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# Stabilisation of Drift-Tearing Modes at the Breakdown of the Constant- $\psi$ Approximation

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STABILISATION OF DRIFT-TEARING MODES AT THE BREAKDOWN OF  
THE CONSTANT- $\psi$  APPROXIMATION

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ABSTRACT

The growth rate of the linearised drift-tearing mode is found to be a non-monotonic function of the stability parameter  $\Delta'$ . It reaches a maximum for  $\Delta' \sim 0.5 \Delta'_1 > 0$  and becomes negative for  $\Delta' \gtrsim \Delta'_1$ , corresponding to the regime where the constant- $\psi$  approximation breaks down. A second mode, identified as the diamagnetic modification of the  $\eta^{1/3}$ -mode near the condition for ideal magnetohydrodynamics marginal stability, becomes unstable for  $\Delta' \gtrsim \Delta'_2 > \Delta'_1$ , leaving a stable window of values of  $\Delta'$ .

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As is well known, a magnetised plasma carrying an inhomogeneous electrical current density is subject to the onset of resistive modes that lead to a change in the topology of the magnetic field [1]. The instability tends to reduce the current density gradient through a process of tearing and reconnection of magnetic field lines and the formation of magnetic island configurations. This fundamental plasma phenomenon has been shown to occur both in astrophysics [2] and in controlled fusion experiments [3]. In particular, in toroidal confinement systems, resistive modes with low poloidal ( $m$ ) and toroidal ( $n$ ) mode numbers are often associated with macroscopic plasma oscillations and loss of confinement. An  $m = 1, n = 1$  helical displacement of a plasma core where  $q(r) < 1$  ( $q = rB_\phi/RB_\theta$ , with  $r$  the distance from the magnetix axis,  $R$  the torus major radius,  $B_\phi$  and  $B_\theta$  the toroidal and poloidal magnetic field components, respectively) is observed preceding the relaxation phase of the sawtooth-like internal oscillations of the plasma temperature [4], while modes dominated by the  $m = 2$  poloidal harmonic are associated with major disruptions where a catastrophic loss of the toroidal current and termination of the plasma discharge suddenly take place [5].

In this Letter, we consider low- $m$  resistive modes of the tearing type in collisional regimes where electron pressure corrections to Ohm's law and ion diamagnetic effects become important [6]. In these regimes, the tearing mode [1] changes its character significantly, going from a purely growing mode where the perturbed current density along the magnetic field is well localised near the rational surface  $r = r_s$  where  $q(r_s) = m/n$ , to an overstable mode. The magnetic energy released by reconnection around  $r = r_s$  ceases to be the main energy source available to the mode. The energy balance is dominated [7] by the energy released by the electron pressure. This energy is propagated and absorbed at a finite distance from the reconnecting surface

by dissipation processes such as electron thermal conductivity [8] or ion viscosity [9], which, if sufficiently small, do not perturb the eigen-mode significantly. The linearised mode frequency in this regime is  $\omega \approx \hat{\omega}_{*e} \equiv \omega_{*e} (1 + \alpha \eta_e)$ , where  $\eta_e = d \ln T_e / d \ln n_e$ ,  $\alpha = 1.71$  for electrons obeying an adiabatic equation of state, and  $\omega_{*e} = [(k_\theta c / eB) (d \ln n_e / dr)]_s$  is the electron drift frequency at  $r = r_s$ , with  $k_\theta = m/r$ , whence the name of drift-tearing mode [8]. Its growth rate is

$$\gamma_{DT} = C_1 m^{2/3} \epsilon_\eta \{ \omega_A^2 / [\hat{\omega}_{*e} (\hat{\omega}_{*e} - \omega_{di})] \}^{1/3} (\Delta')^{4/3} \omega_A, \quad (1)$$

