JOINT EUROPEAN TORUS

JET-P(89)82

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Preprint of a paper to be submitted for publication in the
Proceedings of the Workshop on Electrostatic Turbulence, Cadarache, October 1989

# Correlation Reflectometry: A Possible New Technique For Diagnosing Density Microturbulence.

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## Abstract

We present the principles of a possible new technique for diagnosing density microturbulence - Correlation Reflectometry. A numerical model of a correlation reflectometer is described and the likely limits of the technique identified. Two preliminary correlation reflectometers have been constructed and used on JET and the results obtained are presented.

## Introduction

Evidence of density fluctuations at high frequencies (> several kHz) has been observed on signals obtained with microwave reflectometers for several years [1,2]. The level and frequency spectrum of the fluctuating signals has been observed to change substantially with different plasma conditions (ohmic, L-mode and H-mode)[3]. On the other hand, it has not been possible to obtain quantitative information on the underlying turbulence from the fluctuating signals. Recently we have devised a new technique, Correlation Reflectometry, which may have the potential to provide some information, specifically the correlation length and dispersion curve of the waves characterising the turbulence, and this is described in the present paper.

## Principle

In Correlation Reflectometry two independant reflectometers operating with microwave frequencies $f_1$, $f_2$ probe the plasma along the same line of sight. Either the radiation is in the ordinary mode (**E∥B**) in which case the radiation is reflected at the critical density layer corresponding to the plasma frequency, or in the extraordinary mode (**E⊥B**) and reflected at the upper or lower cutoffs. The reflection layers are at $R_1$ and $R_2$ which are separated by a small distance $\Delta x$, figure 1.

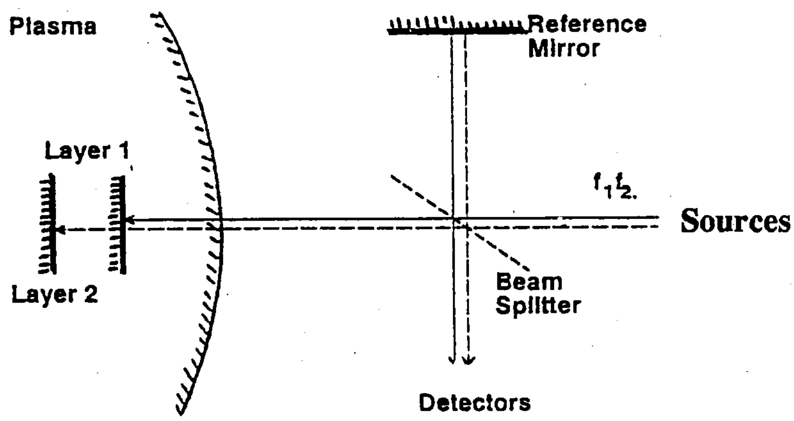


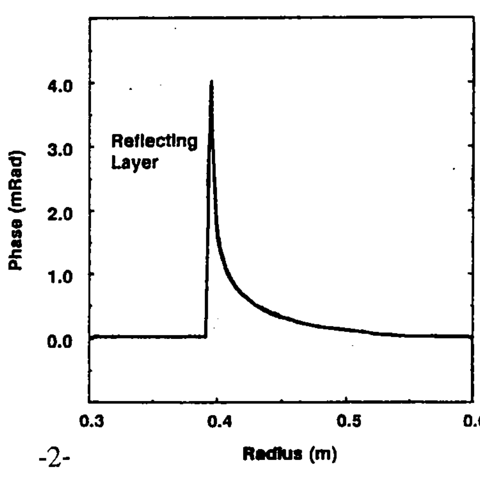

*Figure 1: Schematic of a correlation reflectometer*

For each frequency $f_1$, $f_2$ the measured phase is given by [4]:

$$\phi_i = \frac{4\pi f_i}{c} \int_0^R \mu(R)\, dR + \frac{\pi}{2} + \phi_c \qquad i = 1,2$$

