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A model of magnetic field reconnection in tokamak sawteeth is given. The reconnection rate is determined by electron inertia rather than resistivity and this leads to a faster sawtooth collapse than Kadomtsev reconnection.

Introduction

The concept of fast reconnection in tokamak sawtooth oscillations depends upon localisation of the reconnection process in a narrow layer formed at the $q = 1$ surface. In the Kadomtsev model [1] the width of this layer is determined by the resistivity. However under the conditions of most tokamak experiments resistivity is not the determining factor.

In a full reconnection the helical flux reconnected per unit length is given approximately by

$$\Phi \sim \frac{1}{4}(1-q_o)r_1 B_{\theta 1}$$

where q_o is the axial value of q, $B_{\theta 1}$ is the initial poloidal magnetic field at the $q = 1$ surface and r_1 is the radius of this surface. Thus the electric field generated in the layer is

$$E \sim \frac{(1-q_o)r_1 B_{\theta 1}}{4\tau_c}$$

where τ_c is the time taken for the reconnection. If we use ohm's law to calculate the resulting current density we obtain

$$j \sim \frac{(1-q_o)r_1 B_{\theta 1}}{4\eta\tau_c} .$$

and the corresponding electron drift velocity is

$$v_d \sim \frac{(1-q_o)r_1 B_{\theta 1}}{4\eta n e\tau_c}$$

where η is the electrical resistivity and n the electron density. Thus using $\eta = m/2ne^2\,\tau_e$ where τ_e is the electron collision time and $q(r_1) = B_\phi r_1/B_{\theta 1}R = 1$, where B_ϕ is the toroidal magnetic field and R is the major radius, we obtain

$$v_d \sim (1-q_o)\frac{r_1}{R}\frac{\tau_e}{\tau_c}\omega_c r_1 \quad . \tag{1}$$

where ω_c is the electron cyclotron frequency.

Typical values are $(1 - q_o) = 0.3$, $r_1/R = 0.1$, $\tau_e/\tau_c = 0.1$ $\omega_c = 0.5 \times 10^{12}\ s^{-1}$ and $r_1 = 0.2$ m. Substitution into equation (1) gives

$$v_d \sim c \quad .$$

where c is the velocity of light. It is clear therefore that under typical conditions the usual Ohm's law predicts a drift velocity much in excess of the electron thermal velocity and that consequently, simple resistive dissipation is inappropriate.

The actual behaviour can be understood by recognising that the electrons which carry the current is the layer spend only a short time in the layer. To maintain the required current it is therefore necessary that during their brief stay, they are accelerated to the velocity required to carry the necessary current. To illustrate the relationship between the different reconnection regimes we shall use the Ohm's law

$$\underline{E} + \underline{v} \times \underline{B} = \eta \underline{j} + \frac{m}{ne^2}\underline{v}.\underline{\nabla}\underline{j}$$

where $\underline{v}$ is the plasma velocity. This equation does not describe the transition between the two types of behaviour satisfactorily but is appropriate in the two limits.

We shall find that the acceleration of the electrons constitutes a larger impedance than the collisional resistivity. This broadens the current layer and allows a greater flow of plasma through the layer to give a faster collapse.

The Reconnection Model

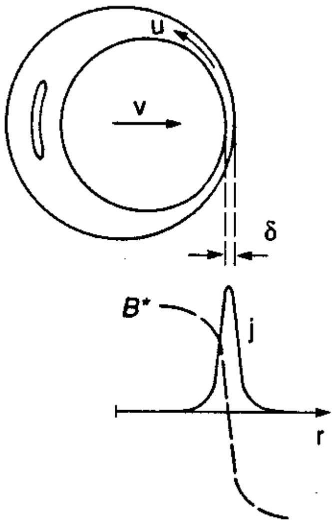

Figure 1. Schematic diagram of reconnection model and the associated current layer.