where  $C_1 = C_0^{4/3} / 2$ ,  $C_0 = (2/\pi) \Gamma(5/4) / \Gamma(3/4) = 0.47$ ,  $\epsilon_\eta \equiv \eta c^2 / (4\pi r_s^2 \omega_A)$  is the inverse magnetic Reynold's number, with  $\eta$  the electrical resistivity along magnetic field lines,  $\omega_A \equiv sV_A / \sqrt{3} R_0$ ,  $s \equiv r_s q'(r_s)$  and  $V_A \equiv B / (4\pi m_i n_i)^{1/2}$ ;  $\omega_{di} = - [(k_\theta c / eB) (d \ln n_i / dr)]_s$  is the ion diamagnetic frequency at  $r = r_s$ ; and  $\Delta' \equiv [d \ln \psi / dr]_{r_s^-}^{r_s^+}$  is the logarithmic jump of the perturbed poloidal magnetic flux function  $\psi$  across the  $r = r_s$  surface. Instability corresponds to  $\Delta' > 0$ . The growth rate (1) is obtained for values of  $\hat{\omega}_{*e} > \gamma_T = C_0^{4/5} m^{2/5} \epsilon_\eta^{3/5} (\Delta')^{4/5} \omega_A$ , where  $\gamma_T$  is the tearing mode growth rate when  $\hat{\omega}_{*e} = 0$ , and for

$$\Delta' \ll \Delta'_1 \sim 2 m \omega_A / [\hat{\omega}_{*e} (\hat{\omega}_{*e} - \omega_{di})]^{1/2} \quad (2)$$

which corresponds to the validity of the constant- $\psi$  approximation as discussed below. We find that, when  $|\hat{\omega}_{*e}|, |\omega_{di}| \gg m^{2/3} \epsilon_\eta^{1/3} \omega_A$  the drift-tearing growth rate has a non-monotonic behaviour as  $\Delta'$  approaches  $\Delta'_1$ ,  $\gamma_{DT}$  reaches a maximum for  $\Delta' \sim 0.5 \Delta'_1$  and becomes negative for  $\Delta' \gg \Delta'_1$ . A second mode becomes unstable with a growth rate

$$\gamma_{R*} = m^2 \epsilon_{\eta} \omega_A^3 / |\hat{\omega}_{*e} \omega_{di}| \quad (3)$$

for values of

$$\Delta' \geq \Delta'_2 \sim (|\hat{\omega}_{*e}| / \omega_A \epsilon_{\eta})^{1/2}, \quad (4)$$

so that a stable window exists for  $\Delta'_1 < \Delta' < \Delta'_2$ . This second root corresponds to the diamagnetic modification of the  $\epsilon_{\eta}^{1/3}$ -mode found near the condition of ideal-MHD marginal stability [10]. For the  $m = 1$  case, this mode is well known under the name of resistive internal kink [11,12].

The existence of a stable window for positive values of  $\Delta'$  in a resistive MHD, diamagnetic plasma was first noted in Ref. [13] and later in Refs. [9] and [14]. However, the behaviour of the two roots approaching the stable window from either sides, and the exact extent of the stable domain, were not discussed in these previous studies. We find that the stable domain is considerably wider than previously thought (see, e.g., region VII of Fig. 2 in Ref. [14]). The stabilisation of the drift-tearing root at the breakdown of the constant- $\psi$  approximation was also noted in Ref. [15]. It was also known that electron thermal conductivity [15] and ion viscosity [16] can suppress the drift-tearing mode and the resistive kink mode, respectively. In this letter, the existence of a stable window is shown to persist even when these additional dissipation processes are vanishingly small.

To proceed with a detailed analysis, we consider the dispersion relation for tearing modes in a resistive MHD plasma with diamagnetic effects [12,17]:

$$\Delta' \delta = - \frac{m\pi}{8} Q \frac{\Gamma[(Q-1)/4]}{\Gamma[(Q+5)/4]} \quad (5)$$

where  $Q = -i\delta^2(\omega - \hat{\omega}_{*e}) / (m^2 \epsilon_{\eta} \omega_A)$ ,  $\delta^4 = -im^2 \epsilon_{\eta} \omega(\omega - \omega_{di}) / [(\omega - \hat{\omega}_{*e})\omega_A]$ ,  $\omega = \omega_r + i\gamma$  is the eigenfrequency and  $m$  is taken to be positive. For the  $m = n = 1$  case, one can relate  $\Delta' = -\pi/\lambda_H$ , where the parameter [11,12]  $\lambda_H$  is



proportional to the negative of the MHD energy functional  $\delta W$  ( $\lambda_H > 0$  is the instability condition for the ideal MHD internal kink mode). The perturbed current density in the resistive layer can be conveniently written in Fourier space [17],  $\tilde{J}(k) = \int_{-\infty}^{\infty} \tilde{J}(x) e^{-ikx} dx$ , where

$$\tilde{J}(k) = J_0 U[(Q-1)/4, -1/2, \delta^2 k^2] \exp(-\delta^2 k^2/2), \quad (6)$$

