JET–P(91)04

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Different Forms of the Plasma Energy Conservation Equation

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Preprint of Paper to be submitted for publication in
Plasma Physics

DIFFERENT FORMS OF THE PLASMA ENERGY CONSERVATION EQUATION

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Abstract

The plasma energy flux is frequently equated to the kinetic energy flux. However, the periodic exchange between kinetic and potential energy in the gyratory motion produces a spurious contribution, which is balanced by a potential energy flux. The total energy flux is more meaningful. This is equivalent to the guiding centre energy flux. The convective part of this is (3/2)ΓT, not (5/2)ΓT as for the kinetic energy flux.

1. INTRODUCTION

This study is intended to resolve differences between expressions used for the energy flux in a magnetised plasma. These differences recently led to a dispute on the separation of the energy flux Q into conductive and convective components. Writing

$$\underline{Q} = \underline{q} + \alpha kT\underline{\Gamma} \tag{1}$$

when $\underline{\Gamma} = \langle n\underline{v}\rangle$ is the particle flux, opinions differ on whether $\alpha = 3/2$ or $5/2$.

The total kinetic energy flux for the j^{th} species may be defined in terms of the velocity distribution relative to the laboratory frame.

$$\underline{Q}_{kj} = \int d^3\mathrm{v}\frac{1}{2}m_j\mathrm{v}^2\underline{\mathrm{v}}f_j \tag{2}$$

and the thermal flux in terms of the particle velocity relative to the mean species velocity, $\underline{u}$

$$\underline{q}_j = \int d^3\mathrm{v}\frac{1}{2}m_j(\underline{\mathrm{v}}-\underline{u})^2(\underline{\mathrm{v}}-\underline{u})f_j \tag{3}$$

A straightforward evaluation of the difference between these two integrals gives

$$\underline{Q}_{kj} = \underline{q}_j + \frac{3}{2}kT\underline{\Gamma}_j + \frac{1}{2}m_j n u^2 \underline{u} + m_j \underline{u} \cdot \int d^3v(\underline{v}-\underline{u})(\underline{v}-\underline{u}) f_j \tag{4}$$

The last term is the rate of working by the pressure tensor. For isotropic pressure

$$\underline{Q}_{kj} = \underline{q}_j + \frac{5}{2}kT\underline{\Gamma}_j + \frac{1}{2}nm_j u^2 \underline{u} \tag{5}$$

This is the justification for $\alpha = 5/2$.

Hazeltine and Ware [4], in a rigorous kinetic analysis, derived an energy flux equal to the guiding centre energy flux plus a term $P_\perp(\hat{n} \times \nabla\Phi)/B$. This is consistent with Eq. (5). Ross [6,7] confirms the relationship in Eq. (5). He points out, however, that the full energy balance equation for electrostatic fluctuations contains a term $\int \underline{E}_\perp \cdot j_\perp d^3r$ which cancels part of Q, so that the same result is obtained by choosing $\alpha = 3/2$ and ignoring the work done by the field on the current. (This cancellation, first shown in [4], will be discussed later.) He concludes that we can take either $\alpha = 5/2$ or $3/2$, provided it is used with the appropriate form of energy balance equation, but that $\alpha = 3/2$ is the more convenient for transport simulation codes [7]. In an Appendix to a recent paper, Balescu [1] gives a careful comparison of different forms of the energy and thermal fluxes, pointing out some inconsistencies.

In the transport analysis for TFTR, Zarnstorff et al. [8] recently found that the conductive heat flux derived from the measured total flux, using Eq. (5), is some-times in the same direction as the temperature gradient, giving a negative thermal diffusivity. To avoid this they used $\alpha = 3/2$.

