in $k_\perp$ space can reasonably be assumed ($k_x \cong k_y \equiv k_\perp$). Then we have:

$$D_I \cong 2\sqrt{\pi}\left(\frac{Z_I n_I}{n_e}\right)^2\left(\frac{cT_e}{eB}\right)^2\left(\frac{\eta_i}{L_{ni}}\right)^2 \frac{e^{-\zeta_{kI}^2}}{|k_\parallel|V_{Ti}}\left(1+\eta_i\,\xi_{kI}^2\right) \qquad (10)$$

Continuing our order of magnitude estimate, we refer to Figs. 2-3 and take $m_I/m_i = 2Z_I = 16$, $Z_I n_I/n_e \sim 0.2$, $k_\parallel L_{ni} \sim 0.1$, $\eta_i \sim 2$, $\zeta_{kI} \sim 2$ and obtain $D_I \sim 2\cdot 10^4 \mathrm{cm}^2/\mathrm{sec}$ for a $T_e = 2$ keV, $B = 3.5$ Tesla, $L_{ni} = 100$cm discharge (note that $D_I \propto T_i^{3/2}B^{-2}L_{ni}^{-2}$, the usual gyro-Bohm scaling). This crude estimate is in the range inferred [18-19] for the q(r) > 1 region of JET (r/a ≥ 0.5) and may represent a minimum effective transport rate for impure plasmas. Note that this diffusion coefficient is one order of magnitude lower than the corresponding quasilinear coefficient inferred for ITG turbulence. Hence an increased threshold for ITG instability (to, e.g., $\eta_i \geq 7$ for $Z_I n_I/n_e = 0.4$, cf. Fig. 1) means that this low level of transport, associated with impurity drift modes, is all that one encounters (in the "anomalous channel") for $\eta_{ic}^{\mathrm{imp}} < \eta_i < \eta_{ic}^{\mathrm{ITG}}$.

Turning our attention to the energy transport, we again use Manheimer's approach [16] and compute the energy flux in the x-direction ("radial") as:

$$Q = \frac{mc}{B}\left\langle \tilde{E}_y \int d^3V \quad \tilde{f}^* \left(\frac{1}{2} V_\parallel^2 + V_\perp^2 \right) \right\rangle \tag{11}$$

This expression (more precise than simply $\vec{E} \times \vec{B}$ convecting the pressure perturbation) is computed by balancing the lowest order terms in the quasilinear energy evolution equation. It contains both the convection of the kinetic energy by $\vec{E} \times \vec{B}$ motion and the cross-field flux of internal energy (per species) $\vec{J} \cdot \vec{E}$. Note that we compute the kinetic energy flux of the plasma $Q = \int d^3V \, V_x (mV^2/2) f(\vec{V})$. As discussed by Stringer [20], this is one of several possible representations for the "energy flux"; each is acceptable provided it is used in the appropriate form of the energy conservation equation. The energy equation which corresponds to Eq. (11) is

$$\frac{3}{2} n \frac{\partial T}{\partial t} + \nabla \cdot \vec{Q} - \vec{J} \cdot \vec{E} = 0 \tag{12}$$

for each species, where $\vec{J} \equiv Ze \int d^3V \, \vec{V} f$. The first term can be evaluated, in the quasilinear approximation and related to the particle flux. For each species, this term is given by the associated particle flux weighted by $(Z\omega/\omega_{*j}) T d\ell n \, n_j/dx$, e.g.

$$\left(\vec{J} \cdot \vec{E} \right)_e \cong -2\sqrt{\pi} \left(T_e \frac{dn_e}{dx} + \frac{n_e}{2} \frac{dTe}{dx} \right) \sum_{k>0} \left| \frac{e\tilde{\phi}_k}{T_e} \right|^2 \left(\frac{k_y c T_e}{eB} \right)^2 \frac{1}{|k_\parallel V_{Te}|} \left(\frac{\omega_k}{\omega_{*e}} \right) \frac{d\ell n \, n_e}{dx} \tag{13}$$

compared to $\nabla \cdot \vec{Q} \sim dQ/dx$, this term is apparently of order $|\omega/\omega_*| (1/k_\perp L_n) \ll 1$ for impurity drift modes.