$J_0$  is a normalisation constant and  $U$  is a Kummer's (confluent hypergeometric) function [18]. Acceptable solutions of Eq. (5) must satisfy the condition  $\text{Re } \delta^2 > 0$  in order for the corresponding eigen-function (6) to be spatially localised. This condition is violated for the drift-tearing root and, for sufficiently large values of  $\omega_{di}$  and  $\hat{\omega}_{*e}$  by the resistive kink root. However, as mentioned previously, for these roots the eigen-function is regularised in the presence of electron thermal conductivity and/or ion viscosity.

The constant- $\psi$  regime corresponds to the limit  $|Q| \ll 1$ . This definition is equivalent to that of Ref. 1, which consists of writing the perturbed poloidal magnetic flux  $\psi$  in the resistive layer (where  $\nabla_{\perp}^2 \psi(x) \propto J(x)$ ) as  $\psi = \psi_0 + \psi_1(x)$ , where  $\psi_0 = \text{constant}$  and  $\psi_1(x)/\psi_0 \sim |\delta|\Delta' \ll 1$ . Then,  $[(d\psi/dx)/\psi_0]_{\text{layer}} \sim \Delta'$ . It can be immediately verified that the condition  $|Q|' \ll 1$  on the solutions of Eq. (5) implies  $|\delta|\Delta' \ll 1$ . In this limit, the dispersion relation reduces to

$$-i(\omega - \hat{\omega}_{*e}) \delta = C_0 m \Delta' \epsilon_{\eta} \omega_A. \quad (7)$$

For the sake of simplicity we set  $\hat{\omega}_{*e} = -\omega_{di}$  in the following numerical computations. First we consider the case where the diamagnetic terms are small compared with the mode growth rate. In this limit Eq. (7) gives the growth rate of the tearing mode,  $\gamma = \gamma_{\Gamma}$ . The constant- $\psi$  regime breaks down

when  $\Delta' \sim (m/\epsilon_\eta)^{1/3}$ . The mode growth rate increases monotonically with  $\Delta'$  and, as  $\Delta' \rightarrow \infty$ , it approaches the value  $\gamma = \gamma_R \equiv m^{2/3} \epsilon_\eta^{1/3} \omega_A$ , corresponding, for  $m = 1$ , to the growth rate of the resistive internal kink.

Before we proceed to show the mode behaviour at finite  $\hat{\omega}_{*e}$ , we point out that in the plane identified by positive values of the parameters  $D \equiv (\Delta' \epsilon_\eta^{1/3} / \pi m^{1/3})^{-1}$  and  $\Omega_* \equiv \hat{\omega}_{*e} / \gamma_R$ , a branch point exists, which has been numerically determined to be near  $D = 2.4$ ,  $\Omega_* = 0.91$ . Following one root of the dispersion relation along a closed path encircling this point will not return the eigen-frequency to its initial value.

Next we consider the behaviour of the growth rate of the tearing mode as  $\hat{\omega}_{*e}$  is increased at a fixed value of  $D > 2.4$ . Starting from  $\gamma = \gamma_T$ , the growth rate is reduced and approaches the value  $\gamma = \gamma_{DT}$  (see Eq. 1), while  $\omega_r$  approaches  $\hat{\omega}_{*e}$ , as  $\hat{\omega}_{*e} > \gamma_T$ . For this root,  $\text{Re } \delta^2$  becomes negative quite soon, for values of  $\hat{\omega}_{*e} \approx 1.4 \gamma_T$ , but we have already indicated how this problem is overcome in the presence of additional dissipation. For larger values of  $\hat{\omega}_{*e}$ , corresponding to  $\hat{\omega}_{*e} / \omega_A \sim 0.6(m/\Delta')$  (see Eq. 12),  $\gamma$  changes sign and the instability is fully suppressed. This behaviour is illustrated by the examples in Fig. 1. Note that as the marginal stability curve is approached,  $|Q| \sim 1$  and the constant- $\psi$  approximation breaks down.

