JOINT EUROPEAN TORUS

JET–P(90)29

T.E. Stringer and JET Team

Inclusion of Poloidal Potential Variation in Neoclassical Transport

“This document contains JET information in a form not yet suitable for publication. The report has been prepared primarily for discussion and information within the JET Project and the Associations. It must not be quoted in publications or in Abstract Journals. External distribution requires approval from the Publications Officer, JET Joint Undertaking, Abingdon, Oxon, OX14 3EA, UK”.

“Enquiries about Copyright and reproduction should be addressed to the Publications Officer, EFDA, Culham Science Centre, Abingdon, Oxon, OX14 3DB, UK.”

The contents of this preprint and all other JET EFDA Preprints and Conference Papers are available to view online free at www.iop.org/Jet. This site has full search facilities and e-mail alert options. The diagrams contained within the PDFs on this site are hyperlinked from the year 1996 onwards.

Inclusion of Poloidal Potential Variation in Neoclassical Transport

T.E. Stringer and JET Team*

JET-Joint Undertaking, Culham Science Centre, OX14 3DB, Abingdon, UK

* *See Appendix 1*

Preprint of Paper to be submitted for publication in
Physics of Fluids B

INCLUSION OF POLOIDAL POTENTIAL VARIATION IN NEOCLASSICAL TRANSPORT

T.E. Stringer
JET Joint Undertaking, Abingdon, Oxon., OX14 3EA, UK.

ABSTRACT

Inclusion of the electrostatic potential is found to increase the electron neoclassical heat flux by an order of magnitude, while producing only a modest increase in the ion heat flux. The analysis is done for the plateau regime, though a similar behaviour is expected in the banana regime. The poloidally varying potential, determined by quasi-neutrality, is predominantly due to ion Landau damping. The resulting $\underline{E}\mathrm{x}\underline{B}$ drift introduces terms into the electron heat flux which are comparable to those previously derived for the ion heat flux. Similar terms enter the particle flux, but here they cancel due to momentum balance.

PACS number: 52.25 Fi.

1. INTRODUCTION

The variation in electrostatic potential over a magnetic flux surface has generally been neglected, arguing that electrons can move along field lines to neutralise any space charge. Hinton and Rosenbluth [1] estimated this variation in the plateau regime to be

$$e\left(\frac{1}{T_{io}}+\frac{1}{T_{eo}}\right)(\Phi-<\Phi>)\sim-\varepsilon\frac{\pi^{1/2}}{2}\frac{\rho_{i\theta}}{L_T}\sin\theta \tag{1}$$

where Φ is the electrostatic potential and angular brackets denotes a flux surface average, $\varepsilon = r/R_o$ is the inverse aspect ratio, $\rho_{i\theta} = \sqrt{2T_i m_i}/eB_\theta$ is the ion Larmor radius in the poloidal magnetic field, and L_T the temperature scale length. This variation was considered negligible because the analysis retains terms of order ε, and $\rho_{i\theta}/L_T << 1$ is assumed.

The mean radial flux of ions and electrons resulting from the above electrostatic potential will now be estimated. We assume the simple geometry with concentric circular flux surfaces, as in Ref. [1], where $B_\phi = B_o/(1 + \varepsilon \cos\theta)$ and the surface element $dS = rR_o (1 + \varepsilon \cos \theta)\, d\phi\, d\theta$. The particle flux then includes the component

$$\frac{1}{4\pi^2 rR}\int n\frac{E_\theta}{B}dS \sim -\frac{n_o}{2\pi r B_o}\oint\frac{\partial\Phi}{\partial\theta}(1+\varepsilon\cos\theta)^2 d\theta$$

$$= -\varepsilon^2\frac{\pi^{1/2}}{4}\frac{n\rho_i^2 \upsilon_{ti}}{(1+1/\tau)r\Theta}\frac{1}{T_i}\frac{dT_i}{dr} \qquad (2)$$

where $\upsilon_{tj} = (2T_j/m_j)^{1/2}$, $\tau = T_{eo}/T_{io}$, and $\Theta = B_\theta/B_\phi$. This has the same scaling as the ambipolar particle flux derived in earlier analyses of the plateau regime, e.g. [1,2], but is larger by a factor $(m_i/m_e)^{1/2}$. It is thus evident that the poloidal variation in potential can be important.

The corresponding estimate derived in Ref. [1] for the potential variation in the banana regime is

$$e\left(\frac{1}{T_i}+\frac{1}{T_e}\right)(\Phi - <\Phi>) \sim -1.81\varepsilon\frac{\rho_{i\theta}}{L_T}\left[\frac{qR\nu_i}{\upsilon_{ti}\varepsilon^{3/2}}\right]\sin\theta \qquad (3)$$

where q is the safety factor and ν_i the ion collision frequency. The square bracketed expression is less than unity over the banana regime. The mean radial flux resulting from this potential differs from that in Eq. (2) by the addition of the bracketed factor and a change in numerical constant. Again this flux exceeds the ambipolar particle flux derived earlier for this regime by a factor $(m_i/m_e)^{1/2}$. For analytic convenience, and continuity with earlier work [3], the following analysis is restricted to the plateau regime. However, a very similar behaviour may be expected in the banana regime, which will be the subject of a later publication.

The present author earlier included a poloidally varying electrostatic potential in a self-consistent derivation of the particle fluxes in the plateau regime [3]. These fluxes included a component similar to Eq. (2), along with other contributions of the same order resulting from the poloidal variation in density. However, the different contributions to the electron flux almost exactly cancelled each other, leaving a net electron flux less than Eq. (2) by a factor $(m_e/m_i)^{1/2}$, i.e. comparable to the flux obtained in other publications [1,2].

The above behaviour can be explained as follows. We first consider the behaviour when the poloidally varying electrostatic potential is neglected. There is then no coupling between the $O(\varepsilon)$ variation in the ion and electron velocity distribution functions. Particle fluxes are related to dissipation, the ion and electron fluxes in the plateau regime resulting from ion and electron Landau damping respectively. The coefficient of the ion Landau damping exceeds that of the electrons by a factor $(m_i/m_e)^{1/2}$. However, ambipolarity, which is imposed on the $O(\varepsilon^2)$ net fluxes, reduces the ion flux to equal that of the electrons. The plasma does this by acquiring an equilibrium flow such that the ion Landau damping is largely self-cancellatory. The ambipolar flux is thus determined by the slower diffusing electrons.

We now consider the behaviour when a poloidally varying potential, determined self-consistently from quasi-neutrality, is included. As will be seen later, the non-dissipative terms in this equation almost cancel, expressing the fact that the ability of ions and electrons to equalise their density over a flux surface is limited mainly by Landau damping. Ion and electron Landau damping both enter this equation but, because of its larger magnitude, ion Landau damping is dominant. This is the reason the potential variations in Eq. (1) is determined by ion parameters.

The $\underline{E}x\underline{B}$ drift resulting from the poloidal electric field thus introduces components proportional to the ion Landau damping into both the ion and electron radial fluxes. The estimate of one of these components in Eq. (2) shows it to be much larger than the ambipolar flux. Why then do these new components cancel themselves? As will be discussed in Section 5, this results from the relationship between particle transport and momentum balance along the magnetic field. It reflects the near-cancellation between the parallel components of the electric field and the pressure gradient in the electron equation of motion.

