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** See Appendix 1*

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## Abstract

The collapse phase of sawtooth oscillations in tokamaks is believed to be due to the rapid growth of an m=1 instability. We show here that the onset of the rapid growth is spontaneous and cannot be described by the increase of the linear growth rate arising from development of the plasma equilibrium. This result calls for a re-appraisal of the sawtooth problem.

## Introduction

The collapse phase of tokamak sawtooth oscillations has for a long time attracted theoretical interest. When these oscillations were discovered by von Goeler, Stodieck and Sauthoff [1], the first difficulty was the rapidity of the collapse. The timescale for the collapse was found in early observations to be $\sim$100μs and this is very much shorter than the characteristic resistive diffusion time in these devices $\sim$100ms. The explanation proposed by Kadomtsev [2] was that observed m=1 instability led to a magnetic reconnection in a narrow layer. The timescale is then shortened to the geometric mean of the resistive timescale and an Alfvén transit time ($\sim$0.1μs). This gave agreement with the experimental results since $(100\text{ms} \times 0.1\mu\text{s})^{1/2} = 100\mu\text{s}$.

The theoretical model then seemed to be as described by Jahns et al [3]. At some point during the ramp phase of the sawtooth a stability boundary is crossed and after that time the equilibrium becomes increasingly unstable giving a growth rate $\gamma(t)$ and a growing perturbation $\xi(t) = \xi_0 \exp \int\gamma(t)dt$. Thus the perturbation grows at a progressively faster rate culminating in a Kadomtsev sawtooth collapse.

This model does not describe JET sawteeth. The discrepancies with Kadomtsev's model have already been reported. It is found that the observed collapse is an order of magnitude too fast [4] and the flow pattern is quite different from that predicted [5,6]. Here, however, we show that there is a more fundamental discrepancy with conventional ideas.

The difficulty concerns the time development of the growth rate. While there are instabilities which have predicted growth rates comparable to those observed, it is the spontaneous onset of the growth rate which presents a problem. In the first treatment of this subject [7] it was pointed out that, because the equilibrium parameters determining the growth rate change very slowly, the fast growth rate observed in the sawtooth collapse could not "switch-on" sufficiently fast. We shall consider this in more detail later but the essential point can be made by a simple argument.

Equilibrium changes in JET are so slow that the time required for the growth rate to change from zero to a typical value observed at the collapse is at least tens of milliseconds. Thus the growth rate observed at the collapse, $\gamma \sim (25\mu s)^{-1}$ would have existed for the final few milliseconds. The displacement at a time $\delta t$ before the collapse would therefore be related to the observed displacement, $\xi_0$, by

$$\xi = \xi_0 \exp\left(-\frac{\delta t}{25\mu s}\right) \quad .$$

Thus, taking the displacement at the observed start of the collapse to be 1cm, the displacement only 1ms earlier would be

$$\begin{aligned} \xi &\sim e^{-40}\text{cm} \\ &\sim 10^{-17}\text{cm} \end{aligned}$$

Since this is of sub-nuclear dimension it is clear that this model of the growth is not correct and that the fast growth rate must have had a sudden onset.

## Experimental Results

We now present experimental results from JET which bear out the above analysis. Using the soft X-ray detector arrays [8], it is possible to form a tomographic reconstruction [9] and to follow the displacement, $\xi$, of the peak emission during the collapse. (It is already known that the time