A different behaviour is found for  $D < 2.4$  when  $\hat{\omega}_{*e}$  is increased. Firstly, the mode oscillation frequency remains always well below  $\hat{\omega}_{*e}$ . The mode growth rate decreases from  $\gamma = \gamma_R$  to  $\gamma = \gamma_{R^*}$  (see Eq. 3) and eventually changes sign, although now this happens for relatively large values of  $\hat{\omega}_{*e}$  such that  $\hat{\omega}_{*e} / \omega_A \sim 0.3 \epsilon_\eta (\Delta')^2$  (see Eq. 4). Also in this case  $\text{Re } \delta^2$  changes sign when  $\gamma$  is still positive. Examples of this behaviour are shown in Fig. 2.

In Fig. 3, the two roots, with  $\gamma = \gamma_{DT}$ ,  $\omega_r = \hat{\omega}_{*e}$  at small  $\Delta'$  and  $\gamma = \gamma_{R^*}$ ,  $\omega_r < \hat{\omega}_{*e}$  at large  $\Delta'$ , are followed as  $\Delta'$  is varied. The two roots do

not connect, as expected in the presence of the branch point. The two roots connect in the third example of Fig. 3 for  $\Omega_* < 0.91$ . As anticipated, the drift-tearing growth rate is a non-monotonic function of  $\Delta'$  for  $\Omega_* \geq 0.75$ . The drift-tearing mode becomes stable for  $\Delta' \sim 0.6m \omega_A / \hat{\omega}_{*e}$ . Instability re-occurs for larger values of  $\Delta'$  through a different branch of the dispersion relation, leaving a stable window of values of  $\Delta'$ . This window opens up for  $\Omega_*$  larger than a minimum value ( $\Omega_* \geq 0.83$ ), a value slightly smaller than the critical value corresponding to the branch point discussed above, and becomes wider as  $\Omega_*$  increases. In Fig. 4 we display the stable domain and instability regimes in the  $(D, \Omega_*)$  plane. Note that a branch cut through the stable domain can be chosen, as illustrated in the figure, so that the tearing, drift-tearing, and the  $\epsilon_\eta^{1/3}$ -modes all lie on the same Riemann sheet.

A heuristic analytical model that accounts for the non-monotonic dependence of  $\gamma_{DT}$  on  $\Delta'$  is obtained by rewriting Eq. (5) in the form

$$C_o \Delta' \delta = m H(Q) [Q/(1 - Q)] \quad (8)$$

where  $H(Q) = \{\Gamma[(Q + 3)/4]/\Gamma(3/4)\} \{\Gamma(5/4)/\Gamma[(Q + 5)/4]\}$ , and by approximating  $H(Q) \approx 1$ . Then we obtain

$$-i(\omega - \hat{\omega}_{*e}) \delta = C_o (\Delta'/m) [m^2 \epsilon_\eta \omega_A + i \delta^2 (\omega - \hat{\omega}_{*e})] \quad (9)$$

The drift-tearing solution is recovered from (9) for  $m^2 \epsilon_\eta \omega_A \gg \delta^2 (\omega - \hat{\omega}_{*e})$  [cf. Eq. (7)], while as  $\Delta' \rightarrow \infty$  the relevant dispersion relation becomes  $m^2 \epsilon_\eta + i \delta^2 (\omega - \hat{\omega}_{*e}) \approx 0$ , yielding the resistive internal kink growth rate. When  $D > 2.4$  and  $\hat{\omega}_{*e} > \gamma_T$  a further simplification can be adopted by setting  $\omega \sim \hat{\omega}_{*e}$  in Eq. (9) everywhere apart from when appearing in the combination