The most frequently used form of the energy conservation equation is the fluid form [2]

$$\frac{\partial}{\partial t}\left(\frac{3}{2}nkT\right) + \nabla \cdot \left(\frac{5}{2}kT\underline{\Gamma} + \underline{q}\right) - \underline{u} \cdot \nabla(nkT) = P_h \tag{6}$$

where P_h is the heat input. This equation may be obtained from the second velocity moment of the kinetic equation, using Eq. (5) and the fluid equation of motion. Düchs [3] points out that the above equation can equally well be written in the mathematically equivalent form

$$\frac{\partial}{\partial t}\left(\frac{3}{2}nkT\right)+\nabla\cdot\left(\frac{3}{2}kT\underline{\Gamma}+\underline{q}\right)+nkT\nabla\cdot\underline{u}=P_h \tag{7}$$

This has the form of an energy conservation equation with a flux q + (3/2)kTΓ. This energy flux, of course, differs from that defined in Eq. (2), and its physical significance is not obvious. In the core region of a large toroidal plasma, the particle source is small (except perhaps when beam injection is used). Then $\nabla.(n\underline{u}) = (\underline{u}.\nabla)n + n\nabla.\underline{u} = 0$. Hence $u.\nabla(nkT) = -(1+\eta)nkT\ \nabla.\underline{u}$, where $\eta = d\ell T/d\ell n\ n$. Since a typical value for η is 2-4, the compressional term in Eq. (6) may be a few times larger than that in Eq. (7). Neither should be omitted [3]. On grounds of physical intuition, and minimisation of computational errors, Düchs favours Eq. (7).

Section 2 examines the simple example of the $\underline{E}\times\underline{B}$ drift of a single charged particle. Since a particle with energy $1/2mv^2$ is convected with velocity E/B the kinetic energy flux of $1/2\ m(v^2 + v_\perp^2)E/B$, is clearly misleading. In fact, the $v_\perp^2$ contribution is cancelled by an opposing potential energy flux. Section 3 expresses the v^2 moment of the Boltzmann equation as a conservation equation for the total energy. When this is transformed to guiding centre variables, it has a simple and physically transparent form.

Since the kinetic energy and total energy fluxes are different, Section 4 considers which corresponds to the energy flux derived from experimental energy analysis codes, and which corresponds to theoretical prediction. In both cases it concludes that the guiding centre total energy flux is usually the appropriate one.

2. IS Q_k A GOOD MEASURE OF ENERGY FLUX?

The kinetic energy flux, defined in Eq. (2), is generally identified with the total energy flux. To illustrate how this can be misleading, consider the simplest example of an ion moving in a uniform magnetic field B along the z-axis, in the presence of an electric field, E = - ∇Φ, in the y-direction. The perpendicular velocity of such an ion varies as

$$v_x = E/B + v_o \cos\zeta$$
$$v_y = -v_o \sin\zeta$$

where ζ = Ωt + α and Ω is the ion cyclotron frequency. The mean kinetic energy flux is

$$Q_{kx} = \frac{1}{2\pi}\oint d\zeta \frac{m}{2}\left(v_o^2 + \frac{E^2}{B^2} + 2v_o\frac{E}{B}\cos\zeta + v_z^2\right)\left(\frac{E}{B} + v_o\cos\zeta\right)$$
$$= \frac{m}{2}\left(2v_o^2 + v_z^2 + \frac{E^2}{B^2}\right)\frac{E}{B} \qquad (8)$$

When extended to an isotropic velocity distribution, this gives the familiar flux, (5/2) $kT\Gamma_x$, assuming $E/B << v_o$.

A single ion has constant total energy, $(v_o^2 + v_z^2 + E^2/B^2)$, and its mean x-velocity is E/B, so Eq. (8) is clearly not the total energy flux. The time variation in $1/2mv^2$, which gives rise to the additional contribution to Eq. (8), results from the conversion between potential and kinetic energy. Since kinetic and potential energies are continually being converted into each other, it is more meaningful to evaluate the total energy flux. For the single ion example

$$Q_{tot} = \frac{1}{2\pi}\oint d\zeta\left(\frac{1}{2}mv^2 + e\Phi\right)v_x$$
$$= \frac{m}{2}\left(v_o^2 + v_z^2 + \frac{E^2}{B^2}\right)\frac{E}{B}$$

since $mv^2/2 + e\Phi = v_o^2 + v_z^2 + E^2/B^2$ is constant during the motion. This agrees with physical intuition. Thus the extra flux, $(mv_o^2/2)E/B$, in the kinetic energy flux is exactly balanced by an opposite flux of potential energy.

An analogous system is a pendulum whose kinetic energy is large enough to produce monotonic rotation about the fixed centre. Evaluation of the kinetic energy flux, $\oint dt\ mv^2\underline{v}/2\tau$, gives a mean kinetic energy flow $mg\ a^2/2\tau$ in a direction perpendicular to gravity, where m is the mass, a the pendulum length, and τ its period. This flow has no physical significance. It is balanced by an equal and opposite flow of potential energy.