The purpose of the present paper is to extend the analysis of Ref. [3] to include an equilibrium temperature gradient, and to evaluate the heat flux. Since it is difficult to see how the cancellation of the ion Landau damping terms in the electron particle flux can be repeated for the higher moment velocity integrals entering the heat flux, the electrostatic field may be expected to produce an electron heat flux comparable to that of the ions.

Also of interest is the effect of the equilibrium temperature gradient on the resonant behaviour, first found in Ref. [4] for a toroidal resistive plasma, and later in Ref. [3] for plasma in the plateau regime. The origin of the resonance is as follows. In the simple geometry used, the equations differ from those for a cylindrical plasma mainly by the vertical ∇B drift of ions and electrons. The poloidal variation in potential is thus the same as in a cylindrical plasma in which vertical ion and electron currents, equal to the ∇B drifts, are externally induced.

The response of such a plasma would be inversely proportional to its dielectric constant, evaluated for a forced wave having the characteristics of the imposed current. Although the charge separation flow is constant in the laboratory frame, as seen by the rotating plasma it has frequency E_r/rB, where E_r is the mean radial electric field. Its wavelength is clearly defined, as discussed in Section 3. The denominator of the potential and density variation found in Ref. [3,4] agrees exactly with the expected dielectric constant.. If the Doppler frequency E_r/rB coincides with the frequency of a natural plasma mode, the real part of the dielectric constant vanishes, and the plasma response is a maximum. The inclusion of a temperature gradient in the plasma model modifies the dielectric constant, introducing the η_i mode. The possibility of a new resonant rotation, corresponding to the additional natural mode, was a further stimulus for this analysis.

Section 2 describes the model used. The linear solution of the kinetic equation is given in Section 3. The quasi-neutrality condition, applied to the poloidal variation in density, defines the electrostatic potential variation. The linear solution is used in Section 4 to derive the quasi-linear particle and heat fluxes, and the physical explanation of their behaviour is discussed in Section 5. Finally an Appendix which discusses the general question of the automatic ambipolarity of neoclassical transport, challenges the commonly held view.

2. THE MODEL

The analysis is an extension of an earlier treatment [3] of neoclassical transport in the plateau regime with self-consistent electrostatic field, to now include a radial temperature gradient and heat fluxes. It uses a direct evaluation of the transport fluxes, rather than the more elegant, but less physically transparent, derivation in terms of parallel viscosity, e.g. in Ref. [5].

Since the underlying physics is the same for all toroidally symmetric geometries, the simplest model, in which magnetic flux surfaces are concentric circular surfaces, is adopted for clarity. Standard coordinates are used, with polar coordinates r and θ centred on the toroidal magnetic axis, while ϕ measures angular distance along this axis. The magnetic field is taken to be

$$\underline{B} = \frac{R_o}{R}\left[o, B_{o\theta}(r), B_{o\phi}\right] \qquad (4)$$

where R_o is the radius of the magnetic axis, and $R = R_o + r\cos\theta$ is the distance from the axis of symmetry. The magnetic surfaces are r = constant.

The basic equation is the guiding centre kinetic equation,

$$\frac{\partial f_j}{\partial t} + (\underline{V}_j \cdot \nabla) f_j + \frac{\partial f_j}{\partial v_\parallel}\frac{dv_\parallel}{dt} + \frac{\partial f_j}{\partial v_\perp^2}\frac{dv_\perp^2}{dt} = 0 \tag{5}$$

for the velocity distribution function for the j^{th} species, $f_j(r,\theta,v_\parallel,v_\perp^2,t)$. $\underline{V}_j$ is the guiding centre velocity. $v_\parallel$ and $v_\perp$ are the components of the particle velocity parallel and perpendicular to the magnetic field. The plateau collisional regime is defined by $\varepsilon^{3/2} << qR/\lambda_{mfp} << 1$, where $\lambda_{mfp} = v_{tj}/\nu_j$ is the collisional mean free path. In this regime the transport is independent of the collision frequency. This was first shown by Galeev and Sagdeev [2], and demonstrated later in several papers where a full collision operator is included, e.g. in Ref. [1]. Since there is no reason to suspect that this is not equally true when poloidal potential is included, the collision operator is omitted from Eq. (5) for simplicity. We consider only the time independent equilibrium state.

For a large aspect ratio toroidal plasma, f_j and the equilibrium electrostatic potential Φ may be expanded as a power series in ε. The lowest order solution is a cylindrical equilibrium, independent of poloidal angle.

$$f_j = f_{oj}(r,v_\parallel,v_\perp^2) + f_{1j}(r,\theta,v_\parallel,v_\perp^2) + ...$$

$$\Phi(r,\theta) = \Phi_o(r) + \Phi_1(r,\theta) + ...$$

The guiding centre velocity of a particle may be written as

$$V_j = v_\parallel \underline{B}/|B| + \underline{v}_{bj} + \underline{v}_o + \underline{v}_1 + ... \tag{6}$$

where

$$\underline{v}_{bj} = -m_j \frac{\left(v_\parallel^2 + v_\perp^2 / 2\right)}{e_j B^2 R}\left(B_\phi \underline{e}_z - B_z \underline{e}_\phi\right)$$

$$\underline{v}_o = \frac{1}{B}\frac{d\Phi_o}{dr} \quad , \quad \underline{v}_1 = -\frac{\nabla\Phi_1 \times \underline{B}}{B^2}$$

$\underline{v}_{bj}$ is the sum of the curvature and magnetic field gradient drifts. $\underline{e}_\phi$ and $\underline{e}_z$ are unit vectors in the toroidal and vertical directions.

As in earlier references [1-5], the zero order electric drift is assumed of the same order as the diamagnetic velocities U_{nj} and U_{Tj} where

$$U_{nj} = \frac{T_{jo}}{e_j B}\frac{1}{n_o}\frac{dn_o}{dr}, U_{Tj} = \frac{1}{e_j B}\frac{dT_{jo}}{dr} \tag{7}$$

This implies $v_1 \sim v_{bj} \sim \varepsilon U_{nj}$. The ratio of the poloidal and toroidal components of the equilibrium magnetic field in tokamak geometry is

$$\Theta = \frac{B_\theta}{B_\phi} = \frac{\varepsilon}{q} = 0(\varepsilon)$$

The zero order distribution function is assumed to be locally Maxwellian and is written in the form

$$f_{\theta j}(r, v_\parallel, v_\perp^2) = \frac{1}{v_{tj}^2}\exp\left[-v_\perp^2 / v_{tj}^2\right] F_{oj}(v_\parallel)$$

$$F_{oj}(v_\parallel) = \frac{n_o(r)}{\pi^{1/2} v_{tj}}\exp\left[-v_\parallel^2 / v_{tj}^2\right] \tag{8}$$

so that density $n = \int dv_{\|} \int dv_{\perp}^2 f$. the condition that the zero order distribution remains Maxwellian, in spite of a preferential loss of a certain class of particles, is the same as the condition that trapped particles are not dominant. This determines the lower collisional limit on the regime. For analytic brevity the ions and electrons are assumed to have no mean motion along the magnetic field. The effect of a parallel flow $\bar{v}_{\|j}$ can readily be included by replacing v_o in the following analysis by $v_o + \Theta\bar{v}_{\|j}$.

