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** See Appendix 1*

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Abstract

It is necessary to operate a divertor with a sufficiently high plasma density at the symmetry point n_s, i.e., adjacent to the main plasma, to satisfy the Lawson criterion and, at the same time, a low enough value of the plasma temperature at the target plate T_t, to minimize sputtering. The possibilities for realizing these two objectives simultaneously are examined. It is found that the combination of high n_s and low T_t can require that the plasma flow be supersonic before reaching the divertor sheath. It is not known if such plasma conditions are stable.

Basic Equations

The basic divertor equations have been considered by various authors, see for example K. Lackner et al.[1] For purposes of illustrating the basic features of subsonic/supersonic divertor solutions, a simplified treatment is considered here. The plasma density n(y), flow velocity v(y), and temperature T(y), where $T_e = T_i$ here, as functions of the distance y measured along $\vec{B}$ from the plate are given by the solutions to the three coupled equations:

$$nv - n_t v_t = \begin{cases} -M_t n_t c_{st} y/L_i & \text{for } y < L_i \\ -n_t v_t & \text{for } y > L_i \end{cases} \tag{1}$$

$$p_s \equiv n(2kT + mv^2) \tag{2}$$

$$T(y)^{7/2} = \begin{cases} T_t^{7/2} + \frac{7(P-P_r)}{2\kappa_o A} y + \frac{7}{4\kappa_o A}\left(\frac{P_r}{L_r} - \frac{P}{L}\right) y^2 & \text{for } y < L_r \\ T_t^{7/2} - \frac{7P_r L_r}{4\kappa_o A} + \frac{7P}{2\kappa_o A} y - \frac{7P}{4\kappa_o AL} y^2 & \text{for } y > L_r \end{cases} \quad (3)$$

where $v_t = M_t c_{st}$

M_t = flow Mach number at the target

$c_{st} = (2kT_t/m)^{1/2}$, the acoustic speed at the target

m = ion mass

L_i = distance from the plate over which the (re-cycle) ionization is assumed to occur uniformly

p_s = total pressure at the symmetry point

P = total power into the Scrape-Off Layer, SOL, which is assumed to occur uniformly over its total length L

P_r = total power radiated from the SOL, assumed to occur uniformly over distance L_r from the plate

A = area of SOL flux tube measured perpendicular to $\vec{B}$

κ_o = electron parallel heat conduction coefficient ($q_\parallel = -\kappa_o T^{5/2}$ dT/dy)

Heat transport along $\vec{B}$ has been simplified by neglecting both convection and the heat flux limit of conduction.

One has the boundary condition at the plate for the heat convected through the sheath, which gives

$$c_{st} = -\frac{2(1 + M_t^2)(P - P_r)}{\gamma M_t A p_s} \quad (4)$$

where γ = sheath heat transmission coefficient.

A solution for n(y), v(y), T(y) is obtained when P, P_r, p_s, L_r, L_i, A, κ_o, γ are specified.

Equations (1) and (2) combine to give, for $y < L_s$,

$$M(y) = \frac{(1 + M_t^2)}{2M_t(1 - y/L_i)} \pm \left[\left\{\frac{(1 + M_t^2)}{2M_t(1 - y/L_i)}\right\}^2 - \frac{T(y)}{T_t}\right]^{1/2} \qquad (5)$$

where $M \equiv v/c_{st}$ (note that this then is the Mach number defined by the target sound speed which differs from the local sound speed).

Differentiation of Eq. (5) gives, for $y < L_s$,

$$\frac{M'}{M} = \frac{T'/T_t + (M^2 - T/T_t)^2 \, (M_t/M)/\left(L_i(1 + M_t^2)\right)}{T/T_t - M^2} \qquad (6)$$

The Bohm criterion for stable sheath existence requires that $M_t \leq -1$. The solutions with $M_t = -1$ are termed 'subsonic' here, those with $M_t < -1$, 'supersonic'.