3. A TOTAL ENERGY BALANCE EQUATION

We will now express the energy conservation equation in terms of kinetic and potential energy. Starting from the kinetic equation for the j^{th} species, its $1/2\ mv^2$ moment is

$$\frac{m_j}{2}\int d^3v v^2\left[\frac{\partial f_j}{\partial t}+(\underline{v}\cdot\nabla)f_j+\frac{e_j}{m_j}(\underline{E}+\underline{v}\times\underline{B})\cdot\frac{\partial f_j}{\partial \underline{v}}\right]=\frac{m_j}{2}\int d^3v v^2\left(\frac{\partial f}{\partial t}\right)_c+P_{ad} \quad (9)$$

where $(\partial f/\partial t)_c$ is the collision operator and P_{ad} is the additional heating. After integrating by parts this may be written

$$\frac{\partial}{\partial t}\left[\int d^3v\frac{1}{2}m_j v^2 f_j\right]+\nabla\cdot\left[\int d^3v\frac{1}{2}m_j v^2\underline{v}f_j\right]-e_j\underline{E}\cdot\int d^3v\underline{v}f_j=P_{jk}+P_{ad} \quad (10)$$

where P_{jk} is the equipartition with other species.

After separating the electric field into an electrostatic part, $-\nabla\Phi$, and an externally induced part, $\underline{E}_A$, Eq. (10) may be expressed as a conservation equation for the total energy. (See the Appendix for details.)

$$\frac{\partial}{\partial t}\left[\int d^3v\left[\frac{1}{2}m_j v^2+e_j\Phi\right]f_j\right]+\nabla\cdot\left[\int d^3v\left[\frac{1}{2}m_j v^2+e_j\Phi\right]\underline{v}f_j\right]$$

$$= P_{jk} + n_j e_j \underline{E}_A \cdot \underline{u}_j + n_j e_j \frac{\partial \Phi}{\partial t} + P_{ad} \quad (11)$$

The physical significance of Eq. (11) is obvious. The change in the total energy, i.e. the sum of the particle kinetic and potential energies, results from the total energy flux, energy exchange with other species, the increase in potential energy, ohmic heating by the induced electric field, plus additional heating. When Eq. (10) is interpreted as a fluid equation, the last term on the left appears as the work done by the electric field on the plasma flow. As may be seen from Eq. (11), that part which results from electrostatic field can be re-expressed as the flux in potential energy.

Equation (11) can be expressed in terms of the guiding centre velocity $\underline{V}_\perp$, where

$$\underline{v}_\perp = \underline{V}_\perp + v_o \underline{e}_\perp, \quad and \quad \underline{e}_\perp = (\cos\zeta, \sin\zeta, 0).$$

As shown in the Appendix, it then takes the form

$$\frac{\partial}{\partial t}\left[n_j\left(\frac{3}{2}kT_j + \frac{1}{2}m_j V_\perp^2\right)\right] + \nabla \cdot \left[n_j\left(\frac{3}{2}kT_j + \frac{1}{2}m_j V_\perp^2\right)\underline{V}_\perp\right]$$
$$= P_{jk} + n_j e_j \underline{u}_j \cdot \underline{E}_A - n_j e_j \underline{V}_\perp \cdot \nabla\Phi + P_{ad}. \quad (12)$$

In the present model, $\underline{V}_\perp = -\nabla\Phi\times\underline{B}/B^2$, and hence $\underline{V}_\perp.\nabla\Phi$ vanishes, leaving the ohmic, $n_j e_j \underline{u}_j.\underline{E}_A$, and additional heating terms. Equation (12) is equivalent to Eq. (29) in Ref. [7].

This analysis does not distinguish between the particle density and the guiding centre density. These differ by a term of order $(\rho_j/r_n)^2$, where ρ_j is the Larmor radius and r_n the density scale length. The guiding centre kinetic energy is usually negligible compared with thermal. Thus if $V_\perp$ is of order the diamagnetic drift, then $m_j V_\perp^2 / kT_j \sim (\rho_j / r_n)^2$.