3. THE LINEAR SOLUTION

The equilibrium equation, Eq. (6) will now be solved to first order in the inverse aspect ratio. Since

$$v_{\|} = [(E - e_j\Phi - \mu B)2/m_j]^{1/2}$$

where the particle energy E and magnetic moment $\mu = m_j v_{\perp}^2/2B$ are constants of the motion, it follows that

$$\frac{dv_{\|}}{dt} = -\frac{1}{mv_{\|}}(\underline{v}\cdot\nabla)(e_j\Phi + \mu B)$$

$$= -\frac{1}{mv_{\|}}\left(\frac{v_o}{r}\frac{\partial}{\partial\theta} + v_{\|}\frac{\partial}{\partial s} + v_r\frac{\partial}{\partial r}\right)(e_j\Phi + \mu B)$$

$$= -\frac{e_j\Theta}{m_j r}\frac{\partial\Phi_1}{\partial\theta} - \frac{\varepsilon}{r}\left(\frac{\Theta v_{\perp}^2}{2} - v_o v_{\|}\right)\sin\theta \qquad (9)$$

where

$$\frac{\partial}{\partial s} = \frac{1}{B}(\underline{B}\cdot\nabla) = \frac{\Theta}{r}\frac{\partial}{\partial\theta}$$

and

$$v_r = -\frac{1}{rB}\frac{\partial \Phi_1}{\partial \theta} - \frac{m_j}{e_j BR}(v_\parallel^2 + v_\perp^2/2)\sin\theta$$

Linearising Eq. (5) with respect to ε, and integrating with respect to θ, gives

$$f_{1j} = \frac{1}{(v_o + \Theta v_\parallel)}\left\{\left[\Phi_1 - \frac{\varepsilon m_j}{e_j}(v_\parallel^2 + v_\perp^2/2)\cos\theta\right]\frac{1}{B}\frac{\partial f_{oj}}{\partial r} - \frac{\varepsilon m_j v_\perp^2}{2T_j}(v_o + \Theta v_\parallel) f_{oj}\cos\theta\right.$$

$$\left. + \left[\frac{e_j m_j}{m_j}\Theta\,\Phi_1 - \varepsilon\left(\frac{\Theta v_\perp^2}{2} - v_o v_\parallel\right)\cos\theta\right]\frac{\partial f_{oj}}{\partial v_\parallel}\right\} \qquad (10)$$

The poloidally varying electrostatic potential is determined by the charge neutrality condition. To apply this the electron and ion densities must be evaluated by integrating Eq. (10) over velocity. Integration over $v_\perp^2$ is trivial. Before integrating over $v_{||}$ it is convenient to replace cosθ by exp(iθ), with the understanding that only the real parts of the equations have physical significance. Integration over $v_{||}$ gives integrals with resonant denominators of the same form as those encountered in microinstability theory, i.e.

$$\frac{1}{n_o}\int_{-\infty}^{\infty}\frac{F_{oj}(v_\parallel)}{v_\parallel - W}v_\parallel^s dv_\parallel = K_s\left(\frac{W}{v_{tj}}\right)$$

These integrals can be expressed in terms of the familiar plasma dispersion function

$$I(z) = 1 - 2z\,\exp(-z^2)\int_o^z \exp(t^2)dt + i\pi^{1/2}z\exp(-z^2) \qquad (11)$$

using the recurrence relation

$$K_s\left(\frac{W}{v_{tj}}\right) = W\,K_{s-1}\left(\frac{W}{v_{tj}}\right) + J_s$$

where $J_s = (s-2)(s-4)\ldots 1(v_{tj}/2)^{(s-1)/2}$ when s is odd, $J_s = 0$ when s is even and

$$K_o\left(\frac{W}{v_{tj}}\right) = \frac{1}{W}\left[I\left(\frac{W}{v_{tj}}\right) - 1\right],\; K_1\left(\frac{W}{v_{tj}}\right) = I\left(\frac{W}{v_{tj}}\right).$$

Integrating Eq. (10) over velocity gives the poloidal variation in density

$$\frac{n_{1j}}{n_o} = \frac{e_j\Phi_1}{T_j}\left[\frac{1}{v_o}\left(U_{nj} - \frac{U_{Tj}}{2}\right)\left(1 - I_j\right) - I_j - \frac{v_o U_{Tj}}{v_{tj}^2\theta^2} I_j\right]$$

(12)

$$+\varepsilon e^{i\theta}\left[\left(1 + \frac{U_{nj}}{v_o} + \frac{U_{Tj}}{2v_o}\right)\left(I_j - 1\right) + 2z_j^2 I_j\left(1 + \frac{U_{nj}}{v_o}\right) + z_j^2\frac{U_{Tj}}{v_o}\left(1 + 2z_j^2 I_j\right)\right]$$

where

$$z_j = -\frac{v_o}{v_{tj}\Theta},\; I_j = I(z_j).$$

Invoking quasi-neutrality, $n_{1i} = n_{1e}$, leads to an equation for Φ_1

$$(F + iL)\Phi_1 = \frac{\varepsilon T_e}{e}\left\{\left(1 + \frac{U_{ni}}{v_o}\right)\left[\left(1 + 2z_i^2\right)I_i - 1\right] - \left(1 + \frac{U_{ne}}{v_o}\right)\left[(1 + 2z_e^2)I_e - 1\right]\right.$$

$$+\frac{U_{Ti}}{2v_o}\left(I_i-1+2z_i^2+4z_i^4I_i\right)-\frac{U_{Te}}{2v_o}\left(I_e-1+2z_e^2+4z_e^2I_e\right)\Big\} \quad (13)$$

where

$$F+iL=\left(1+2z_e^2\frac{U_{Te}}{v_o}\right)I_e-\left(\frac{U_{ne}}{v_o}-\frac{U_{Te}}{2v_o}\right)(1-I_e)$$

$$+\tau\left(1+2z_i^2\frac{U_{Ti}}{v_o}\right)I_e-\tau\left(\frac{U_{ni}}{v_o}-\frac{U_{Ti}}{2v_o}\right)(1-I_i) \quad (14)$$

F is obtained by replacing I_j in Eq. (14) by I_{oj}, the real part of I(z) which omits the Landau term in Eq. (11), while

$$L=-\frac{\pi^{1/2}}{v_{ti}\Theta}\left[v_o+U_{ne}-U_{Te}\left(1+2z_e^2\right)/2\right]-\frac{\tau\pi^{1/2}}{v_{ti}\Theta}\left[v_o+U_{ni}-U_{Ti}\left(1+2z_i^2\right)/2\right] \quad (15)$$

As discussed in the Introduction, the response of the plasma to the toroidal drift may be expected to vary inversely as the plasma dielectric constant. The poloidal variation driven by the toroidal drift has the form of an m = 1 stationary wave whose parallel wave number is $(qR)^{-1} = \Theta/r$. The rotating plasma sees it as having the Doppler frequency $-v_o/r$. If these mode parameters are substituted into the dispersion equation for electrostatic waves in a slab [6], one obtains exactly F + iL as defined in Eq. (14).