$\omega - \hat{\omega}_* e$ . This leads to the approximate growth rate

$$\gamma_{DT} \approx C_1 (\Delta'_1)^{2/3} (\Delta')^{4/3} \epsilon_\eta \omega_A [1 - (8 C_0^{4/3}/3) (\Delta'/\Delta'_1)^{2/3}], \quad (10)$$

where the first term at the right-hand-side corresponds to the growth rate in Eq. (1). Following the analysis of Ref. [7] (see in particular Section V), the stabilisation of the drift-tearing mode as  $\Delta'$  is increased can be interpreted as related to the peculiar form of the mode energy balance. As we noted before, the energy released by the electron pressure is convected away and absorbed at a finite distance from the reconnecting surface. The rate at which this energy is convected away, rather than the specific absorption process (provided the latter is sufficiently weak), determines the growth rate of the drift-tearing mode. This rate is controlled by  $\Delta'$  and, as  $\Delta'$  approaches  $\Delta'_1$ , it increases, eventually draining all the energy available for the mode growth. On the contrary, in the energy balance of the resistive internal kink mode, the magnetic energy remains important as a source of excitation energy even in the presence of large diamagnetic effects (see Ref. [7], Table 1).

Values of  $D$  and  $\Omega_*$  in the stable domain of Fig. 4 are relevant in particular to  $m = 1, n = 1$  modes in a magnetically confined, toroidal plasma with low values of the poloidal beta parameter. For these modes, the stable domain can be accessed when the magnetic shear is reduced near the  $q = 1$  surface even if  $q(0)$  is significantly less than unity, as  $\Omega_* \propto s^{-2/3}$  and  $D \propto (1 - q_0)/s^{5/3}$  [19], with  $s \equiv r_s q'(r_s)$  the local magnetic shear parameter.

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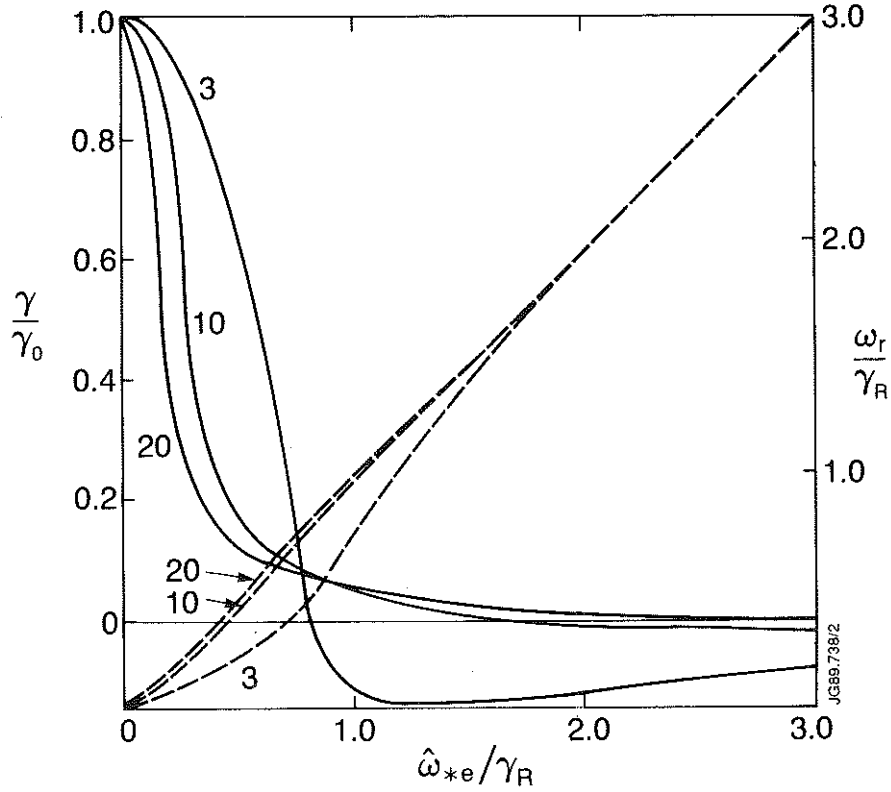


Fig. 1 Growth rates (solid curves) and oscillation frequencies (dashed curves) versus  $\hat{\omega}_e$  for different values of  $D \equiv (\Delta' \epsilon_\eta^{1/3} / \pi m^{1/3})^{-1}$  as indicated near each curve. Here  $\gamma_0$ , the value of  $\gamma$  at  $\hat{\omega}_e = 0$ , is well approximated by  $\gamma_r$  (in the text) for  $D \gg 1$ ;  $\gamma_r = m^{2/3} \epsilon_\eta^{1/3} \omega_A$ .