We now consider the relationship between this guiding centre energy equation and the fluid-type equations in Eqs. (6) and (7). Although the total species energy is the most natural quantity for the guiding centre description, it is not so for the fluid description. The key difference is that the total particle energy can be expressed in guiding centre variables in the simple form, (see Eq. (A2) in the Appendix)

$$\frac{1}{2}m_j v^2 + e_j\Phi = \frac{1}{2}m_j\left(V_\perp^2 + v_o^2 + v_z^2\right) + e_j\Phi_{GC}. \tag{13}$$

The fast variation in v^2, due to the gyration, gives rise to a kinetic energy flux $nkT(\underline{E}\times\underline{B})/B^2$, as shown in Eq. (8). The fast variation in Φ, seen by the gyrating particle, contributes to the potential energy flux,

$$\begin{aligned}\int d^3v e_j \Phi \underline{v} f_j &= \frac{e_j}{2\pi}\int dv_z dv^2 \oint d\zeta\left[\Phi_{GC} + \frac{v_o}{\Omega B}\nabla\Phi\cdot\underline{B}\times\underline{e}_\perp\right]\left(\underline{V}_\perp + v_o\underline{e}_\perp\right) \\ &= ne_j\Phi_{GC}\underline{V}_\perp - nkT\frac{\underline{E}\times\underline{B}}{B^2}\end{aligned} \tag{14}$$

In the guiding centre model the fast variation cancels out in the total energy. In the fluid description, however, the guiding centre potential has no role, and it loses the simplifying relation, Eq. (13). Now the fast variation in v^2 contributes to the kinetic energy flux. The balancing potential energy flux reappears in the term $-e_j\,\underline{E}\cdot\underline{u}_j$ in Eq. (10). After combining with the bulk kinetic energy, this gives rise to the $\underline{u}\cdot\nabla p$ term in Eq. (6). This is the physical origin of the cancellation between $e_j\,\underline{E}\cdot\underline{u}_j$ and the divergence of the additional energy flux, $n\underline{u}kT$, pointed out by Ross [6,7].

The alternative form of fluid energy equation in Eq. (7), favoured by Düchs [3], is closest to the guiding centre energy equation. We can now see that the flux, $3/2\ kT\Gamma + q$, which enters Eq. (7) corresponds approximately to the total energy flux. (An exact equivalence cannot be expected, because the thermal flux is relative to the mean fluid velocity in one case, and the mean guiding centre velocity in the other.)

4. MOST SUITABLE FORM OF ENERGY FLUX

The guiding centre energy flux due to fluctuating fields, when averaged over a flux surface, can be split into convective and conductive components as follows

$$\left\langle \frac{3}{2} n_j k T_j \underline{V}_j \right\rangle = \frac{3}{2} \left\langle \tilde{n}_j \underline{\tilde{V}}_j \right\rangle k \overline{T}_j + \frac{3}{2} \overline{n}_j \left\langle k \tilde{T}_j \underline{\tilde{V}} \right\rangle$$

$$= \frac{3}{2} k \overline{T}_j \underline{\Gamma}_j + \underline{q}_j$$

where angular brackets or an overbar denote an average over a flux surface, and a tilde denotes the variation over the surface.

The separation of the kinetic energy flux into convective and conductive components is useful if it is easier to separately predict the conductive heat flux and the particle flux. However, this is not generally true. Thus neoclassical analysis evaluates the guiding centre drift of the total energy. For example, Hinton and Rosenbluth [5] evaluate

$$Q_j = \frac{1}{S} \int dS \left(\epsilon - e_j \overline{\Phi} \right) f_j \underline{V}_\perp \cdot \underline{e}_r \qquad (15)$$

where ϵ is the total particle energy, $\underline{V}_\perp$ is the guiding centre velocity, and $\overline{\Phi}$ is the flux surface averaged potential. (The effect of a mean electrostatic potential is excluded from Q_j, since it is already taken account of in the $n_j e_j \underline{V}_\perp \cdot \nabla \overline{\Phi}$ term in Eq. (12)). Thus the neoclassical flux can be substituted directly for the total energy flux in Eq. (14). The energy flux resulting from drift instabilities is generally estimated from an integral of the type $< \int d^3v \frac{1}{2} m_j v_o^2 (\tilde{E}_\theta \tilde{f}_j / B)$. This again corresponds to the total energy flux.