The ratio L/F is of order $\tau(v_o + U_{ni})/v_{ti}\Theta$. Since the measured radial electric field is typically of order $\nabla p/ne$, giving $v_o \sim U_{nj}$, this ratio is about $U_{ni}/v_{ti}\Theta \sim \rho_i/\Theta L_n$. This is of order 1/10 in tokamaks, expressing the fact that the driven wave is rather weakly damped by ion Landau damping. If the values of the wave parameters coincide with those for a natural plasma mode, then F = 0. The excitation of the driven mode is then limited only by damping or by nonlinear effects [7].

4. EVALUATION OF THE TRANSPORT

In first order the particle and heat fluxes across magnetic surfaces vanish when integrated over a surface. To second order in ε a net radial flux results from products of first order quantities. The surface element is $dS = rR_o(1 + \varepsilon \cos\theta)\, d\theta\, d\phi$. The mean radial fluxes of particles (Γ_j) and heat (q_j) are given by

$$\Gamma_j = \frac{1}{S}\int dS \int_{-\infty}^{\infty} dv_\parallel \int_{-\infty}^{\infty} dv_\perp^2 f_j v_{rj}$$

$$= -\frac{1}{2\pi r B_o}\int d\theta \int dv_\parallel \int dv_\perp^2 \left(f_{oj} + f_{1j}\right)\left[\frac{\partial\Phi_1}{\partial\theta} + \frac{\varepsilon m_j}{e_j}\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)\sin\theta\right](1 + \varepsilon\cos\theta)^2 \quad (16)$$

$$= -\frac{1}{2\pi r B_o}\int d\theta \left[\frac{\partial\Phi_1}{\partial\theta}\left(2\varepsilon n_o \cos\theta + n_{1j}\right) + \frac{\varepsilon m_j}{e_j}\int dv_\parallel \int dv_\perp^2 \left(v_\parallel^2 + \frac{v_\perp^2}{2}\right) f_{1j}\sin\theta\right] \quad (17)$$

$$q_j = -\frac{1}{2\pi r B_o}\int d\theta \left[3\varepsilon n_o T_{oj}\frac{\partial\Phi_1}{\partial\theta}\cos\theta\right.$$

$$\left. + \frac{m_j}{2}\int dv_\parallel \int dv_\perp^2 \left\{ f_{1j}\frac{\partial\Phi_1}{\partial\theta} + \frac{\varepsilon m_j}{e_j}\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right) f_{1j}\sin\theta\right\}\left(v_\parallel^2 + v_\perp^2\right)\right] \quad (18)$$

Evaluating each product term in Eq. (17) leads, after much analysis, to the following expression for the particle flux

$$\Gamma_j = -\frac{\varepsilon^2 \pi^{1/2} n_o}{8\tau\theta} v_{tj} \rho_j^2 \left\{ \left[\frac{n_o'}{n_o} - \frac{e_j E_{or}}{T_{oj}} \right] \right.$$

$$\left[1 + \frac{\left[\left(1 + 2z_j^2\right) F - \beta_j P \right]^2 + \left[\left(1 + 2z_j^2\right) L - \beta_j P \right]^2}{F^2 + L^2} \right] \exp(-z_j^2)$$

$$+ \frac{T_{oj}'}{T_{oj}} \left[3 - \frac{\left(2z_j^2 F - \beta_j P\right)^2 + \left(2z_j^2 L - \beta_j Q\right)^2}{2(F^2 + L^2)} \right] \exp(-z_j^2)$$

$$- \frac{v_{tj}}{v_o} \beta_j \left(1 - I_{oj}\right) \frac{(PL - QF) T_{oj}}{(F^2 + L^2) T_{oj}} \tag{19}$$

where $\rho_j = \sqrt{2T_j m_j}/e_j B$ is the Larmor radius of the j^{th} species, and $\beta_j = e_j T_e / eT_j$ equals τ for ions and -1 for electrons. P and Q are proportional to the real and imaginary parts of the right hand of Eq. (13), defined by

$$\Phi_1 = \frac{\varepsilon T_e}{e} \left(\frac{P + iQ}{F + iL} \right) \exp(i\theta) \tag{20}$$

P may be obtained by replacing I_j by I_{oj} on the right hand of Eq. (13), while

$$Q = -\frac{\pi^{1/2}}{v_{ti}(\Theta)} \left[\left(v_o + U_{ni}\right)\left(1 + 2z_i^2\right) + \frac{1}{2} U_{Ti} \left(1 + 4z_i^4\right) \right] \exp\left(-z_i^2\right)$$

$$+ \frac{\pi^{1/2}}{v_{te}(\Theta)} \left[\left(v_o + U_{ne}\right)\left(1 + 2z_e^2\right) + \frac{1}{2} U_{Te} \left(1 + 4z_e^2\right) \right] \exp\left(-z_e^2\right) \tag{21}$$

The expression for q_j is appreciably longer than Eq. (19), partly because the extra factor $(v_{||}^2 + v_\perp^2)$ doubles the number of product terms. It can be simplified by noting that $z_j = 0(U_{nj}/v_{tj}\Theta) = \rho_j/L_n\Theta$. Thus in all conditions of experimental interest, $z_e << z_i << 1$. Contributions of order z_j^2 and higher can therefore be neglected. The heat flux then simplifies to

$$q_j = -\frac{\varepsilon^2 \pi^{1/2} n_o T_{oj}}{4\tau\Theta} v_{tj}\rho_j^2 \left[3\left(\frac{n_o'}{n_o} + \frac{5}{2}\frac{T_{oj}'}{T_{oj}} - \frac{e_j E_r}{T_{oj}} \right) \right.$$

$$\left. + \frac{1}{2}\left(\frac{T_e}{T_j}\right)^2 \left(\frac{PF + LQ}{F^2 + L^2}\right)\left(\frac{n'}{n} + \frac{T_{oj}'}{T_{oj}} - \frac{e_j E_r}{T_{oj}} \right) \right]$$

$$-\frac{\varepsilon^2 n_o T_j T_e}{reB}\left[\left(\frac{QF - PL}{F^2 + L^2}\right) - \frac{3\pi^{1/2} v_o}{v_{tj}\Theta}\left(\frac{PF + LQ}{F^2 + L^2}\right) \right] \qquad (22)$$

The above fluxes may be further simplified by taking the values of F, L, P, and Q in the limit $z_e << z_i << 1$. In this limit

$$I_{oj} = 1 - 2z_j^2 + o(z_j^4), \qquad F = 1 + \tau + o(z_i^4), \qquad P = o(z_i^4)$$

$$L = -\frac{\pi^{1/2}\tau}{v_{ti}\Theta}(v_o + U_{ni} - U_{Ti}/2), \qquad Q = -\frac{\pi^{1/2}\tau}{v_{ti}\Theta}(v_o + U_{ni} + U_{Ti}/2)$$

Substituting these limiting forms in Eqs. (19) and (22) gives

$$\Gamma_j = -\frac{\varepsilon^2 \pi^{1/2} n_o}{4r\Theta} \nu_{tj} \rho_j^2 \left[\frac{n_o'}{n_o} + \frac{3}{2}\frac{T_{oj}'}{T_{oj}} - \frac{e_j}{T_{oj}}\left(E_{or} - B_\theta \bar{v}_{\parallel j}\right)\right] \tag{23}$$

$$q_j = -\frac{3\varepsilon^2 \pi^{1/2} n_o}{4r\Theta} T_{oj} \nu_{tj} \rho_j^2 \left[\frac{n_o'}{n_o} + \frac{5}{2}\frac{T_{oj}'}{T_{oj}} - \frac{e_j}{T_{oj}}\left(E_{or} - B_\theta \bar{v}_{\parallel j}\right)\right]$$

$$+\frac{\varepsilon^2 \pi^{1/2} n_o}{4r\Theta} T_{oj} \left(\frac{\tau}{1+\tau}\right) \nu_{ti} \rho_i^2 \left[\frac{n_o'}{n_o} + \frac{T_{oi}'}{2T_{oi}} - \frac{e}{T_{oi}}\left(E_{or} - B_\theta \bar{v}_{\parallel i}\right)\right] \tag{24}$$

The fluxes have been generalised to include parallel flows, using the substitution specified in Section 2.