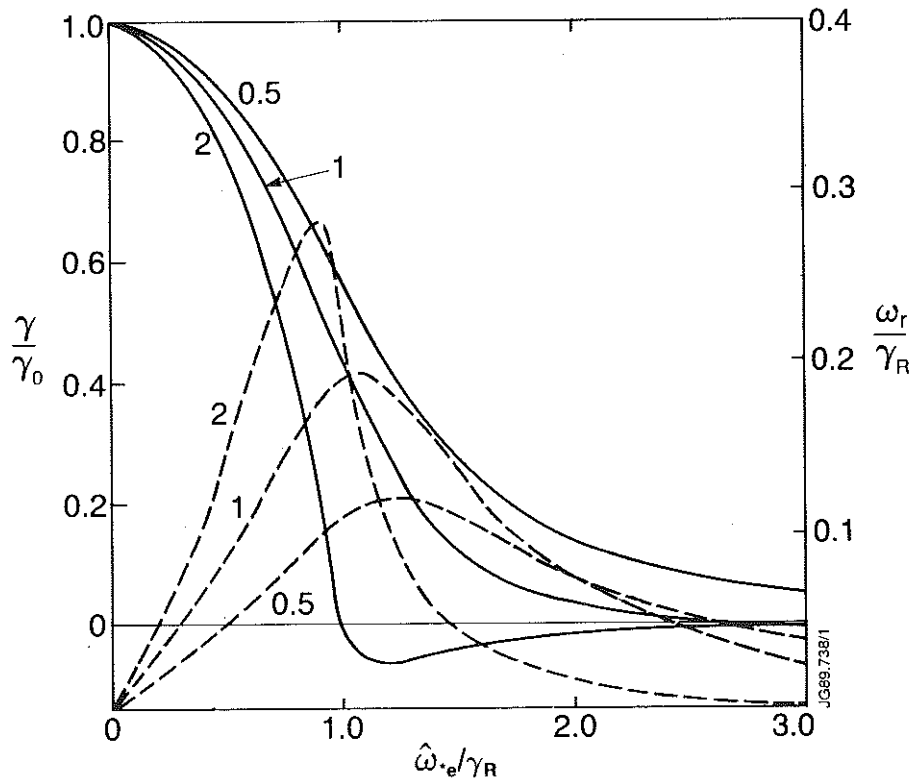


Fig. 2 Growth rates (solid curves) and oscillation frequencies (dashed curves) versus  $\hat{\omega}_e$  for different values of  $D \equiv (\Delta' \epsilon_\eta^{1/3} / \pi m^{1/3})^{-1}$  as indicated near each curve.  $\gamma_0$ , the value of  $\gamma$  at  $\hat{\omega}_e = 0$ , is well approximated by  $\gamma_r = m^{2/3} \epsilon_\eta^{1/3} \omega_A$  for  $D \rightarrow 0$ .