Subsonic Solutions

Setting $M_t = -1$, one may note that when the quantity in the square root of Eq. (5) is negative, no solution is possible. This sets a lower limit to the permitted value of T_t:

$$T_t^{min} = \left((P - P_r)L_i/(2\kappa_o A)\right)^{2/7} \qquad (7a)$$

and an upper limit to the permitted value of p_s using Eq. (4):

$$p_s^{max} = (8m)^{1/2}(P-P_r)/(A\gamma\left((P-P_r)L_i/2\kappa_o A\right)^{1/7}) \qquad (7b)$$

where Eqs. (7) were obtained by expanding $(1-y/L_i)^2$ and $T(y)/T_t$ in the square root expression of Eq. (5) for small y. For all $p_s < p_s^{max}$ one finds the usual fluid result that at the sheath edge $M_t' = \infty$. When $p_s = p_s^{max}$, however, M_t' becomes finite. Then if values of $p_s > p_s^{max}$ are imposed (together with the $M_t = -1$ boundary condition) the point where the local Mach number reaches −1 moves to positive y-values, with imaginary values for smaller y, as already mentioned. This effect is illustrated in Fig. 1 for the example of $P = 2\times10^7$ W, $P_r = 10^7$ W, $A = 0.2\ m^2$, L = 40 m, $L_i = 1$ m, $L_r = 0.4$ m, $\kappa_o = 6.1\times10^{68}\ [WJ^{-7/2}]$ (i.e., $Z_{eff} = 2$), $\gamma = 10$, D^+ ions. For these conditions Eq. (7b) gives $p_s^{max} = 480.85\ N/m^2$. As can be seen from Fig. 1, for $p_s \lesssim 300\ N/m^2$ the M(y) profiles are virtually constant. As p_s increases above $\approx 300\ N/m^2$, however, the M(y) profile near the plate begins to change rapidly, particularly as $p_s \to p_s^{max}$. At $p_s = p_s^{max}$ the profile abruptly loses the property that $M_t' = \infty$. For $p_s > p_s^{max}$, $M \to -1$ for $y > 0$ with no real values of M occurring for smaller y.

One may note from Eq. (7b) that for strong radiation, $P_r \to P$, one finds $p_s^{max} \to 0$, i.e., strong radiation, and low T_t values require very low densities for subsonic solutions.

Table 1 gives solutions for Case A with $P = 40$ MW, $L = 30$ m, $A = 0.04\ m^2$, $L_r = 5$ m, $L_i = 1$ m, $\kappa_o = 6.1\times10^{68}\ WJ^{-7/2}$, $\gamma = 10$ and Case B which is the same except $A = 0.5\ m^2$, $L_i = L_r = 3$ m. The results shown are for the extreme cases of subsonic flow where $p_s = p_s^{max}$, $T_t = T_t^{min}$, thus also $n_s = n_s^{max}$.

Table 1. Subsonic solution results for the extreme situation of $p_s = p_s^{max}$.
Cases A and B defined in text.

Case	P_r [MW]	p_s^{max} [N/m^2]	T_t [eV]	T_s [eV]	n_t [m^{-3}]	n_s [m^{-3}]
A	0	6270	42.5	161	2.3×10^{20}	1.22×10^{20}
A	20	3460	34.9	160	1.6×10^{20}	6.9×10^{19}
A	39	270	14.2	160	3.0×10^{19}	5.3×10^{18}
B	0	615	28.3	78	6.8×10^{19}	5.0×10^{19}
B	20	340	23.2	77	4.6×10^{19}	2.8×10^{19}
B	39	26	9.9	76	8.3×10^{18}	2.2×10^{18}

As may be noted, the combination of high radiated power (and low T_t) together with high n_s is not compatible with a subsonic solution. One may also note that T_t is not a very strongly decreasing function of increasing P_r/P: as P_r/P increases from 0 to 0.975, T_t only decreases by a factor of ~3. The reason that radiative cooling is so relatively ineffective is that as P_r/P increases, n_t is forced to decrease substantially for a subsonic solution, preventing a strong decrease in T_t.

Supersonic Solutions

Subsonic plasma flows to surfaces are experimentally widely observed. There do not appear to be experimental observations reported of supersonic flows to surfaces and it is not known whether such solutions would be stable. If a smooth transition to supersonic flow is to occur then the numerator and the denominator of Eq. (6) must pass through zero simultaneously.

It can be shown that this will occur when y equals y_1, where

$$y_1 = -\alpha + (\alpha^2 + \beta)^{1/2} \tag{8}$$

where $\alpha \equiv 4a/9b - L_i/9$

$\beta \equiv (aL_i - 7T_t^{7/2})/9b$

$a \equiv 7(P - P_r)/(2\kappa_o A)$

$b \equiv \left(7/(4\kappa_o A)\right)(P_r/L_r - P/L)$

The value of M_t is then given by

$$-M_t = \left(T(y_1)/T_t\right)^{1/2}\left(1 - y_1/L_i\right) - \left[\left(T(y_1)/T_t\right)\left(1 - y_1/L_i\right)^2 - 1\right]^{1/2} \tag{9}$$

Supersonic solutions can exist up to arbitrarily is large values of p_s. For illustration consider P = 40 MW, P_r = 39 MW, L = 30m, A = 0.04 m^2, L_r = 5m, L_i = 1m; results are given in Table 2.