The radial electric field, which so far has been treated as an arbitrary parameter, will now be determined by the ambipolar condition $\Gamma_i = \Gamma_e$. In studies of neo-classical transport in the banana regime it is sometimes stated that this transport is automatically ambipolar and independent of the electric field [8,9]. Since the reasoning is quite general, should this conclusion not apply also to the plateau regime? The Appendix discusses the basis for this conclusion and argues that automatic ambipolarity is a simple result of the assumption that the equilibrium is stationary, and is in no way a result of neoclassical processes. The question of whether neoclassical ambipolarity is automatic or not is of more than academic interest. If it is, then the neoclassical particle transport is ambipolar even in the presence of another non-ambipolar loss mechanism, such as magnetic field ergodicity. If it is not, then the ambipolar condition must include all loss mechanisms, and the neoclassical transport may be non-ambipolar in order to balance the other non-ambipolar loss.

Assuming there is no other loss mechanism, we impose the condition $\Gamma_i = \Gamma_e$. Since the coefficient in Eq. (23) is a factor $(m_i/m_e)^{1/2}$ larger for the ions than the electrons, an ambipolar electric field must be set up so that the bracketed terms in the ion flux almost cancel, i.e.

$$\frac{eE_{or}}{T_{oi}} \approx \frac{n_o'}{n_o} + \frac{3}{2}\frac{T_{oi}'}{T_{oi}} + \frac{eB_\theta}{T_{oi}}\bar{v}_{\parallel i} \tag{25}$$

The ambipolar particle flux is then determined by the electrons

$$\Gamma_a = -\frac{\varepsilon^2 \pi^{1/2} n_o}{4r\Theta} v_{te}\rho_e^2 \left[\frac{n_o'}{n_o}\left(1+\frac{1}{\tau}\right) + \frac{3}{2}\left(\frac{T_{oi}'+T_{oe}'}{T_{oe}}\right) + \frac{B_\theta}{n_o} j_{o\parallel}\right] \tag{26}$$

The corresponding ion and electron heat fluxes are

$$q_i = -\frac{\varepsilon^2 \pi^{1/2} v_{ti}}{4r\Theta}\rho_i^2 n_o \left(3 + \frac{\tau}{(1+\tau)}\right)\frac{dT_i}{dr} \tag{27}$$

$$q_e = -\frac{\varepsilon^2 \pi^{1/2} v_{ti}}{4r\Theta}\rho_i^2 \frac{n_o \tau^2}{(1+\tau)}\frac{dT_i}{dr} \tag{28}$$

5. DISCUSSION OF THE TRANSPORT FLUXES

The transport fluxes which are obtained when the electrostatic potential is neglected may be obtained from Eqs. (19) and (22) by putting P = 0 = Q. These fluxes agree with those derived by Galeev [2]. The fluxes are proportional to $v_{tj}\rho_j^2$, i.e. the electron flux is less than the ion flux by a factor $(m_e/m_i)^{1/2}$. This is because the net transport depends on the in-phase components of velocity and density, and these result from Landau damping. In the absence of an electrostatic potential there is no coupling between ions and electrons in the first order, and consequently the ion and electron fluxes are proportional to their respective Landau terms (the imaginary parts of Eq. (11)).

When a poloidally varying electrostatic potential is included, the ExB drift introduces a new component of radial velocity. The component in phase with density results from the total Landau damping, in which the ion Landau damping is dominant. Since the ExB drift is common to both ions and electrons, one might expect the electrons to acquire a flux proportional to ion Landau damping. Detailed inspection of the particle flux shows that the electrons do indeed have such a component, but it is exactly cancelled by another term.

The physical origin of this cancellation may be seen as follows. Equation (17) for the mean flux may easily be recast in the following form

$$\Gamma_j = -\frac{1}{2\pi r B e_j}\oint d\theta\left[e_j(n_o + n_1)\frac{\partial \Phi_1}{\partial \theta} + \frac{\partial p_{j\parallel}}{\partial \theta} + \frac{1}{2}\frac{\partial}{\partial \theta}\left(p_{j\perp} - p_{j\parallel}\right)\right](1+\varepsilon\cos\theta)^2 \qquad (29)$$

where the poloidally varying part of $p_{||}$ is $m_j \int dv_{||}^2\, v_{||}^2\, f_{1j}$. From the parallel component of the equation of motion, the first two terms in the integrand balance, except for a small inertial residue. Thus particle diffusion is proportional to $\langle (p_{j\perp} - p_{j||}) \sin\theta \rangle$, which can be shown to be proportional to the Landau damping of the j^{th} species. The above form for Γ_j is the basic form for the particle flux when the radial velocity is obtained from the perpendicular component of the fluid equation of motion, as in Ref. [10].

It is obvious that the above cancellation in the particle flux, resulting from momentum balance, can no longer occur when the integrand is multiplied by $(v_{||}^2 + v_\perp^2)$ to obtain the heat flux. Terms proportional to $v_{ti}\rho_i^2$ in Eq. (24) for q_j come from the electrostatic potential. The effect on the ion heat flux of this additional contribution is typically small. When the temperatures are equal it increases the flux in Eq. (27) by a factor 7/6. Its effect on the electron heat flux, however, is large, increasing it from about 1/60 times the ion heat flux to about one sixth.

Although the present analysis gives an electron heat flux which is an order of magnitude larger than earlier analyses, it is still an order of magnitude less than that observed experimentally. The result might thus seem of academic interest. To make it of practical importance requires a large enhancement of the electrostatic field, and hence the transport of both species. As discussed in the Introduction, such an enhancement would result if the rotation velocity, v_o/r, coincides with the frequency of a natural mode. However, resonance (defined by F = 0) occurs only at much higher electric fields than that observed. This is because the natural slab modes have phase velocities larger than sound speed, i.e. $\omega/v_{ti}\, k_{||} > 1$. For a rotating plasma the corresponding condition is $|v_o|/v_{ti}\Theta > 1$, which requires $|v_o| >> U_{ni}$, i.e. $E_r >> \nabla p/ne$. However, the full range of possibilities has not yet been fully investigated. For example, propagation of the η_i-mode in a toroidal plasma, as compared with a slab plasma, is strongly affected by the ∇B drift. Such effects are neglected in the linear analysis leading to F + iL in Section 3.

6. CONCLUSIONS

The variation in electrostatic potential over a flux surface, which is neglected in earlier neoclassical analyses, has been shown to make significant contributions to the radial transport. The analysis has been done for the plateau regime, though similar behaviour is expected in the banana regime.