The constant $\phi_c$ depends on the different free space path lengths in the arms of the reflectometer. Fluctuations in the plasma will have two principal effects on the phase: they will cause a variation in the local value of the refractive index $\mu(R)$, and in the position of the reflecting layer $R_i$. Our calculations show that the change of phase is dominated by the fluctuations near the reflecting layer. For example in figure 2 we show the calculated change of phase of a reflectometer signal plotted against the position of an assumed fluctuation ($\Delta n/n = 0.001$). Typical JET conditions are considered and the WKB approximation is used to calculate $\Delta\phi$.

*Figure 2: Change of phase of a reflectometer signal due to a small density perturbation located at different radial positions. The reflecting layer is at 0.39 m.*



We represent the fluctuations as a broadband spectrum of electrostatic waves. Each wave has a frequency $\omega$, wavenumber k and a finite coherence length $l_c$, where $l_c$ is defined as the length of the coherent wavetrain to the $e^{-1}$ amplitude points. The magnitude of the fluctuating phase in the reflectometer caused by such waves is:

$$\Delta\phi_1(t) = \mu^* \frac{4\pi f_1}{c} \sum_m \Delta L_{m1} \sin\left( \omega t + \delta_{m1}\right)$$

where $\Delta L_{m1}$ is the amplitude of the movement of the layer at $R_1$ caused by the wave m and $\delta_{m1}$ is the phase of the wave. A similar expression exists for $\Delta\phi_2$ by replacing the subscript 1 with 2. $\mu^* = (1/2\Delta L).\int_{2\Delta L}\mu(R)\ dR$ is the average refractive index over $\Delta L$ and, for smoothly varying density profiles, is a constant for different values of $\Delta L$. To analyse the data spectral analysis techniques are used. First the autopower spectrum $G_1(\omega)$ and $G_2(\omega)$ of each signal is calculated

$$G_i(\omega) = \lim_{\Delta\omega\to\infty} \frac{1}{\Delta\omega}\left\{ \lim_{T\to\infty} \frac{1}{T}\int \Delta\phi_i(t,\omega,\Delta\omega).\Delta\phi_i(t,\omega,\Delta\omega)\ dt\right\} \qquad i = 1,\ 2$$

This is a measure of the relative power at each frequency in the signal where T is the time slice over which the analysis is performed [5]. The power that is common to both reflectometers is determined by taking the crosspower spectrum $G_{12}(\omega)$. This may be written as a complex number:

$$G_{12}(\omega) = C_{12}(\omega) + i.Q_{12}(\omega)$$

$C_{12}(\omega)$ is the coincident spectral density (the cospectrum) and it is a measure of the power that is common to both signals. It is defined as:

$$C_{12}(\omega) = \lim_{\Delta\omega\to\infty} \frac{1}{\Delta\omega}\left\{ \lim_{T\to\infty} \frac{1}{T}\int \Delta\phi_1(t,\omega,\Delta\omega).\Delta\phi_2(t,\omega,\Delta\omega)\ dt\right\}$$

$Q_{12}(\omega)$ is the quadrature spectral density (the quadspectrum) and it is similar to the cospectrum except that one of the signals is shifted by $\pi/2$ at each frequency. It is a measure of the out of phase power that is common to both signals. The phase difference between signals that are common to both channels is given by the crossphase spectrum:

$$\Theta_{12}(\omega) = \tan^{-1}\left\{ Q_{12}(\omega)/\ C_{12}(\omega) \right\}$$

The phase difference between the two signals is due to a propagation of the wave between $R_1$ and $R_2$ and thus this may be related to the wavenumber as follows:

$$\Theta_{12}(\omega) = k(\omega).\Delta x$$

Thus if $\Theta_{12}(\omega)$ and $\Delta x$ can be calculated then the *dispersion relation* of the waves may be determined. Strictly this technique measures k along the line of sight of the reflectometer. There is in fact an ambiguity of $\pm \pi$ in the determination of $\Theta_{12}(\omega)$ due to the constants in the expression for $\phi$. In practice this is not a serious limitation because physically $k = 0$ when $\omega = 0$ and so $\Theta_{12}(\omega = 0) = 0$.