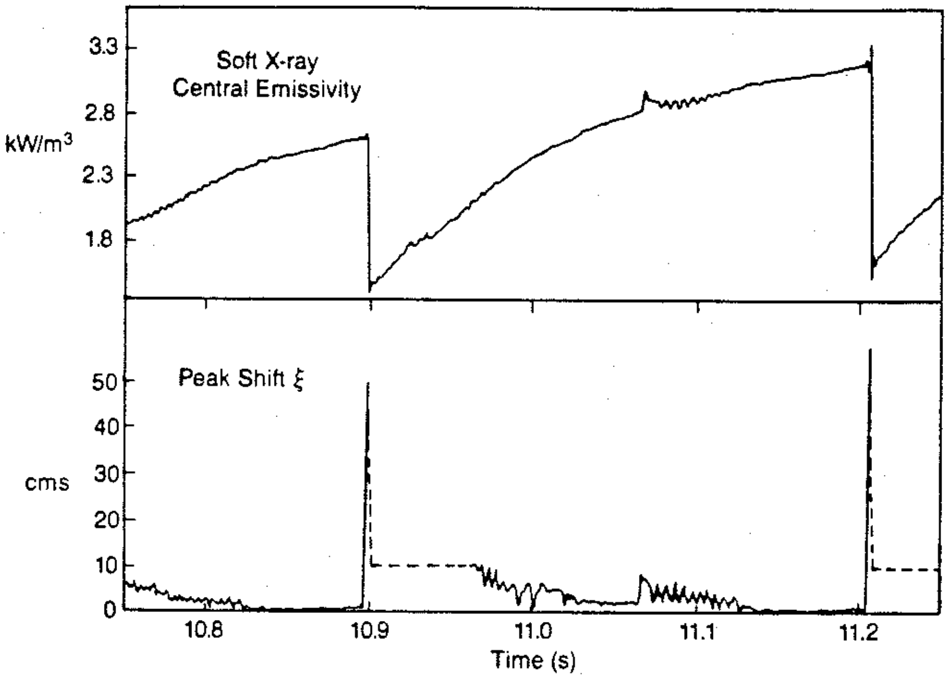


Figure 1. Showing the time behaviour of the displacement, ξ, of the peak soft X-ray emission, and its sudden increase at the time of the sawtooth collapse as indicated on the trace of the central soft X-ray emission. (The position of the peak emission is not well defined immediately following the collapse phase and is not therefore shown on the graph. The fluctuations which appear during the ramp phase have an m=1 structure but do not appear to directly influence the sawtooth behaviour).

development of this displacement coincides with that of the externally measured perturbed magnetic field [10].) Figure 1 shows $\xi$ as a function of time during a complete sawtooth oscillation. The abruptness of the growth is apparent from this figure, but to address the central question of the time development of the growth rate, a quantitative analysis is necessary.

We define $\xi$ as the magnitude of the displacement of the point of maximum X-ray emission from its average position taken over a period of 1ms preceeding the collapse. The measured displacement during this time was constant within the experimental errors. The growth rate is then defined by

$$\gamma(t) = \frac{1}{\xi}\frac{d\xi}{dt}$$

and is given by the slope of the graph of $\ln \xi$ against time. Figure 2 shows three such graphs. They are taken from different discharges and give an indication of the level of variability. The initial noise level is $\sim$1cm and the final displacement is $\sim$50cm. The dashed line has a slope corresponding to a growth rate of $(25\mu s)^{-1}$. It is seen that the growth from 1cm to 50cm occurs at approximately this rate. It is difficult to know how long this rapid growth rate has existed. However, a reasonable upper bound can be obtained by extrapolating the growth back into the noise to calculate the time at which the displacement would equal the Debye length. This time is approximately 100μs earlier.

We see from these results that the instantaneous growth rate in the sawtooth collapse reaches $(25\mu s)^{-1}$, and that this fast growth rate arises suddenly, that is on a timescale comparable to $\gamma^{-1}$ rather than on the much longer timescale required for equilibrium change.

It should perhaps be commented that some sawteeth have a slow growing precursor. However this does not affect the issue, firstly because there remain the cases without an observed precursor, and secondly because a sudden increase in growth rate from a small to a large value poses the same problem as that described above.

We now return to the question of the timescales required to increase the growth rate of specific instabilities through the time evolution of the equilibrium.

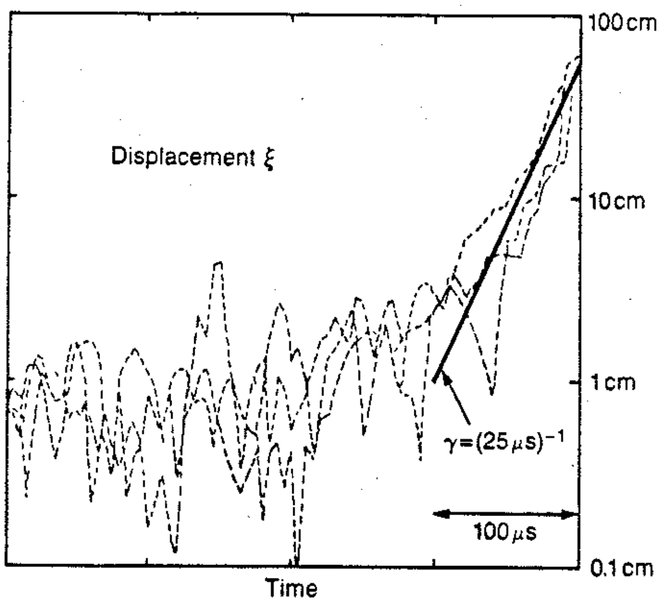


Figure 2. Graphs of the magnitude of the displacement, ξ, of the peak soft X-ray emission for three sawtooth collapses taken from different discharges. The initial noise level is ∿1cm and the growth rises out of this noise with a growth rate of $\sim(25\mu s)^{-1}$, which increases the displacement to ∿50cm in ∿100μs. (The plasma current was 3.5MA, the toroidal magnetic field 2.9T and I.C.R.H. of 4-5MW was applied.)