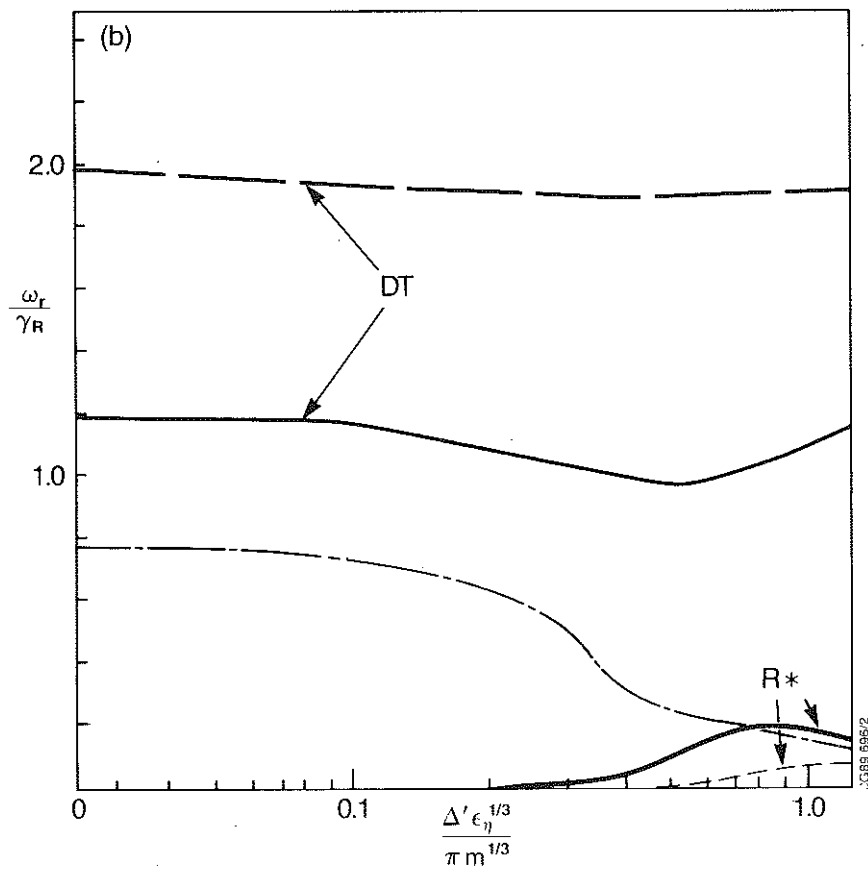
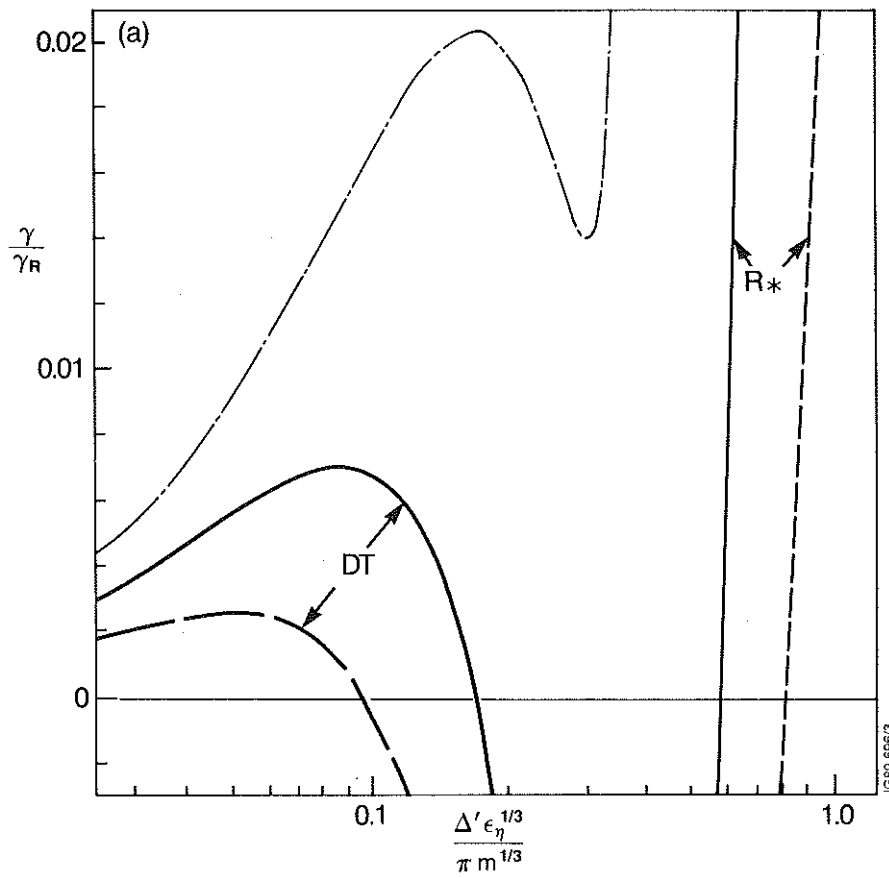


Fig. 3 (a) Normalised growth rates versus  $\Delta'$  for  $\omega_e/\omega_A = 2$  (dashed curves), 1.2 (solid curves), and 0.8 (dashed-dotted curves). The symbols 'DT' and 'R\*' identify the drift-tearing and the resistive kink ( $m = 1$ ) branches, respectively. (b) Corresponding normalised oscillation frequencies versus  $\Delta'$ .