Earlier analyses found the ion and electron heat fluxes to depend on ion and electron Landau damping respectively. Consequently, the electron heat flux was smaller by a factor $(m_e/m_i)^{1/2}$. The electrostatic potential, determined self-consistently from charge neutrality, results from both ion and electron Landau damping. The resulting $\underline{E}\times\underline{B}$ drift contributes terms proportional to ion Landau damping to both the ion and electron heat fluxes. While the increase in total ion heat flux is relatively modest, the smaller electron heat flux is increased by an order of magnitude.

Comparable terms, proportional to the ion Landau damping, enter the evaluation of the ion and electron radial particle fluxes. However, these terms nearly cancel. This can be attributed to momentum balance along the magnetic field. Here the variation in electrostatic potential is almost balanced by the electron pressure gradient. This ensures that the n_o ($\underline{E}$x$\underline{B}$) flux is almost cancelled by the curvature drift flux. As a consequence of this cancellation, the ambipolar particle diffusivity is an order of magnitude less than the electron thermal diffusivity.

Even after the large increase in the neoclassical electron heat flux, it is still an order of magnitude less than that observed. An enhancement of the neoclassical fluxes to the measured levels could conceivably result from the resonant response when the poloidal frequency equals the frequency of a natural plasma mode. However, the rotation frequencies at which such resonances can occur appear to be higher than the observed rotation.

ACKNOWLEDGEMENTS

The author wishes to thank Prof. D.F. Düchs for his encouragement, and Prof. R. Balescu for helpful criticism.

APPENDIX 1

Is Neoclassical Diffusion Automatically Ambipolar?

In the analysis in References 8 and 9, which leads to automatic ambipolarity, the particle energy, $K = m_j v^2/2 + e_j\Phi$, and magnetic moment, $\mu = m_j v^2/2B$, are used as velocity coordinates. The linearised equation, including a collision term, C(f), then takes for form

$$\frac{\partial f_1}{\partial t} + v_\parallel \underline{b}\cdot\nabla f_1 + V_{jr}\frac{\partial f_o}{\partial r} = C(f) \qquad (A.1)$$

This is multiplied by $v_{||}/\Omega_{\theta j}$. where $\Omega_{\theta j} = e_j B_\theta/m_j$ and integrated over velocity and over the magnetic surface. The velocity space element is now $d^3v = 2\pi B d\mu dk/m_j^2 v_{||}$. An integration by parts converts $\oint d\theta \int d^3v\, (v_{||}^2/\Omega_{\theta j})\, \underline{b}\cdot\nabla f_1$ into the mean radial flux across a magnetic flux surface, using the relation [8,9]

$$v_{bjr} = \frac{m_j}{e_j} v_\parallel (\underline{b}\cdot\nabla)\left(\frac{v_\parallel}{B_\theta}\right)$$

where $\underline{b}$ is a unit vector along the magnetic field. This leads to the equation

$$\Gamma_j \equiv < \int d^3v\, v_{bjr} f_j > = < -\frac{1}{\Omega_{\theta j}} \int d^3v\, v_\parallel C_j(f) >$$

$$+\frac{\partial}{\partial t} < \frac{1}{\Omega_{\theta j}} \int d^3v\, v_\parallel f_j > + \frac{1}{R\Omega_{\theta j}}\frac{1}{r}\frac{\partial}{\partial r} r < \int d^3v\, v_\parallel R v_{bjr} f_j > \qquad (A.2)$$

where angular brackets denote averaging over a magnetic surface, i.e $<A> = (2\pi)^{-1} \int A(1 + \varepsilon \cos\theta)\, d\theta$.

The second term on the right of Eq. (A.2) is dropped in Refs. [8,9] on the assumption that the system has reached a quasi-stationary equilibrium, while the last term, which is the radial convection of parallel momentum, can be assumed small. The particle flux is thus given by the first term, which is the parallel momentum transferred to the j^{th} species by collisions. Like particle collisions contribute no net momentum, while transfer due to unlike collisions must be equal but opposite for the two species. We thus find equal fluxes of ions and electrons without explicitly imposing the quasi-neutrality condition.

To clarify the physical meaning of Eq. (A.2), we will compare it with the ϕ component of the momentum equation. The most general form of the momentum equation is obtained from the first moment of the kinetic equation,

$$nm_j\left(\frac{\partial}{\partial t}+\underline{u}_j\cdot\nabla\right)\underline{u}_j=-\nabla\cdot\underline{\underline{P}}_j+nZ_je\left(\underline{E}+\underline{u}_j\times\underline{B}\right)+\underline{R}_j \tag{A.3}$$

where $\underline{u}_j = \int d^3v\, \underline{v}\, f_j$ is the fluid velocity, $P_{j\alpha\beta} = m_j \int d^3v\, v_\alpha v_\beta f_j$ and $\underline{R}_j = m_j \int d^3v\, \underline{v}\, C_j(f)$. We will assume that the friction perpendicular to $\underline{B}$ can be written as a resistivity, i.e. $\underline{R}_j = R_{j||}\underline{b} - \eta_\perp \underline{j}_\perp$. We separate the perpendicular current into its mean and poloidally varying parts, $\bar{j}_\perp + \tilde{j}_\perp$, where $\bar{j}_\perp = B_o^{-1}\, dp_o/dr$ is the mean dia-magnetic current. From $\nabla\cdot j = 0$ it follows that $\tilde{j}_\perp = 0(\Theta\, \tilde{j}_{||})$, where $\Theta = B_\theta/B_\phi = 0(\varepsilon)$. Hence $\tilde{j}_\perp$ may be neglected.

To obtain the radial flux we take the ϕ component of Eq. (A.3), multiply by $(1 + \varepsilon\cos\theta)^2$, and integrate over θ. The $n\, \underline{u}_j \times \underline{B}$ term thus becomes $B_{\theta o}\, \Gamma_j$, where $B_{\theta o} = (1 + \varepsilon\cos\theta)\, B_\theta$ is independent of θ. We can divide Γ_j into its classical and neo-classical parts, the latter resulting entirely from toroidal effects

$$\Gamma_j=-\frac{\eta_\perp \bar{n}}{B_o^{\,2}}\frac{dp_o}{dr}+\bar{n}\left(\frac{\overline{E\times B}}{B^2}\right)_r+\Gamma_{Tj} \tag{A.4}$$

We then arrive at the following relation

$$\frac{m_j}{Z_j e} < (1+\varepsilon\cos\theta)n\left(\frac{\partial}{\partial t}+\underline{u}\cdot\nabla\right)u_{j\phi} >= \Gamma_{Tj}B_{\theta o} + < (1+\varepsilon\cos\theta)R_{j\parallel} > B_\phi / B Z_j e \quad \text{(A.5)}$$

A more detailed derivation of this equation is given in Ref. [11].