As $\Delta x$ increases the power common to both signals decreases because $l_c$ is finite. This effect may be quantified in the coherence function $\gamma_{12}(\omega)$ which is a measure of the power common to both signals normalised to the total power in each signal.

$$\gamma^2_{12}(\omega) = \frac{|G_{12}(\omega)|^2}{G_1(\omega).G_2(\omega)}$$

$|G_{12}(\omega)|$ is the magnitude of the crosspower spectrum. The *correlation length* of the fluctuation is found by varying the probing frequencies, and thus $\Delta x$, and by analysing the variation of $\gamma_{12}(\omega)$ as a function of $\Delta x$.

## Numerical Model

In order to explore the possible measurement capability of a correlation reflectometer we have developed a numerical model of such a device. In the model the plasma density and magnetic field are of the form $n_e(\rho) = n_0\left(1 - \rho^\alpha\right)^\beta$ and $B(R) = B_0.R_0/(R_0 + R)$ respectively where $n_0$, $\alpha$, $\beta$, $B_0$ and $R_0$ can be independantly chosen. The density fluctuations are described as

$$\tilde{n}(t,r) = \sum_m A_m.\sin\left\{\omega_m t - k_m r + \delta_m\right\}.\exp\left\{-(r_{0m} - r)^2/l^2_{cm}\right\}$$

where $A_m$, $\omega_m$, $k_m$ and $\delta_m$ are the amplitude, frequency, wavenumber and phase of mode m. $r_{0m}$ is the position of maximum amplitude and $l_{cm}$ is the coherence length. A large number of modes may be assumed to exist simultaneously. Using the WKB approximation the phase change measured by two independant reflectometers is calculated and the time dependences of the two reflectometer signals are constructed. The functions $G_1(\omega)$, $G_2(\omega)$, $\gamma^2_{12}(\omega)$ and $\Theta_{12}(\omega)$ are calculated. The dispersion curve and coherence lengths are derived and compared with the initially assumed values.

A typical example is shown in figure 3. Here $n_0 = 3.10^{19} m^{-3}$, $\alpha = 2.0$, $\beta = 0.5$, $B_0 = 2.6$ T and $R_0 = 3.05$ m. The assumed power spectrum, dispersion relation and coherence length of the waves describing the density fluctuations are shown. We assume the plasma is probed with two reflectometers operating at 76 and 77 GHz in the extraordinary mode with $\Delta x \approx 1.4$ cm. The curves that would be measured are also shown and compare well with the assumed values.