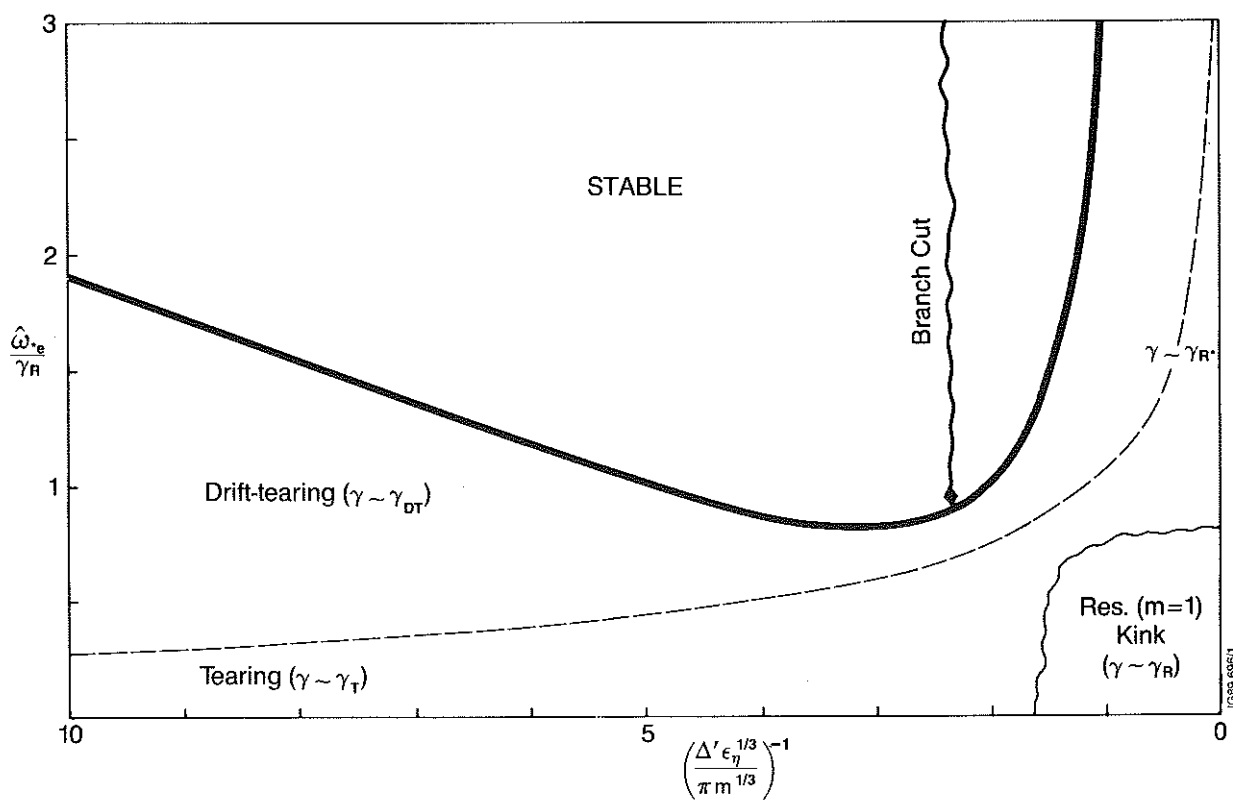


Fig. 4. Stability plane identified by the parameters  $\hat{\omega}_e/\gamma_R$  and  $(\Delta' \epsilon_\eta^{1/3}/\pi m^{1/3})^{-1}$ . The stable domain lies above and to the left of the solid line;  $\text{Re } \delta^2 < 0$  above and to the left of the dashed line. The branch point at  $(\Delta' \epsilon_\eta^{1/3}/\pi m^{1/3})^{-1} \approx 2.4$ ,  $\hat{\omega}_e/\gamma_R \approx 0.9$ , is also shown, together with the associated branch cut.

## APPENDIX 1.

### THE JET TEAM

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