Table 2. Supersonic solutions.
Case P = 40 MW, P_r = 39 MW, L = 30m, A = 0.04 m^2, L_r = 5m, L_i = 1m.

p_s [N/m^2]	T_t [eV]	M_t	y_1 [m]	$T(y_1)$ [eV]	n_t [m^{-3}]	T_s [eV]	n_s [m^{-3}]
364	10	-1.65	0.136	17.0	3.06×10^{19}	153	7.4×10^{18}
714	5	-2.76	0.159	17.3	5.17×10^{19}	153	1.5×10^{19}
1780	2	-4.72	0.161	17.3	1.2×10^{20}	153	3.6×10^{19}
3560	1	-6.83	0.161	17.3	2.3×10^{20}	153	7.3×10^{19}

The achievement of a high density, $n_s \gtrsim 5\times10^{19}$ m^{-3}, highly radiative, $P_r/P \gtrsim 98\%$, low $T_t \sim 1$ eV, divertor plasma requires a stable, highly supersonic flow for these input assumptions.

Relation to Other Studies

Chodura[2] has noted the possibililty of supersonic solutions in the presence of finite heat conductivity along $\vec{B}$. Neuhauser et al[3] has noted that with regard to numerical solutions of divertor cases "... supersonic flow near the target is frequently obtained at low divertor temperature". Harbour and Morgan[4] obtained some supersonic cases in their numerical analysis for NET, but imposed the boundary condition at the sheath that $|M_t| = 1$ for all cases, which may explain why only slightly supersonic velocities were obtained. There does not appear to be any previous quantitative examination of the limits for subsonic and supersonic divertor solutions.

It is worth noting that the possibility of a n_s-limit, which here would be putatively associated with the subsonic-to-supersonic transition in solution type, is not to be confused with possible density limits[1,3,5] associated with high re-cycling or impurity radiation losses causing P_r to rise as T_t decreases. The foregoing analysis cannot accommodate such an effect since P_r/P is fixed. As shown in the last section, when T_t is lowered without limit, n_s increases without limit. The n_s-limit of Lackner et al can be readily obtained with a small change to the present formulation if one replaces the assumption of a fixed P_r, with their assumption that

$$P_r = -\varepsilon n_t M_t c_{st} A \tag{10}$$

where ε = non-sheath energy loss per recycling ion. Thus Eq. (4) is replaced by

$$c_{st} = -\frac{2P(1 + M_t^2)}{\gamma M_t A p_s (1 + \varepsilon/\gamma k T_t)} \qquad (11)$$

Results are shown in Fig. 2 for P = 40 MW, γ = 10, L = 30 m, L_i = 1 m, A = 0.4 m^2 and ε is taken from an approximation by Harrison et al[6]

$$\varepsilon = 17.5 + \left(5 + \frac{37.5}{T_t}\right) \log_{10}\left(\frac{10^{21}}{n_t}\right) \qquad (12)$$

with ε and T in eV and n_t in m^{-3}. An n_s-limit occurs around $T_t \simeq 2$ eV, giving a maximum $n_s \simeq 10^{20}$ m^{-3}. The subsonic-to-supersonic transition in solution type occurs at a substantially higher $T_t \simeq 22$ eV, and lower n_s, $\simeq 3\times10^{19}$ m^{-3} and thus, if this transition does in fact constitute a limit to stable solutions, it would set lower n_s and higher T_t limits.

One may note that it is not possible to radiate all the power away, given the assumptions of the present analysis. McCracken has discussed this in a recent analysis,[7] which includes both impurity radiation and hydrogen re-cycle energy losses, i.e., above a certain density it is not possible to obtain energy balance. If supersonic divertor solutions are not stable then achievable values of P_r/P may be further reduced, e.g., for the case shown in Fig. 2, P_r/P would be constrained below $\simeq 0.1$.

Conclusion

Achievement of a divertor plasma which combines high density at the symmetry point together with low temperature at the target plate can require highly supersonic plasma flow. It is not known whether such solutions are stable.