Thus when comparing predicted and experimental heat fluxes, the total heat fluxes should generally be used. The experimental flux can be derived from the guiding centre energy equation, Eq. (12) (neglecting the $1/2\ m_j\ V_\perp^2$ bulk energy), or approximately from the second fluid energy equation, Eq. (7). Since experimentally the particle convection and heat flux may be individually controlled, by adjusting the sources, there may be advantages in separating the total flux into convective and conductive components.

5. CONCLUSIONS

1. The quantity of most physical interest is the total (kinetic and potential) energy flux. Most theoretical estimates of the transport fluxes, in particular those based on guiding centre equations, are for the total energy flux. Hence comparison with these theories should use the experimental total energy flux.
2. For the cases considered in this paper, using the guiding centre model the total energy flux, when expressed as a sum of conductive and convective components, is $q_j + (3/2)\, kT_j\Gamma_j$. The kinetic energy flux is $q_j + (5/2)\, kT_j\Gamma_j$.
3. The three forms of energy conservation equation considered here are all mathematically correct. The guiding centre form is physically obvious, easy to use, and gives the total energy flux.
4. The first form of the fluid energy equation, which is frequently used in interpretive energy analysis codes, gives the kinetic energy flux if the $\underline{u}_j \cdot \nabla p_j$ term is properly evaluated.
5. Comparing the second form of the fluid energy equation with the guiding centre equation shows that the energy flux derived from this equation corresponds closely to the total energy flux.

ACKNOWLEDGEMENTS

I am grateful to A. Ware for raising this question and for valuable comments, and to D. Ross for helpfully pointing out several weaknesses in an earlier version. I also thank D.F. Düchs for useful discussions, and F. Tibone and P. Thomas for helpful suggestions.

REFERENCES

[1] BALESCU, R., (1990), Phys. Fluids B2, 2100.

[2] BRAGINSKII, S.I., (1965), Transport Process in Plasmas, in Reviews of Plasma Physics, ed. M.A. Leontovich, Vol. 9, pp 205-309, Consultants Bureau, N.Y.

[3] DÜCHS, D.F., (1989), "3/2 or 5/2 for Convective Thermal Transport?", JET-R(89)13.

[4] HAZELTINE, R.D. and WARE, A.A., (1976), Phys. Fluids 19, 1163.

[5] HINTON, F.L. and ROSENBLUTH, M.N., (1973), Phys. Fluids 16, 836.

[6] ROSS, D.W., (1989), Comments Plasma Phys. Cont. Fusion 12, 155.

[7] ROSS, D.W., (1990), "On Standard Forms for Transport Equations and Fluxes - Part II", Univ. of Texas Report FRCR # 357.

[8] ZARNSTORFF, M. et al., (1988), 12th Int. Conf. on Plasma Phys. and CNFR (Nice) Vol. I, 183.

APPENDIX

Derivation of a Guiding Centre Energy Equation

In Eq. (10) we make the following substitutions

$$\underline{E} = -\nabla\Phi + \underline{E}_A$$

$$\nabla\Phi \cdot \int d^3 v \underline{v} f_j = \nabla \cdot \left[\Phi \int d^3 v \underline{v} f_j\right] - \Phi \int d^3 v (\underline{v} \cdot \nabla) f_j$$

The last term can be simplified to

$$\int d^3 v (\underline{v} \cdot \nabla) f_j = -\int d^3 v \left[\frac{\partial f_j}{\partial t} + (\underline{E} + \underline{v} \times \underline{B}) \cdot \frac{\partial f_j}{\partial \underline{v}} - \left(\frac{\partial f_j}{\partial t}\right)_c\right] = -\int d^3 v \frac{\partial f}{\partial t}$$

The integral of the collision term must vanish, since collisions do not change the particle density. Equation (10) can then be written as

$$\frac{\partial}{\partial t}\left[\int d^3 v \left(\frac{1}{2} m_j v^2 + e_j \Phi\right) f_j\right] + \nabla \cdot \left[\int d^3 v \left(\frac{1}{2} m_j v^2 + e_j \Phi\right) \underline{v} f_j\right]$$

$$= P_{jk} + n_j e_j \underline{E}_A \cdot \underline{u}_j + P_{ad} + n_j e_j \frac{\partial \Phi}{\partial t} \qquad \text{(A1)}$$