Figure 1 illustrates the basic geometry. The plasma core undergoes an m = 1 instability and moves sideways with a velocity v. Reconnection takes place in a layer of thickness δ and the plasma flows along this layer into the magnetic island with a velocity u. The length of the layer is characterised by the radius, r_1, of the q = 1 surface. The sideways displacement of the plasma produces an electric field in the reconnection region and the rate of reconnection is determined by the impedance to the resulting current layer. In a perfectly conducting plasma the electric field would be zero and no reconnection would take place. In Kadomtsev's model the electric field is determined by the resistivity through $E = \eta j$. In the present model the electric field is that necessary to provide the acceleration of the electrons.

The magnetic field involved in reconnection is the helical component B^*. This is the magnetic field perpendicular to the m = 1, n = 1 helix. In the original symmetric equilibrium $B^* = (1 - q) B_\theta$.

To see how the electric field arises, consider the helical flux between the moving magnetic axis and the $B^* = 0$ surface at the centre of the layer. As the axis moves toward the $B^* = 0$ surface, flux is continually removed by reconnection. From Faraday's law this changing flux produces the required electric field. The rate of change of this flux per unit length is given by the velocity of the plasma multiplied by the helical magnetic field at the edge of the layer, that is vB^*_{edge}. Thus the electric field on the surface where $B^* = 0$ is of order vB^*_{edge} and this electric field characterises the electric field in the layer.

The Flow Rate

From continuity, the flow into the layer is equal to the outward flow along the layer, that is

$$v r_1 \sim u\delta \tag{1}$$

The pressure in the layer is $p \sim B^{*2}/2\mu_o$, where B^* now represents B^*_{edge}. Equating this pressure to $\frac{1}{2}\rho u^2$ gives

$$u \sim B^*/\sqrt{\mu_o \rho} \ . \tag{2}$$

Relations (1) and (2) gives

$$v \sim \frac{\delta}{\tau_A} \tag{3}$$

where

$$\tau_A = \frac{r_1}{B^*/\sqrt{\mu_o \rho}} \ .$$

The reconnection time is

$$\tau = \frac{r_1}{v} \ ,$$

and so, using relation (3)

$$\tau \sim \frac{r_1}{\delta}\tau_A \ . \tag{4}$$

Calculation of the reconnection time now requires an expression for the layer thickness, δ.

The Reconnection Time

We obtain δ by considering the component of Ohm's law parallel to the helix,

$$E + v_r B^* = \eta j + \frac{m}{ne^2} v_r \frac{dj}{dr} \quad , \tag{5}$$

where v_r is the radial plasma flow velocity and j is the helical current density.

We first obtain Kadomtsev's result using equation (5) and keeping only the resistive term on the right. Then, recalling that $E \sim vB^*$, we have

$$vB^* \sim \eta j \quad .$$

Ampere's law gives $j \sim B^*/\mu_o\delta$, and hence

$$\delta \sim \frac{\eta}{\mu_o v}$$

so that, using relation (3) to eliminate v, we obtain the layer thickness

$$\delta \sim \left(\frac{\tau_A}{\tau_R}\right)^{1/2} r_1 \quad , \tag{6}$$

where τ_R is the resistive diffusion time

$$\tau_R = \frac{r_1^2}{\eta / \mu_o} \quad .$$

Substitution of relation (6) into relation (4) gives the Kadomtsev reconnection time

$$\tau_K \sim \left(\tau_A \tau_R\right)^{1/2} \quad . \tag{7}$$

We now consider the case where the second term on the right of equation (5) is much larger than ηj, so that

$$vB^* \sim \frac{m}{ne^2} v \frac{j}{\delta}$$

and, using Ampere's law again, we obtain the layer thickness

$$\delta \sim \frac{c}{\omega_p} \tag{8}$$

where c is the velocity of light and the plasma frequency ω_p is given by

$$\omega_p^2 = \frac{ne^2}{\varepsilon_o m} \quad .$$

Substitution of relation (8) into relation (4) then gives the new reconnection time