The expressions for the energy fluxes of the three species in the limits $b_I < b_i \sim 1$, $k_\parallel V_{Te} \gg |\omega|$, $|\omega_{*j}| > |\omega|$ and $k_\parallel V_{Ti} > |\omega| > k_\parallel V_{TI}$ follow:

$$Q_e \cong -2\sqrt{\pi} \left(2T_e \frac{dn_e}{dx} + n_e \frac{dTe}{dx} \right) \sum_{k>0} \left| \frac{e\tilde{\phi}_k}{T_e} \right|^2 \left(\frac{k_y c T_e}{eB} \right)^2 \frac{1}{|k_\parallel| V_{Te}} \tag{14a}$$

$$Q_i \cong -2\sqrt{\pi} \sum_{k>0} \left| \frac{e\tilde{\phi}_k}{T_e} \right|^2 \left(\frac{k_y c T_e}{eB} \right)^2 \frac{\Gamma_0(b_i)}{|k_\parallel| V_{Ti}} \left[2T_i \frac{dn_i}{dx} \left(1 - b_i \delta_i + \zeta_{ki}^2 \right) + n_i \frac{dT_i}{dx} \Delta_{ki} \right] \tag{14b}$$

$$Q_I \cong -2\sqrt{\pi} \sum_{k>0} \left|\frac{e\tilde{\phi}_k}{T_e}\right|^2 \left(\frac{k_y c T_e}{eB}\right)^2 \frac{e^{-\zeta_{kI}^2}}{|k_\parallel| V_{TI}} \zeta_{kI}^2 \left[2T_i \frac{dn_I}{dx} + n_I \frac{dT_I}{dx} \zeta_{kI}^2\right] \qquad (14c)$$

where $\Delta_{ki} \equiv -(1 - 2b_i\delta_i)(2 - b_i)/(1 - b_i\delta_i) + (\zeta_{ki}^2/2)(3 - 2b_i\delta_i)$ is positive in the region $0.8 \le b_i \le 5.0$, namely the region of interest. Thus, we find that these impurity drift modes entail outward energy fluxes for all three species: particle thermal energy is carried out. In contrast, note that inward particle and energy fluxes have been theoretically predicted before for collisionless trapped electron dynamics in ITG turbulence [21-22].

IV Conclusions

The linear stability theory of collisionless drift-type modes in multi-ion plasmas with temperature gradients is analyzed in a shearless slab geometry. Electrostatic perturbations are assumed (i.e. low-β plasmas). It is shown that the presence of one impurity species can cause the appearance of a new instability, co-existent with the well-known ion temperature gradient (ITG) instability, but propagating in the opposite direction, $\omega \sim \omega_{*e}(Z_I n_I/n_e)(d\ell n\, n_I/d\ell n\, n_i)$. This impurity drift mode is driven by the free energy in the primary (not impurity) ion temperature gradient, ∇T_i, and has a threshold lower than the modified threshold for ITG instability. Indeed, the substantial increase in the threshold value (η_i-critical) of the ITG mode by the presence of impurities and the simultaneous instability of waves propagating in the ω_{*e} and ω_{*i} directions (when both threshold conditions are exceeded) fit in naturally with observed trends in toroidal experiments. In particular, we note that experiments [11-13] aimed at verifying the "standard" threshold for ITG instability returned negative verdicts: no qualitative changes in transport occurred for the range $\eta_i = 1 \rightarrow 2$. Since impure plasmas have $\eta_{ic}^{ITG} \ge 3$ for $Z_I n_I/n_e \ge 0.2$, this may provide an answer to this paradox: observed lower rates of transport were due to the impurity drift modes and modified ITG threshold was in fact never reached. We note that there exists an alternative explanation [23] for the failure of standard ITG theory: toroidal ITG may be dominated by trapped ion dynamics which would also raise η_{ic} upward.

When the impurity thermal speed is much lower than that of primary species, the instability tends to be robust (see Section III-a). This case is characterized by impurities contributing three fluid-like terms to the particle balance (quasi-neutrality), the dominant of which is $\vec{V}_E \cdot \nabla n_I$. This case will be of relevance in the outermost plasma region where the cold impurities have been introduced and have not had the opportunity to thermalise. In the main bulk of the plasma, however, one may expect that $T_i \cong T_I$ and thus the impurity response to the perturbation is kinetic (for all but the heaviest, $A_i \gtrsim 50$, impurities). In that case, the same effects that are responsible for the appearance of the instability (which is a variant of the ITG instability and obeys the same physics), namely parallel compression of the primary ion fluid and/or Landau damping, appear also in a stabilizing fashion in the impurity response. In other words, given that the instability appears when the drift wave (ω_{*i}) couples to the ion sound wave ($k_{\|}V_{Ti}$), it also necessarily entails coupling (albeit more weakly) to the impurity sound wave ($k_{\|}V_{TI}$). These considerations explain why, for equal temperatures, the instability resides only in a finite region of multi-dimensional parameter space (η_i, $Z_I n_I/n_e$, $k_{\|}L_{ni}$, $k_{\perp}\rho_i$). We have three main frequency benchmarks: (ω_{*i}, $k_{\|}V_{Ti}$) whose combination determine at what frequency the destabilization from the primary ions is greatest (cf. Section III-b) and $-\ \omega_{*I}(Z_I^2 n_I/n_e)$ which is the impurity drift wave "natural" frequency. Unless these two imposed values for ω are congruent, the mode cannot grow. Furthermore, coupling to the impurity sound mode $\omega \sim k_{\|}V_{TI}$ introduces stabilization due to impurity Landau damping, hence there is a lowest frequency ($\leftrightarrow$ lowest $\omega_* \propto k_{\perp}\rho$) below which the instability disappears.