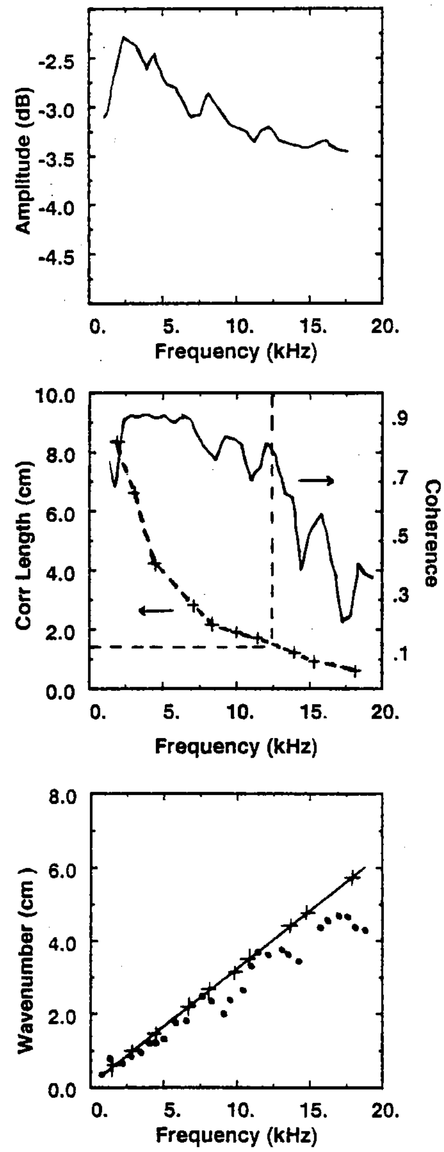



*Figure 3.* *(a) Assumed amplitude spectrum of the density fluctuations.*

*(b) Assumed correlation length and the calculated coherence function that would be measured. Note that when $l_c \approx 1.4$ cm (i.e $\Delta x$) the coherence begins to fall*

*(c) Assumed dispersion relation (−+−) and the curve that would be measured with a correlation reflectometer (• • •).*

## Limitations

There are a number of effects that limit the information that may be obtained using this technique.

i) The finite value of $\Delta x$ means that only waves with $l_c > \Delta x$

can be investigated.

ii) The radiation is not reflected at a single plane but over a finite distance $\Delta R$ and so there is a minimum value for $l_c \geq \Delta R$.

iii) The finite value of $\Delta R$ means that only waves with $k \leq 2\pi.(\Delta R)^{-1}$ can be investigated.

iv) The finite size of the data set means that for coherence to be meaningful $\gamma_{12}$ must be greater than a mimimum value ($\gamma_r$) determined by the number of data points.

Other limits of conventional reflectometry, for example accessability due to electron cyclotron absorbtion, also apply [6].

## Implementation

Thus far two correlation reflectometers have been constructed and used on JET. In the first radiation from a fixed frequency Gunn oscillator at 49 GHz is combined with radiation from a backward wave oscillator operating in the range 51 - 57 GHz, and the plasma is probed along the major radius in the extraordinary mode using an oversized waveguide, constructed and normally used for ECE measurements (figure 4). A specially designed diplexer which has a sharp edge ( > 10 dB/GHz) at 50 GHz is used to separate the two reflected beams and Schottky diodes are used to detect the radiation. In the second experiment radiation from two Gunn oscillators at 76.0 GHz and 77.0 GHz are combined and frequency separation and detection is carried out by heterodyne detection with a broad band mixer and appropriate I.F. filters.