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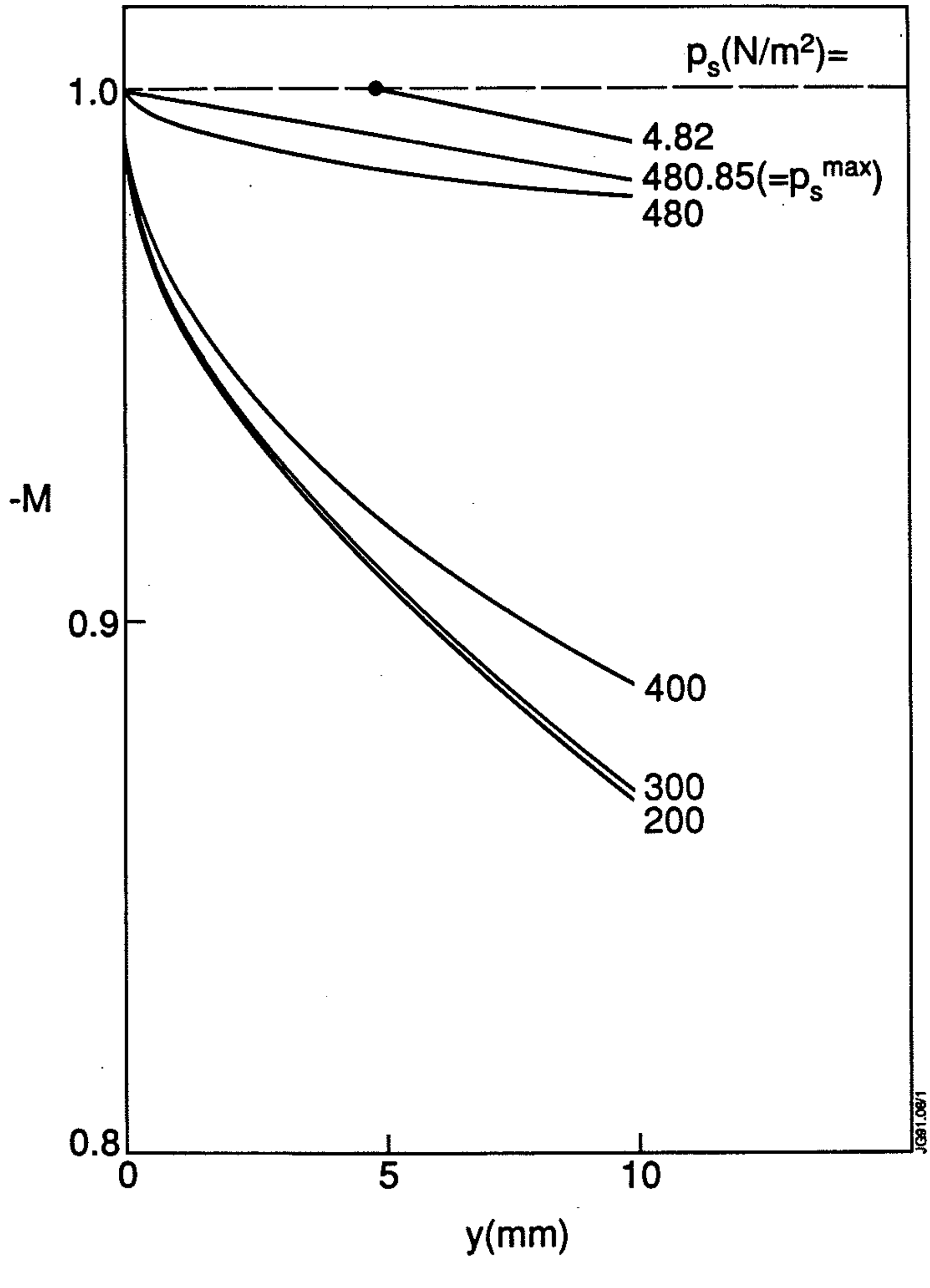
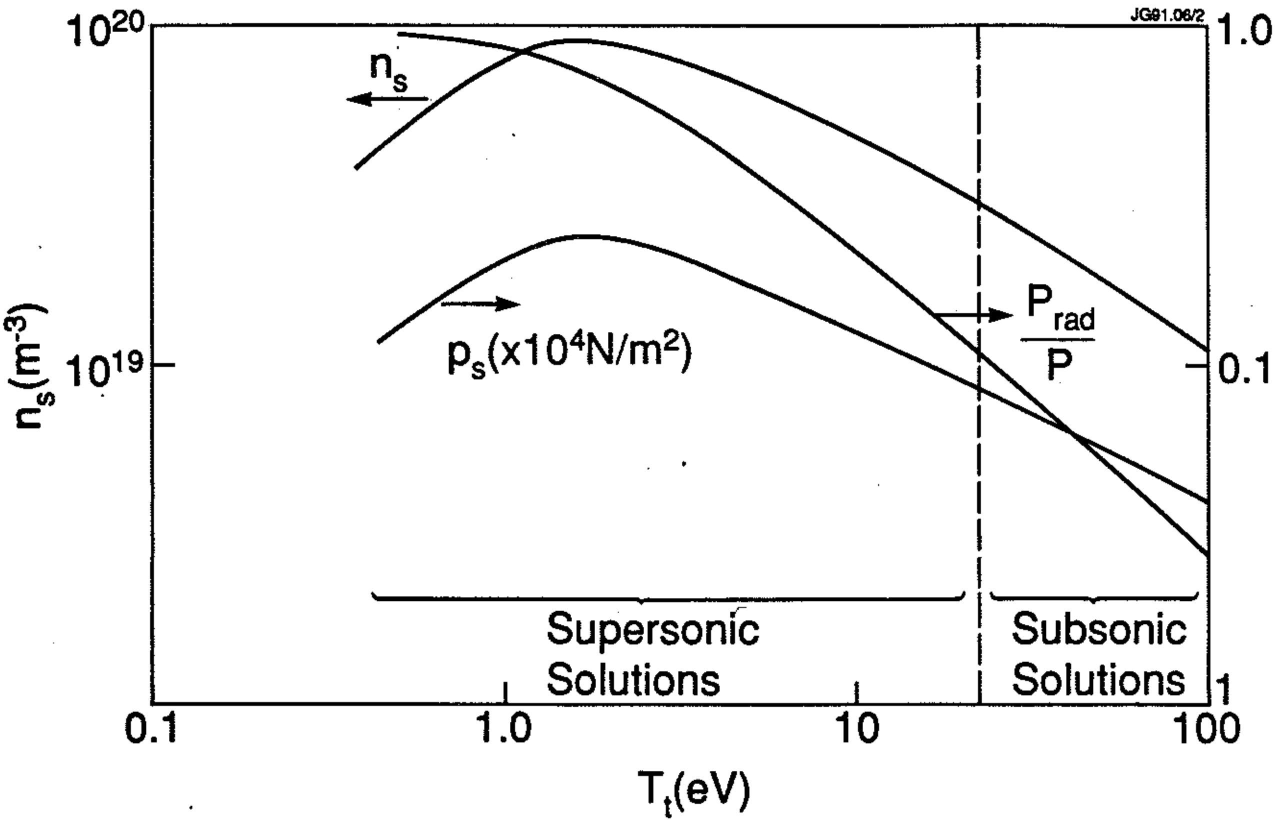
References