We now transform this into guiding centre variables, by expressing the particle perpendicular velocity as a gyration superimposed on a mean drift

$$\underline{v}_\perp = \underline{V}_\perp + v_o \underline{e}_\perp, \quad \textit{where} \quad \underline{e}_\perp = (\cos\zeta, \sin\zeta, 0) \quad \textit{and} \quad \zeta = \Omega t + \alpha$$

The variation in potential energy is - $\underline{r} \cdot \underline{E}$ where the particle position $\underline{r}$ is given by

$$\underline{r} = \underline{r}_o + \int \underline{V}_\perp dt + \frac{v_o}{\Omega B} \underline{B} \times \underline{e}_\perp$$

The particle energy is then

$$\frac{1}{2} m_j v^2 + e_j \Phi = \frac{1}{2} m_j \left(V_\perp^2 + v_o^2 + v_z^2\right) + m_j v_o \left(\underline{V}_\perp - \frac{\underline{E} \times \underline{B}}{B^2}\right) \cdot \underline{e}_\perp + e_j \Phi_{GC} \qquad \text{(A2)}$$

where

$$\Phi_{GC} = \Phi(r_o) - \underline{E} \cdot \int \underline{V}_\perp dt$$

is the potential at the particle guiding centre.

To obtain an energy balance equation comparable to the fluid equations, the particle energy, integrated over the velocity distribution, can be written in terms of a temperature.

$$\int dv_z dv_o^2 \oint d\zeta \left(\frac{1}{2} m_j v^2 + e_j \Phi\right) f_j = \frac{1}{2} n_j m_j V_\perp^2 + \frac{3}{2} nkT + n_j e_j \Phi_{GC} \qquad \text{(A3)}$$

since $\oint d\zeta \underline{e}_\perp = 0$. The velocity integral is over all particles whose guiding centres lie within an element of real space around the point $\underline{r}$. Thus $\Phi_{GC} \equiv \Phi(r)$. Strictly speaking, in Eq. (A3) n is the guiding centre density while $kT = \overline{m(v_o^2 + v_z^2)/3}$ is the thermal energy of particles whose guiding centres are at $\underline{r}$. To avoid the considerable analytic detail involved in retaining finite Larmor radius (FLR) effects in the guiding centre and fluid equations (which are not essential for the present problem), such terms will be neglected. Thus we do not distinguish between particle and guiding centre densities.

The total energy flux is

$$Q_{tot} = \int dv_z dv_o^2 \oint d\zeta \left[\frac{1}{2} m_j \left(V_\perp^2 + v_o^2 + v_z^2\right) + m_j v_o \left(\underline{V}_\perp - \frac{\underline{E} \times \underline{B}}{B^2}\right) \cdot \underline{e}_\perp + e_j \Phi\right] \left(\underline{V}_\perp + v_o \underline{e}_\perp\right) f_j$$
$$= n_j \left[\frac{1}{2} m_j V_\perp^2 + \frac{3}{2} kT + e_j \Phi\right] \underline{V}_\perp + nkT \left(\underline{V}_\perp - \frac{\underline{E} \times \underline{B}}{B^2}\right)$$

The last term vanishes, since the guiding centre drift is wholly due to the electric field. The total energy flux thus equals the energy density multiplied by the guiding centre velocity. When expressed in terms of the guiding centre velocity $V_\perp$, Eq. (A1) therefore becomes

$$\frac{\partial}{\partial t}\left[n_j \left(\frac{3}{2} kT_j + \frac{1}{2} m_j V_\perp^2 + e_j \Phi\right)\right] + \nabla \cdot \left[n_j \left(\frac{3}{2} kT_j + \frac{1}{2} m_j V_\perp^2 + e_j \Phi\right) \underline{V}_\perp\right]$$

$$= P_{jk} + n_j e_j \underline{E}_A \cdot \underline{u}_j + P_{ad} + n_j e_j \frac{\partial \Phi}{\partial t}.$$

Because the $O(\rho_j/r_n)^2$ difference between guiding centre and particle density is neglected, the divergence operator pruces the same results in either description. By invoking the continuity equation for guiding centres, this may be rewritten as Eq. (12).

APPENDIX 1.

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CR 88.49.1A