Quasilinear particle and energy transport have been examined, under the assumption that the saturated spectrum has frequencies and wavelengths of the same order of magnitude as the linearly unstable spectrum. We find an inward electron particle pinch when $\eta_e \geq 2$ and an outward impurity particle flow. The energy fluxes are outward for all three species (as expected, since ∇T_i is the source of free energy for the instability).

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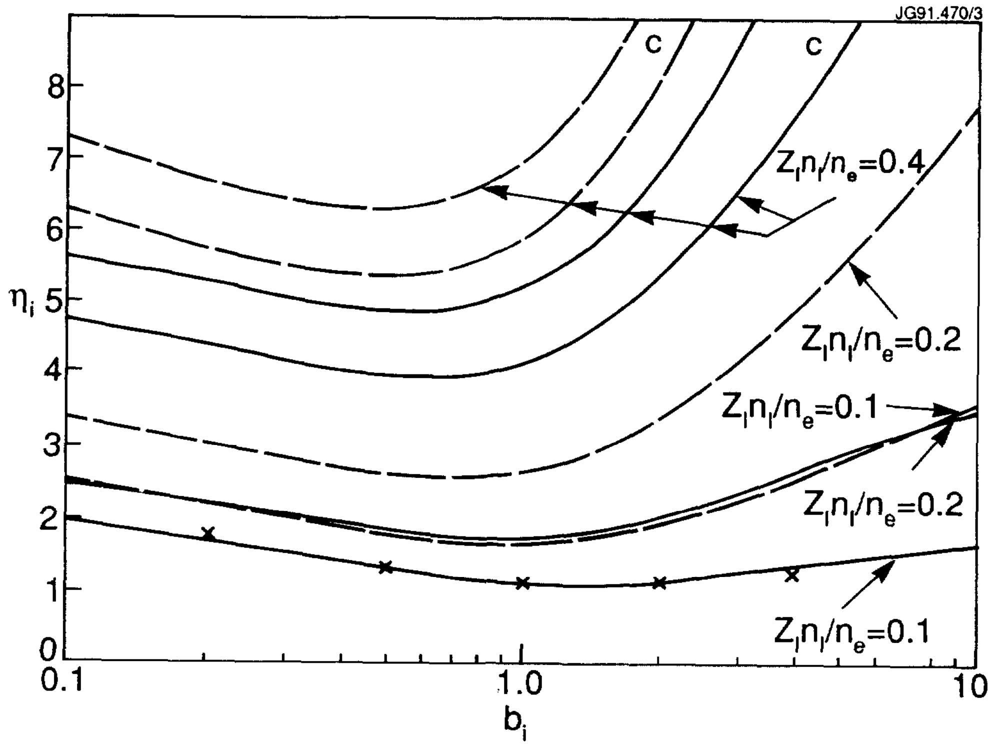

Fig. 1 Marginal stability diagram for ion temperature gradient modes (ITG, dashed lines) and impurity drift modes (solid lines) for the case of fluid impurities. Crosses indicate marginal stability points for a pure plasma ($n_I = 0$, cf. [3]). Here $\eta_i \equiv (d\ell n\, T_i/dx)/(d\ell n\, n_i/dx)$ and $b_i \equiv k_\perp^2 T_i/m_i\Omega_i^2$. Fully ionized impurities with $m_I/m_i = 2Z_I$ (hydrogen = main ion) are considered with $d\ell n\, n_I/d\ell n\, n_i = Z_{eff}$ and $k_{\|}L_{ni} = 0.1$. Impurities with $T_I << T_i = T_e$ are taken as examples. The charge state is $Z_I = 8$ (oxygen) for all curves, except for two (marked "c") where $Z_I = 6$ (carbon).