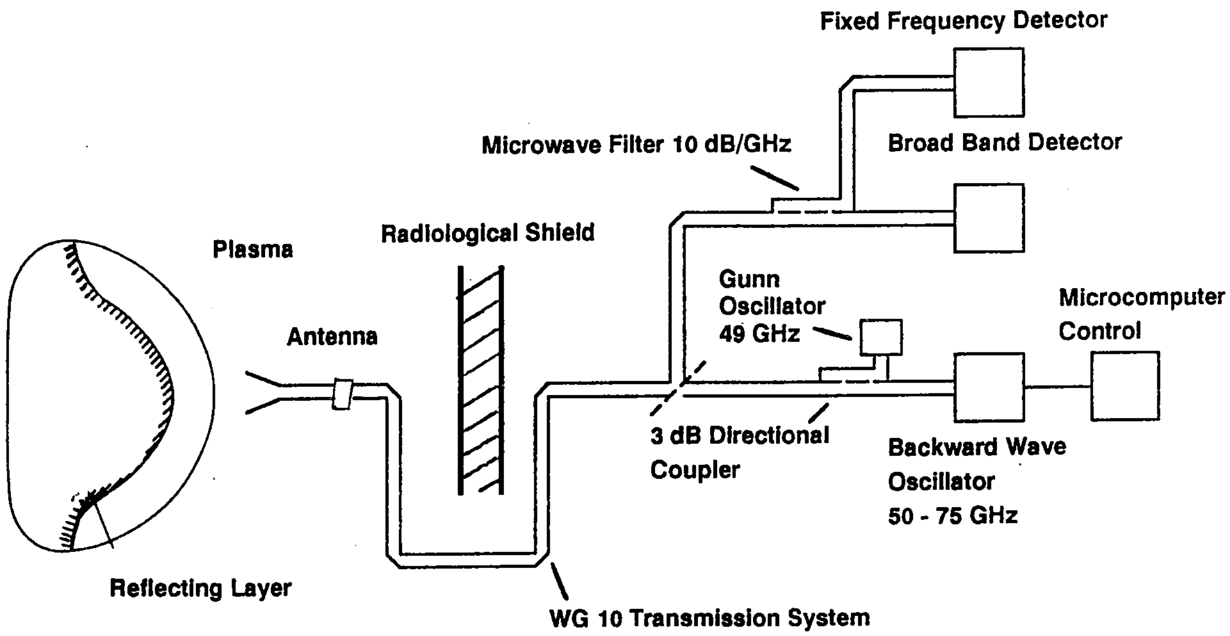



*Figure 4: Schematic of a Correlation Reflectometer on JET.*

## Results

Measurements have been made for a variety of plasma conditions (ohmic, L mode, H mode) with different experimental configurations of the correlation reflectometers. However, because the ECE waveguide channel has not normally been available for these studies and because the reflectometers themselves have been in a state of development, systematic scans have not been possible. Nevertheless some patterns have emerged. We find that usually under ohmic conditions little or no correlation is observed even when the interlayer distance is as small as ≈ 1cm (figure 5a). On the other hand, clear correlation is observed under additional heating conditions but usually there is no frequency dependence on the observed cross-phase (figure 5b). However under some conditions a clear frequency variation of the cross phase is observed (figure 5c).

*Figure 5 (a) Ohmic plasma, $\Delta x \approx 1$ cm, $R = 3.65$ m. The straight line on the coherence plot is the significance level ($\gamma_r$)*

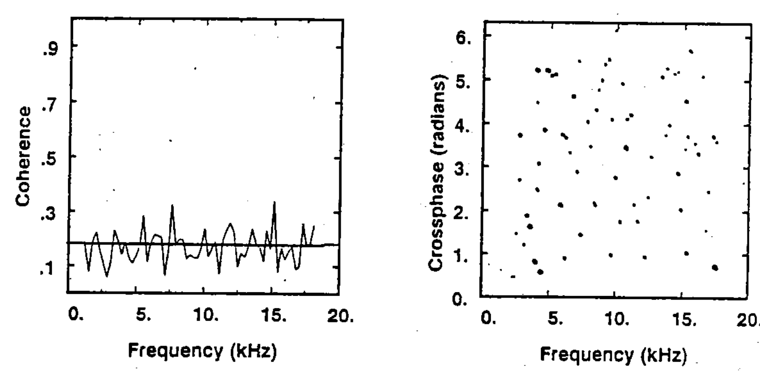



*Figure 5 (b) Beam heated L–mode plasma. $\Delta x \approx 1.2$ cm, $R = 3.8$ m.*