1. K. Lackner, R. Chodura, M. Kaufmann, J. Neuhauser, K. G. Rauh, W. Schneider, Plasma Phys. and Controlled Fusion 26 (1984), 105.
2. R. Chodura in "Physics of Plasma-Wall Interactions in Controlled Fusion", Eds. D. E. Post and R. Behrisch, Plenum Press, 1986.
3. J. Neuhauser, B. Braams, M. Krech, U. Ritschel, W. Schneider and R. Wunderlich, Contrib. to Plasma Phys. 30 (1990), 95.
4. P. J. Harbour and J. G. Morgan, CLM-R234, 1982.
5. K. Borrass, EUR-FU/80/89-95 and Proc of the 17th European Physical Society Meeting, Amsterdam, 1990.
6. M.F.A. Harrison, P. J. Harbour and E. S. Hotston, Nucl. Techn./Fusion 3 (1983), 432.
7. G. M. McCracken and J. M. Pedgley, "Estimates of the Radiated Power in a Tokamak Divertor", to be published.



1. Spatial variation of the deuterium plasma flow velocity (Mach number M) near the plate for various upstream pressures, p_s. Subsonic boundary condition at the target $M_t = -1$ imposed. Specific case shown, $P = 2\times10^7$ W, $P_r = 10^7$ W, $A = 0.2$ m^2, $L = 40$ m, $L_i = 1$ m, $L_r = 0.4$ m, for which the critical pressure $p_s^{max} = 480.85$ N/m^2. For $p_s > p_s^{max}$ no solution exists with real M close to the plate. For $p_s < p_s^{max}$, $M_t' = \infty$, a property which is abruptly lost when $p_s = p_s^{max}$.



2. Upstream density n_s versus target temperature T_t resulting from different driving pressures p_s, for the specific case of P = 40 MW, L = 30 m, L_i = 1 m, A = 0.4 m^2 and energy loss due to deuterium re-cycle (i.e., rather than fixed P_r). Subsonic and supersonic solutions.

APPENDIX 1.

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