Since $u_{j\phi}$ and $u_{j||}$ are effectively the same, and Ω_θ^{-1} varies as $1 + \varepsilon \cos\theta$, Eqs. (A.2) and (A.5) may be seen to be equivalent. When the electron and ion equations, in either form (A.2) or (A.5), are subtracted the collision terms cancel because collisions conserve momentum, and we get

$$(Z_i\Gamma_{Ti} - \Gamma_{Te})eB_{\theta o} =< (1+\varepsilon\cos\theta)n\frac{d}{dt}\left(m_i u_{i\phi} + m_e u_{e\phi}\right)>. \quad \text{(A.6)}$$

Earlier authors assumed the inertial terms to be of higher order and concluded that the diffusion is automatically ambipolar, independent of the quasi-neutrality condition. However, it can be seen that Eq. (A.6) is no more than the conservation of total momentum about the axis of symmetry, i.e.,

$$< R\, j_r B_\theta >= \frac{d}{dt} < Rn\left(m_i u_{i\phi} + m_e u_{e\phi}\right)>. \quad \text{(A.7)}$$

It says nothing specific about neoclassical diffusion. All that we can conclude is that the toroidal flow reaches its stationary value only when the diffusion has become exactly ambipolar.

When there is no other loss process, neoclassical diffusion must conserve quasi-neutrality. This certainly requires the diffusion to be approximately ambipolar, though a small residual j_r, enough to produce significant $j_r B_\theta$ acceleration, is still permitted. The final diffusion rates are the same whether ambipolarity is attributed to quasi-neutrality or is assumed to be automatic. However, the results can be different when there are other loss mechanisms, as discussed in Section 3.

A simple example of neoclassical banana diffusion, where quasi-neutrality does not imply ambipolarity, will now be considered. We again consider a pure electron-ion plasma, but now introduce a conducting wire along the magnetic axis which is maintained at a fixed potential. This wire is assumed to be a perfect emitter of particles, so that no charged sheath forms around it. Since the system is still axisymmetric, the conclusion that momentum conservation during collisions requires the flux across any magnetic surface to be ambipolar, if it were correct, should still apply. This is consistent with quasi-neutrality only if the emission from the wire vanishes, i.e. $E_r(0)$ is zero. However, as we shall see below, such a solution is possible only after the plasma has acquired a specific parallel flow. During the finite growth time of this flow, the diffusion is non-ambipolar.

We can use the diffusion fluxes for the banana regime first derived by Galeev and Sagdeev [5], but including the effect of parallel flow. To reduce analytic detail we shall take the temperature to be homogeneous. Then

$$\Gamma_j = -1.46\,\varepsilon^{1/2}\frac{\bar{v}_j n}{\Omega_{j\theta}B_\theta}\left[\frac{T_j}{e_j}\frac{n_j'}{n_j} + B_\theta u_{j\parallel} - E_r\right]. \tag{A.8}$$

where $\bar{v}_j$ is the collision frequency for the j^{th} species. The coefficient of the ion term exceeds that for the electrons by $0(m_i/m_e)^{1/2}$. In the absence of a central conductor a radial electric field develops to reduce the ion rate to that of the electrons, but now the plasma does not have the freedom to do this. Since charge must not accumulate within the plasma, the flux across a magnetic surface is independent of r, i.e., $r(\Gamma_i - \Gamma_e) \approx r\,\Gamma_i = C$. This gives an expression for $E_r = - d\Phi/dr$ which, when integrated between r = o and a, with boundary conditions $\Phi(o) = \Phi_o$, $\Phi(a) = 0$, determines the flux C.

$$C(t) = D\left[\Phi_o + \frac{T_i}{Ze}\ell n\frac{n_o}{n_a} - \int_o^a dr\, u_{i||}B_\theta\right] \quad \text{(A.9)}$$

where $D = 0.685 \int_o^a dr \frac{\Omega_{i\theta}B_\theta}{\epsilon^{1/2}\, r\bar{v}_j n}$ and n_o and n_a are the densities at the centre and the wall.

The radial current produces mass acceleration in the ϕ-direction, $n\, m_i\, du_\phi/dt = e\, B_\theta C/r$. To allow the various integrals to be evaluated, we take the following profiles: $n(r) = n_o(1 - r^2/b^2)$, $B_\theta = B_{\theta a}r/a$, $u_{i||} \approx u_\phi = V(t)\,(1 - r^2/b^2)^{-1}$. The integrals are then easily evaluated, and the differential equation for V(t) gives

$$V(t) = -[1 - \exp(-\alpha t)]\left[\Phi_o + \frac{T_i}{Ze}\ell n\left(\frac{n_o}{n_a}\right)\right]\frac{2a}{b^2 B_{\theta a}} / \ell n(1 - a^2/b^2) \quad \text{(A.10)}$$

where $\alpha = -0.73\, \bar{v}_j(b/r)^{1/2}\, \ell n[1 - a^2/b^2]\cdot[\tanh^{-1}(a/b)^{1/2} - \tan^{-1}(a/b)^{1/2}]^{-1}$. For $t > \alpha^{-1}$ the mass flow tends to the asymptotic value at which Γ_i is zero. When the electron diffusion is taken into account, the asymptotic value is that for which $\Gamma_i = \Gamma_e$.

Although once again we reach the ambipolar state, the neoclassical diffusion is non-ambipolar over a finite time interval, $\tau = 0(\alpha^{-1})$, demonstrating that it is not automatically ambipolar. For typical tokamak parameters this time interval would be of the order of milliseconds. The approach to ambipolarity results not from any property of neoclassical processes, but simply because the $j_r B_\theta$ driven acceleration continues until $j_r = 0$.

REFERENCES

[1] F.L. Hinton, M.N. Rosenbluth, Phys. Fluids, 16, 836 (1973).

[2] A.A. Galeev, R.Z. Sagdeev, Sov. Phys. JETP 26, 233 (1968).

[3] T.E. Stringer, Phys. Fluids, 13, 810 (1970).

[4] T.E. Stringer, Phys. Rev. Lett., 22, 770 (1969).

[5] S.P. Hirshman, D.J. Sigmar, Nucl. Fusion, 21, 1079 (1981).

[6] B. Coppi, M.N. Rosenbluth, R.Z. Sagdeev, Phys. Fluids, 10, 582 (1967).

[7] R.D. Hazeltine, E.P. Lee, M.N. Rosenbluth, Phys. Fluids, 14, 361 (1971).

[8] P.H. Rutherford, Phys. Fluids., 13, 482 (1970).

[9] P.H. Rutherford, L.M. Kovrizhnikh, M.N. Rosenbluth, F.L. Hinton, Phys. Rev. Lett., 25, 1090 (1970).

[10] J.W. Connor, T.E. Stringer, Phys. Fluids, 14, 2184 (1971).

[11] T.E. Stringer, Plasma Phys., 14, 1063 (1972).

APPENDIX 1.

THE JET TEAM

JET Joint Undertaking, Abingdon, Oxon, OX14 3EA, U.K.