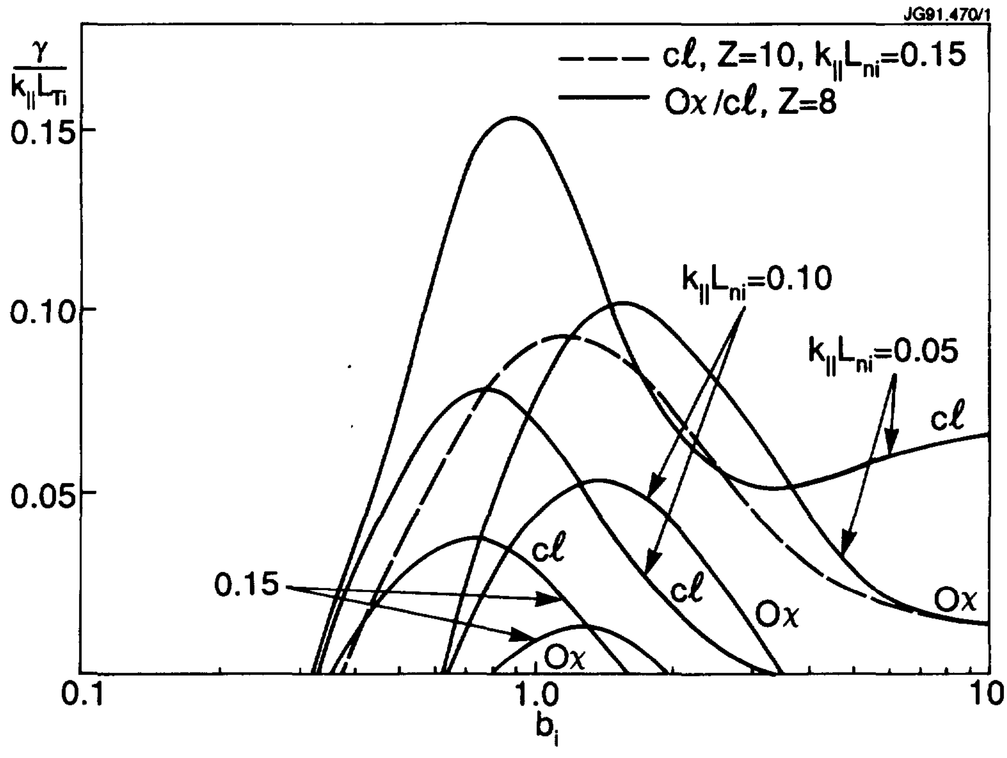

Fig. 2 Growth rates of impurity drift modes for the fully kinetic (primary ions + impurities) case: $T_I = T_i = T_e$. Solid lines show Z = 8 oxygen and (for comparison with a heavier species) chlorine, the dashed line shows chlorine with Z = 10 and $k_{||}L_{ni}$ = 0.15. In all cases we take $\eta_i = 2$, $Z_I n_I/n_e = 0.2$ and $\sigma_I \equiv L_{ni}/L_{nI} = Z_{eff} = 1.6$.

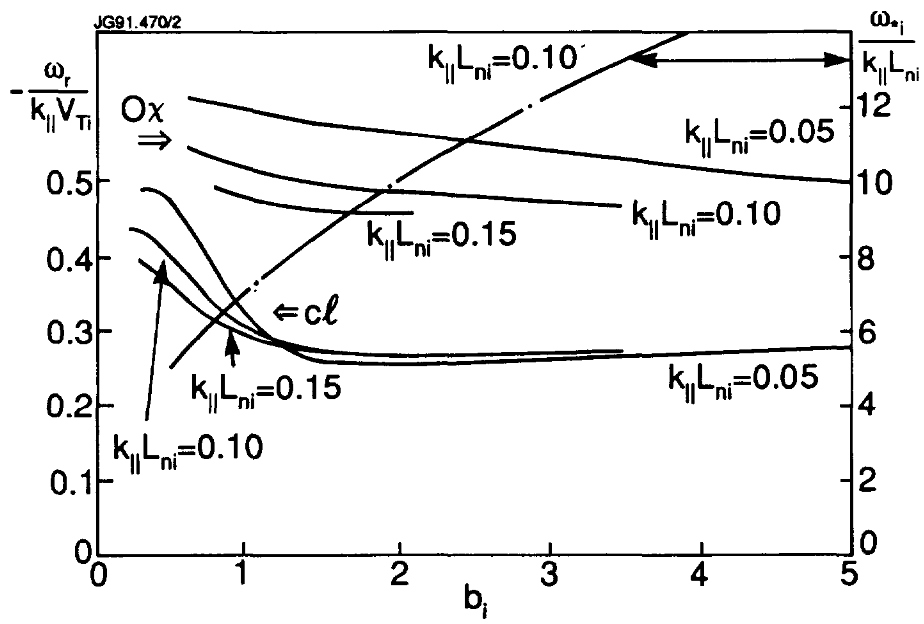

Fig. 3 Parallel phase velocities ($\omega_r \equiv Re(\omega)$) of unstable impurity drift modes for the fully kinetic species, cf. Fig. 2. Also shown for comparison is the normalized primary ion diamagnetic frequency.

Appendix I

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