## Equilibrium Change

Most of the proposed instabilities depend on a change of the safety factor q and this in turn depends on resistive diffusion of the current. This is influenced by neo-classical resistivity which produces an effect localised around the axis, a subject to which we shall return later. Apart from this, the diffusion is driven by development of temperature from its flattened profile after the sawtooth collapse. The resulting change $\delta q$ in a time $\delta t$ is given by

$$\frac{\delta q}{q} \sim \frac{\delta j}{j} \sim \frac{\nabla^2(\delta\eta . j)}{\mu_0 j}\,\delta t \sim \frac{\delta\eta}{\mu_0}\,\frac{\delta t}{4r_1^2} \qquad (1)$$

where j is the current density, $\eta$ is the resistivity and $r_1$ is the radius of the q=1 surface. The change in the resistivity is related to the change in temperature, $\delta T$, and we can write

$$\frac{\delta\eta}{\eta} \sim \frac{3}{2}\,\frac{\delta T}{T} \sim \frac{3}{2}\,\frac{\Delta T}{T}\,\frac{\delta T}{\Delta T} \sim \frac{3}{2}\,\frac{\Delta T}{T}\,\frac{\delta t}{\tau_s} \qquad (2)$$

where T is the central electron temperature and $\Delta T$ is the change in T during the sawtooth period $\tau_s$. Combining relations (1) and (2) and using $q \sim 1$ we have approximately

$$\delta q \sim \frac{\Delta T}{T}\,\frac{(\delta t)^2}{\tau_s \tau_R}$$

where $\tau_R = \mu_0 r_1^2/4\eta$. Typical values for JET are $\Delta T/T \sim 10^{-1}$, $\tau_s \sim 10^{-1}$s and $\tau_R \sim 1$s, so the time required for a change $\delta q$ is

$$\delta t \sim \sqrt{\delta q}\ .$$

For the quasi-interchange instability the change in q to achieve a growth rate of $(100\mu s)^{-1}$ is $\delta q \sim 10^{-2}$ [7], giving $\delta t \sim 100$ms. This is very much longer than the sawtooth collapse time and is in fact comparable to the full sawtooth period.

For the resistive kink instability the growth rate depends on dq/dr at the q = 1 surface. The required value of $\delta q^*(=\frac{1}{2} r_1 q_1')$ to obtain the $(100\mu s)^{-1}$ growth rate is at least $10^{-1}$ [7], and this gives an even larger $\delta t$ than that for the quasi-interchange. These estimates substantiate the account given in the introduction and give timescales for the growth of the growth rate which are too long by orders of magnitude to explain the experimental results.

The effect of neoclassical resistivity is to enhance the rate of fall of q on axis, typically by a factor 5, and the rate of increase of $q_1'$ by a smaller factor. This would not significantly affect the large discrepancy between the timescales.

Another equilibrium quantity which appears in the stability analysis of ideal modes is $\beta_p$ defined by

$$\beta_p = \frac{2\mu_0}{r_1{}^2 B_{\theta 1}{}^2} \int_0^{r_1} (-dp/dr) r^2 dr$$

where R is the major radius of the plasma, $B_{\theta 1}$ is the poloidal magnetic field at the q=1 surface and p is the plasma pressure. The change in $\beta_p$ is related to the change in temperature by

$$\delta\beta_p \sim \frac{n\ \delta T}{B_{\theta 1}{}^2/2\mu_0}$$

and so, putting $dT/dt = \Delta T/\tau_s$, the time required for the change $\delta\beta_p$ is

$$\delta t \sim \frac{B_{\theta 1}{}^2/2\mu_0}{nT} \frac{\tau_s}{(\Delta T/T)} \delta\beta_p \ .$$

The change in $\beta_p$ to produce the observed growth rate [7] is $\delta\beta_p \sim 10^{-1}$, and so for typical values, say $nT/(B_{\theta 1}{}^2/2\mu_0) \lesssim 1$ and $\Delta T/T \sim 10^{-1}$, the required time for the change is of the order of the whole sawtooth period, $\tau_s$. Again the slowness of the change precludes it from explaining the observed change in the growth rate.

## Discussion

From the above discussion of the development of the growth rate, it is seen that the sudden appearance of the fast growth rate cannot be understood in terms of simple linear theory. The typical time required to develop a linear growth rate of the required magnitude through equilibrium changes ($\sim$100ms) is a thousand times longer than the experimental time ($\sim$100μs). While more accurate numerical calculations might reduce this discrepancy the basic problem will remain.