J.M.Adams[1], F.Alladio[4], H.Altmann, R.J.Anderson, G.Appruzzese, W.Bailey, B.Balet, D.V.Bartlett, L.R.Baylor[24], K.Behringer, A.C.Bell, P.Bertoldi, E.Bertolini, V.Bhatnagar, R.J.Bickerton, A.Boileau[3], T.Bonicelli, S.J.Booth, G.Bosia, M.Botman, D.Boyd[31], H.Brelen, H.Brinkschulte, M.Brusati, T.Budd, M.Bures, T.Businaro[4], H.Buttgereit, D.Cacaut, C.Caldwell-Nichols, D.J.Campbell, P.Card, J.Carwardine, G.Celentano, P.Chabert[27], C.D.Challis, A.Cheetham, J.Christiansen, C.Christodoulopoulos, P.Chuilon, R.Claesen, S.Clement[30], J.P.Coad, P.Colestock[6], S.Conroy[13], M.Cooke, S.Cooper, J.G.Cordey, W.Core, S.Corti, A.E.Costley, G.Cottrell, M.Cox[7], P.Cripwell[13], F.Crisanti[4], D.Cross, H.de Blank[16], J.de Haas[16], L.de Kock, E.Deksnis, G.B.Denne, G.Deschamps, G.Devillars, K.J.Dietz, J.Dobbing, S.E.Dorling, P.G.Doyle, D.F.Düchs, H.Duquenoy, A.Edwards, J.Ehrenberg[14], T.Elevant[12], W.Engelhardt, S.K.Erents[7], L.G.Eriksonn[5], M.Evrard[2], H.Falter, D.Flory, M.Forrest[7], C.Froger, K.Fullard, M.Gadeberg[11], A.Galetsas, R.Galvao[8], A.Gibson, R.D.Gill, A.Gondhalekar, C.Gordon, G.Gorini, C.Gormezano, N.A.Gottardi, C.Gowers, B.J.Green, F.S.Griph, M.Gryzinski[26], R.Haange, G.Hammett[6], W.Han[9], C.J.Hancock, P.J.Harbour, N.C.Hawkes[7], P.Haynes[7], T.Hellsten, J.L.Hemmerich, R.Hemsworth, R.F.Herzog, K.Hirsch[14], J.Hoekzema, W.A.Houlberg[24], J.How, M.Huart, A.Hubbard, T.P.Hughes[32], M.Hugon, M.Huguet, J.Jacquinot, O.N.Jarvis, T.C.Jernigan[24], E.Joffrin, E.M.Jones, L.P.D.F.Jones, T.T.C.Jones, J.Källne, A.Kaye, B.E.Keen, M.Keilhacker, G.J.Kelly, A.Khare[15], S.Knowlton, A.Konstantellos, M.Kovanen[21], P.Kupschus, P.Lallia, J.R.Last, L.Lauro-Taroni, M.Laux[33], K.Lawson[7], E.Lazzaro, M.Lennholm, X.Litaudon, P.Lomas, M.Lorentz-Gottardi[2], C.Lowry, G.Magyar, D.Maisonnier, M.Malacarne, V.Marchese, P.Massmann, L.McCarthy[28], G.McCracken[7], P.Mendonca, P.Meriguet, P.Micozzi[4], S.F.Mills, P.Millward, S.L.Milora[24], A.Moissonnier, P.L.Mondino, D.Moreau[17], P.Morgan, H.Morsi[14], G.Murphy, M.F.Nave, M.Newman, L.Nickesson, P.Nielsen, P.Noll, W.Obert, D.O'Brien, J.O'Rourke, M.G.Pacco-Düchs, M.Pain, S.Papastergiou, D.Pasini[20], M.Paume[27], N.Peacock[7], D.Pearson[13], F.Pegoraro, M.Pick, S.Pitcher[7], J.Plancoulaine, J-P.Poffé, F.Porcelli, R.Prentice, T.Raimondi, J.Ramette[17], J.M.Rax[27], C.Raymond, P-H.Rebut, J.Removille, F.Rimini, D.Robinson[7], A.Rolfe, R.T.Ross, L.Rossi, G.Rupprecht[14], R.Rushton, P.Rutter, H.C.Sack, G.Sadler, N.Salmon[13], H.Salzmann[14], A.Santagiustina, D.Schissel[25], P.H.Schild, M.Schmid, G.Schmidt[6], R.L.Shaw, A.Sibley, R.Simonini, J.Sips[16], P.Smeulders, J.Snipes, S.Sommers, L.Sonnerup, K.Sonnenberg, M.Stamp, P.Stangeby[19], D.Start, C.A.Steed, D.Stork, P.E.Stott, T.E.Stringer, D.Stubberfield, T.Sugie[18], D.Summers, H.Summers[20], J.Taboda-Duarte[22], J.Tagle[30], H.Tamnen, A.Tanga, A.Taroni, C.Tebaldi[23], A.Tesini, P.R.Thomas, E.Thompson, K.Thomsen[11], P.Trevalion, M.Tschudin, B.Tubbing, K.Uchino[29], E.Usselmann, H.van der Beken, M.von Hellermann, T.Wade, C.Walker, B.A.Wallander, M.Walravens, K.Walter, D.Ward, M.L.Watkins, J.Wesson, D.H.Wheeler, J.Wilks, U.Willen[12], D.Wilson, T.Winkel, C.Woodward, M.Wykes, I.D.Young, L.Zannelli, M.Zarnstorff[6], D.Zasche[14], J.W.Zwart.

PERMANENT ADDRESS

1. UKAEA, Harwell, Oxon. UK.
2. EUR-EB Association, LPP-ERM/KMS, B-1040 Brussels, Belgium.
3. Institute National des Récherches Scientifique, Quebec, Canada.
4. ENEA-CENTRO Di Frascati, I-00044 Frascati, Roma, Italy.
5. Chalmers University of Technology, Göteborg, Sweden.
6. Princeton Plasma Physics Laboratory, New Jersey, USA.
7. UKAEA Culham Laboratory, Abingdon, Oxon. UK.
8. Plasma Physics Laboratory, Space Research Institute, Sao José dos Campos, Brazil.
9. Institute of Mathematics, University of Oxford, UK.
10. CRPP/EPFL, 21 Avenue des Bains, CH-1007 Lausanne, Switzerland.
11. Risø National Laboratory, DK-4000 Roskilde, Denmark.
12. Swedish Energy Research Commission, S-10072 Stockholm, Sweden.
13. Imperial College of Science and Technology, University of London, UK.
14. Max Planck Institut für Plasmaphysik, D-8046 Garching bei München, FRG.
15. Institute for Plasma Research, Gandhinagar Bhat Gujat, India.
16. FOM Instituut voor Plasmafysica, 3430 Be Nieuwegein, The Netherlands.
17. Commissiariat à L'Energie Atomique, F-92260 Fontenay-aux-Roses, France.
18. JAERI, Tokai Research Establishment, Tokai-Mura, Naka-Gun, Japan.
19. Institute for Aerospace Studies, University of Toronto, Downsview, Ontario, Canada.
20. University of Strathclyde, Glasgow, G4 ONG, U.K.
21. Nuclear Engineering Laboratory, Lapeenranta University, Finland.
22. JNICT, Lisboa, Portugal.
23. Department of Mathematics, Univeristy of Bologna, Italy.
24. Oak Ridge National Laboratory, Oak Ridge, Tenn., USA.
25. G.A. Technologies, San Diego, California, USA.
26. Institute for Nuclear Studies, Swierk, Poland.
27. Commissiariat à l'Energie Atomique, Cadarache, France.
28. School of Physical Sciences, Flinders University of South Australia, South Australia SO42.
29. Kyushi University, Kasagu Fukuoka, Japan.
30. Centro de Investigaciones Energeticas Medioambientales y Techalogicas, Spain.
31. University of Maryland, College Park, Maryland, USA.
32. University of Essex, Colchester, UK.
33. Akademie de Wissenschaften, Berlin, DDR.

CR 88.49.1A