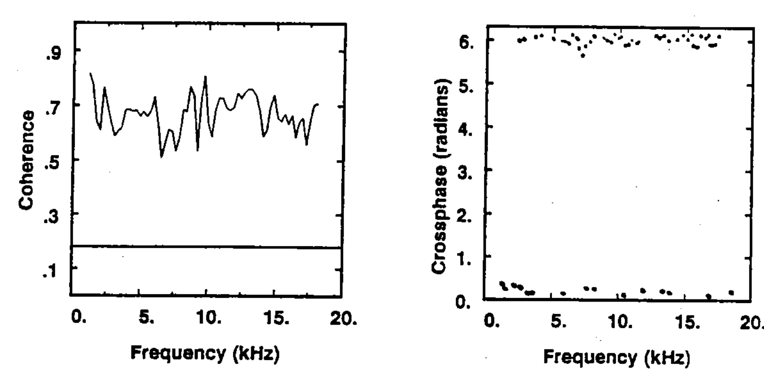
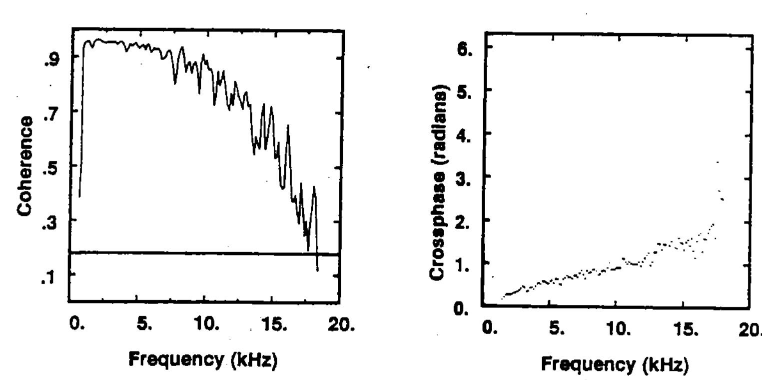


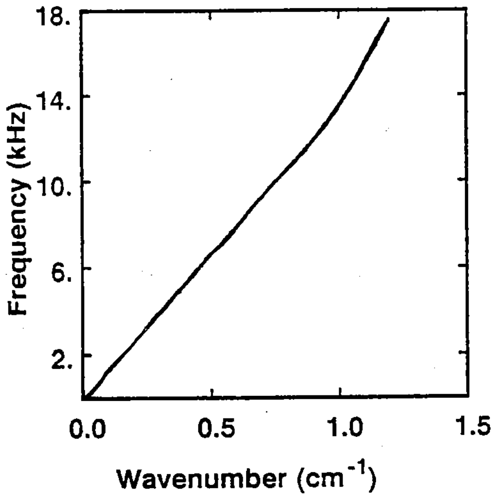

*Figure 5 (c) Beam heated L–mode plasma after beams. $\Delta x \approx 1.4$ cm, $R = 3.85$ m.*



## Discusson

There are several aspects about the results which are difficult to explain. In particular the lack of coherence under ohmic conditions even when $\Delta x \approx \Delta R \approx 1$ cm, and the high correlation under additional heating conditions without a frequency variation on the crossphase. On the other hand, the results that show a frequency variation on the crossphase appear to comply with the expectations of our theory of correlation reflectometry developed above and in these cases it is possible to derive a dispersion curve (eg figure 6). This implys an outward going radial wave with a phase velocity $\approx 9.10^2 m.s^{-1}$, but this velocity is low relative to those normally associated with microturbulence. However, there are potentially important effects which have not yet been considered in detail, for example plasma rotation, and before we can draw conclusions on the character of the turbulence from these measurements these effects must be properly considered. Our current work is aimed in this direction.

*Figure 6: Dispersion relation derived from data in figure 5c.*




## References


1. E. Mazzucato.'Density Fluctuations in the A.T.C'. Matt-1151, Report of The Plasma Physics Laboratory, Princeton University, Sept 1975.
2. F. Simonet.'Étude de la Réflectometrie Hyperfréquence'. Thèse de Doctorate D'État, University de Nancy 1; 1985.
3. Prentice et al.'Results from Jet Reflectometer Systems', 30th Annual Meeting of the Division of Plasma Physics of the American Physical Society, Hollywood, Florida. October 1988, Abstract 6U 1.
4. V.L. Ginzburg.'The Propagation of Electromagnetic Waves in Plasmas' Pergamon Press 1964
5. Bendat and Piersol.' Random Data: Analysis and Measurement Techniques', Wiley, 1971.
6. A.E. Costley. 'Diagnosis of Fusion Plasmas using Reflectometry' Proc Course and Workshop, Basic and Advanced Diagnostic Techniques for Fusion Plasmas, EUR 10797, Vol 2, 379 -396, Varenna, 1986.