An explanation in general terms might be that the rapid growth does correspond to the release of energy arising from an ideal or resistive mhd mode, but that the onset of the rapid growth does not correspond to the equilibrium passing through marginal stability for that mode. Thus it could be that the mhd stability boundary is passed at an earlier time but that a weak effect provides a "fragile" stability. The final rapid growth would then occur when a second stability boundary is crossed, this boundary being determined by the weak effect. The free energy accumulated during the period of fragile stability would then suddenly become available to drive the fast instability. This behaviour is illustrated diagrammatically in Figure 3.

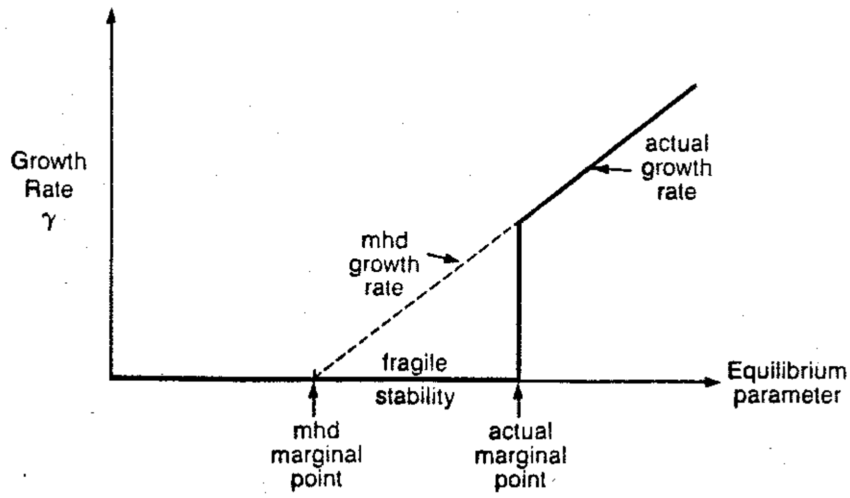


Figure 3. Illustrating a type of behaviour which would be consistent with the experimental observations. A weak stabilising effect provides a fragile stability beyond the mhd stability boundary. This allows the build up of free energy which is then suddenly released when the actual stability boundary is reached.

At first sight an attractive possibility is that a stabilising contribution from the inner layer of the eigenfunction allows stability up to the second marginal point but that, once the displacement amplitude of the instability exceeds the layer width, the full free energy of the instability is released. Unfortunately, this explanation appears to be unacceptable. In many JET sawteeth with the normal fast collapse there are m=1 temperature oscillations which are independent of the sawtooth collapse. These oscillations often last for a substantial part of the sawtooth period and often continue through the collapse [11]. The m=1 structure associated with these fluctuations must involve a magnetic island much larger than the layer width (∿mm), and the linear stabilisation arising from the layer would therefore not occur.

Another possibility is that a small amplitude perturbation produces a geometric change which allows the plasma to escape the toroidal stabilising effect and hence releases the larger free energy of the cyclindrical m=1 ideal mode. However, again there is an objection. Since this effect would be entirely mhd, it should appear in toroidal simulations of the m=1 instability [12], but such behaviour is not seen.

In a report of recent sawtooth simulations by Aydemir et al. [12] it is suggested that our interpretation of the onset of the sawtooth collapse is incorrect. However the evidence offered is that, in the numerical simulations, the fast collapse is the result of many e-foldings taking place over a long period. This of course is the conventional model which it is our purpose to challenge. In the reported simulation the growth-rate is within a factor of two of its maximum value for a third of the sawtooth period. In the JET sawteeth reported here this behaviour would lead to a growth by a factor in excess of exp $(\frac{1}{3} \times 300\text{ms}/2 \times 25\mu\text{s}) = e^{2000}$, or an initial perturbation of less than $10^{-800}$ cm. Scaling to a more realistic value of S only makes matters worse because the time for equilibrium changes scales as S whereas the resistive kink growth rate scales only as $S^{1/3}$.

Summarising, the basic conclusion is that the sawtooth collapse in JET cannot be understood in terms of conventional stability theory. Almost all of the instablility growth occurs at a fast growth rate which arises spontaneously on a timescale comparable to the collapse time, and cannot be explained in terms of the dependence of the growth rate on the slowly changing equilibrium parameters. This result demonstrates that the physics of the sawtooth collapse is quite different from that at present assumed and calls for a re-appraisal of the problem.

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# APPENDIX 1.

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