$$\tau \sim \frac{r_1 \omega_p}{c} \tau_A \quad . \tag{9}$$

Numerical Values

Let us now consider the reconnection times predicted by relations (7) and (9) for typical experimental conditions. For $n = 3 \times 10^{19}\ m^{-3}$, we have $c/\omega_p = 1 \times 10^{-3}m$, and taking $B^* \sim B_{\theta 1}\ (1-q_o)$, with $B_\phi r_1 / B_{\theta 1} R = 1$, we obtain

$$\tau_A = 0.3 \times 10^{-6} \frac{R}{(1-q_o)B_\phi} \quad s \quad .$$

Taking JET as an example of a large tokamak, $B_\phi = 3T$ and $R = 3m$, and using $1-q_o = 0.3$ gives $\tau_A = 1 \times 10^{-6}$ s.

For a temperature of 3 keV and $Z_{eff} = 2$, $\eta/\mu_o = 1 \times 10^{-2}\ m^2\ s^{-1}$ and taking $r_1 = 0.3m$, then $\tau_R = 10s$. Using these values, relations (7) and (9) give the reconnection times

$$\tau_K = 3\ ms$$
$$\tau = 300 \mu s \quad .$$

Comparing these values with the experimental collapse time of ~ 100 μs it is clear that τ gives a closer value than τ_K. If further we accept the evidence that the change in q_o at reconnection is only a fraction of $1-q_o$, so that only partial reconnection takes place, there seems to be the possibility of agreement with τ.

For a smaller tokamak having dimensions one fifth of those of JET and a temperature of 1 keV, the corresponding values are

$$\tau_K = 100 \mu s$$
$$\tau = 10 \mu s \quad .$$

The observed fast collapse times for smaller tokamaks are typically somewhat less than 100 μs, in reasonable agreement with the Kadomtsev model. However we must bear in mind the roughness of the analysis behind both figures.

We shall now derive a general expression for τ encompassing both types of behaviour.

General Expression for τ

Using equation (5), but now with both terms on the right,

$$vB^* \sim \eta j + \frac{m}{ne^2} v \frac{j}{\delta} \quad ,$$

and following the same procedure as before, we obtain the layer thickness

$$\delta^2 \sim \frac{\tau_A}{\tau_R} r_1^2 + \frac{c^2}{\omega_p^2} \quad .$$

Then, substituting δ into relation (4), we obtain the reconnection time

$$\tau \sim \frac{\tau_A}{\left(\frac{\tau_A}{\tau_R} + \frac{c^2}{r_1^2 \omega_p^2} \right)^{1/2}} \quad . \tag{10}$$

Relation (10) shows that the expected reconnection time is the shorter of the two times considered. Thus from the numerical comparisons discussed above we would not expect to see the Kadomtsev reconnection time but rather the shorter reconnection time limited by electron inertia, as given by relation (9).

Discussion

Relation (10) does not adequately represent the transition between the two cases. In the transitional regime electrons entering the layer will initially obey the resistive Ohm's law. Then, as this drift velocity increases they undergo the runaway process in which the resistive drag becomes a decreasing function of velocity. Equation (5) then no longer describes the behaviour, which will in fact be very complex. However the generalised relation (10) for τ does represent the two limiting cases and provides a basis for comparison.

It is interesting to conjecture whether the model described here could lead to a clarification of some of the other puzzles associated with the sawtooth collapse [2]. For example it might be that the sudden appearance of fast growth is related to the transition from the slow resistive to the faster inertial behaviour. The observation of incomplete reconnection could be understood if the fast current carrying electrons were at some point retained in the layer. This would make the layer highly conducting and virtually halt reconnection.

The analysis presented here can only be used to indicate the type of behaviour to be expected in sawtooth reconnection and the quantitative estimates are obviously subject to great uncertainty. However it is clear that the physics of the layer needs re-consideration. The present model constitutes a first attempt at calculating the consequences. It seems quite possible that additional physics might be involved, one obvious possibility being that of velocity space instabilities.

Summary

A model of reconnection for tokamak sawteeth has been outlined. The impedance to reconnection is electron inertia rather than resistivity. This mechanism predicts a shorter reconnection time than the Kadomtsev model, as observed in large tokamaks.

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